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INTRODUCTION :-

- ⇒ Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion. Thus this branch of science deals with the static, kinematics and dynamic aspects of fluids.
- ⇒ The study of fluids at rest is called fluid static.
- ⇒ The study of fluid in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

PROPERTIES OF FLUIDS :-Density or Mass Density :-

- ⇒ Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density.
- ⇒ It is denoted by the symbol ρ (rho).
- ⇒ The unit of mass density in SI unit is $\text{Kg per cubic meter}$ (Kg/m^3).
- ⇒ The density of liquid may be considered as constant while that of gases changes with the variation of pressure and temperature.

mathematically, mass density is written as

$$\rho = \frac{\text{mass of Fluid}}{\text{Volume of Fluid}}$$

The value of density of water is 1 gm/cm^3
or 1000 kg/m^3

Specific weight or weight density:-

Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w .

$$\text{Thus mathematically, } w = \frac{\text{weight of Fluid}}{\text{Volume of fluid}}$$

$$= \frac{(\text{mass of Fluid}) \times (\text{Acceleration due to gravity})}{\text{Volume of Fluid}}$$

$$= \frac{\text{mass of Fluid} \times g}{\text{Volume of Fluid}}$$

$$= \rho \times g$$

$$w = \rho \times g$$

$$\left. \begin{array}{l} \therefore \frac{\text{mass of fluid}}{\text{Volume of Fluid}} \end{array} \right\}$$

The value of specific weight density (w) for water is 9.81×1000 Newton / m^3 in SI unit.

Specific volume:-

Specific volume of a fluid is the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called Specific volume. mathematically it is expressed as.

$$\begin{aligned} \text{Specific Volume} &= \frac{\text{Volume of fluid}}{\text{mass of Fluid}} \\ &= \frac{1}{\frac{\text{mass}}{\text{Volume of Fluid}}} = \frac{1}{\rho} \end{aligned}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg . It is commonly applied to gases.

Specific Gravity:-

Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the ~~weight density~~ standard fluid. For liquid, the standard fluid is taken water and for gas the standard fluid is taken air.

Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S .

Mathematically, S (for liquid) = $\frac{\text{weight density (density) of liquid}}{\text{weight density (density) of water}}$

S (for gases) = $\frac{\text{weight density (density) of gas}}{\text{weight density (density) of air}}$

Thus weight density of a liquid = $S \times \text{weight density of water}$

= $S \times 1000 \times 9.81 \text{ N/m}^3$

The density of liquid = $S \times \text{Density of water}$
 = $S \times 1000 \text{ kg/m}^3$

In the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water. For example, the specific gravity of mercury is 13.6.

hence density of mercury = 13.6×1000
 = 13600 kg/m^3

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calculate the specific weight, density and specific gravity of one litre of a liquid which weight 7 N.

Solution:-

given,

$$\text{Volume} = 1 \text{ lit} = \frac{1}{1000} \text{ m}^3 \quad \left[\begin{array}{l} 1 \text{ lit} = \frac{1}{1000} \text{ m}^3 \\ 1 \text{ lit} = 1000 \text{ cm}^3 \end{array} \right]$$

$$\text{weight} = 7 \text{ N}$$

$$\text{Specific Weight (w)} = \frac{\text{weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left[\frac{1}{1000} \right] \text{ m}^3}$$

$$= \boxed{7000 \text{ N/m}^3}$$

$$\text{Density (P)} = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3$$

$$= \boxed{713.5 \text{ kg/m}^3}$$

$$\text{Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$= \frac{713.5}{1000} \quad \left[\begin{array}{l} \therefore \text{Density of water} \\ = 1000 \text{ kg/m}^3 \end{array} \right]$$

$$= \boxed{0.7135}$$

a) Calculate the density, specific weight and weight of one liter of petrol of specific gravity = 0.7

Solution!

Given,

$$\text{Volume} = 1 \text{ liter} = 1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3$$

$$\text{Sp. gravity } S = 0.7 = 0.001 \text{ m}^3$$

$$\text{density } (\rho) = S \times 1000 \text{ kg/m}^3$$

$$= 0.7 \times 1000 = \boxed{700 \text{ kg/m}^3}$$

$$\text{specific weight } (w) = W = \rho \times g$$

$$= 700 \times 9.81 \text{ N/m}^3$$

$$= \boxed{6867 \text{ N/m}^3}$$

weight (W)

we know that specific weight = $\frac{\text{weight}}{\text{volume}}$

$$= W = \frac{W}{0.001} = 6867$$

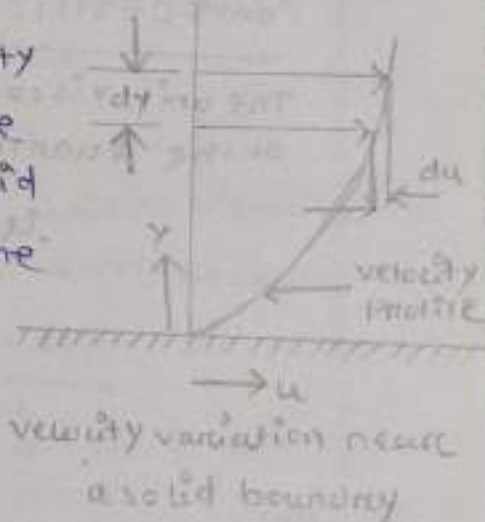
$$= \frac{W}{0.001}$$

$$W = 6867 \times 0.001$$

$$= \boxed{6.867 \text{ N}}$$

viscosity

viscosity is defined as the property of a fluid which offers resistance to a movement of one layer of fluid over another adjacent layers of the fluid. when two layers of a fluid a distance 'dy' apart, move one over the other at different velocity. Say u and u+du as shown in fig. the viscosity with relative velocity causes a shear stress acting between the fluid layers.



The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. it is denoted by symbol τ (Tau).

Mathematically, $\tau \propto \frac{du}{dy}$

$T = \mu \frac{du}{dy}$

or

where μ (called mu) is the constant of proportionality and is known as the coefficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represent the rate of shear strain or rate of shear deformation or velocity gradient.

From equation (1.2) we have

$$\mu = \frac{\tau}{\left[\frac{du}{dy} \right]}$$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

unit of viscosity :-

The unit viscosity is obtained by putting the dimensions of the quantities in equation

$$\mu = \frac{\text{shear stress}}{\frac{\text{change of velocity}}{\text{change of distance}}} = \frac{\text{Force/Area}}{\left(\frac{\text{length}}{\text{Time}}\right) \times \frac{1}{\text{length}}}$$

$$= \frac{\text{Force} (\text{length})^2}{\frac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{length})^2}$$

In MKS system, force is represented by kgf and length by meter (m), in cgs system, force is represented by dyne and length by cm and in SI system force is represented by Newton (N) and length by meter (m)

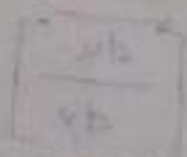
$$\therefore \text{MKS unit of viscosity} = \frac{\text{kgf} \cdot \text{sec}}{\text{m}^2}$$

$$\text{cgs unit of viscosity} = \frac{\text{dyne} \cdot \text{sec}}{\text{cm}^2}$$

In the above express N/m^2 is also known as pascal which is represented by Pa. Hence $\text{N/m}^2 = \text{Pa}$
 $= \text{pascal}$

$$\therefore \text{SI unit of viscosity} = \text{Ns/m}^2 = \text{Pa} \cdot \text{s}$$

$$\text{SI unit viscosity} = \frac{\text{Newton} \cdot \text{sec}}{\text{m}^2} = \frac{\text{Ns}}{\text{m}^2}$$



The unit of viscosity in cgs is also called poise which is equal to $\frac{\text{dyne-sec}}{\text{cm}^2}$

The numerical conversion of the unit of viscosity from MKS unit to cgs unit is given below

$$\frac{1 \text{ kgf-sec}}{\text{m}^2} = \frac{9.81 \text{ N-sec}}{\text{m}^2}$$

$$\begin{aligned} \text{But one Newton} &= 1 \text{ kg (mass)} \times 1 \left(\frac{\text{m}}{\text{sec}^2} \right) (\text{acc}) \\ &= (1000 \text{ gm}) \times (100 \text{ cm}) \\ &\quad \text{sec}^2 \end{aligned}$$

$$= \frac{1000 \times 100 \text{ gm-cm}}{\text{sec}^2}$$

$$= 1000 \times 100 \text{ dyne} \left\{ \begin{aligned} &= \text{dyne} \\ &= \text{gm} \times \frac{\text{cm}}{\text{sec}^2} \end{aligned} \right.$$

$$\therefore \frac{1 \text{ kgf-sec}}{\text{m}^2} = \frac{9.81 \times 100000 \text{ dyne-sec}}{\text{cm}^2}$$

$$= \frac{9.81 \times 100000 \text{ dyne-sec}}{100 \times 100 \times \text{cm}^2}$$

$$= \frac{98.1 \text{ dyne-sec}}{\text{cm}^2} = 98.1 \text{ poise}$$

$$\left\{ \begin{aligned} &= \frac{\text{dyne-sec}}{\text{cm}^2} = \text{poise} \end{aligned} \right.$$

Thus for solving numerical problem, if viscosity is given in poise, it must be divided by 98.1 to get its equivalent numerical value in MKS.

$$\text{But } \frac{\text{one kgf-sec}}{\text{m}^2} = \frac{9.81 \text{ Ns}}{\text{m}^2} = 98.1 \text{ Poise}$$

$$\therefore \frac{\text{one Ns}}{\text{m}^2} = \frac{98.1}{9.81} \text{ poise} = 10 \text{ poise}$$

$$\text{or one poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$$

Alternate method,

$$\text{one poise, } \frac{\text{dyne} \times \text{s}}{\text{cm}^2} = \left(\frac{1 \text{ gm} \times 1 \text{ cm}}{\text{s}^2} \right) \times \frac{\text{s}}{\text{cm}^2}$$

$$\text{But dyne} = \frac{1 \text{ gm} \times 1 \text{ cm}}{\text{s}^2}$$

$$\therefore \text{one poise} = \frac{1 \text{ gm}}{\text{s cm}} = \frac{1}{1000} \frac{\text{kg}}{\text{s} \frac{1}{100} \text{ m}}$$

$$= \frac{1}{100} \times 100 \frac{\text{kg}}{\text{s m}} = \frac{1}{10} \frac{\text{kg}}{\text{s m}} \text{ or,}$$

Note

$$1 \frac{\text{kg}}{\text{s m}} = 10 \text{ poise}$$

- ⇒ In SI unit second is represented by 's' and not by 'sec'
- ⇒ If viscosity is given in poise it must be divided by 10 to get its equivalent numerical value in SI unit. Sometimes a unit of viscosity as centipoise is used where,

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise} \text{ or } 1 \text{ cp} = \frac{1}{100} \text{ p}$$

The viscosity of water at 20°C is 0.01 poise or 1 cp (cp = centipoise, p = poise)

Kinetic viscosity! -

It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol (ν) called 'nu'. Thus mathematically

$$\nu = \frac{\text{viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

The unit of kinematic viscosity is obtained

$$\text{as } \nu = \frac{\text{units of } \mu}{\text{unit of } \rho}$$

$$= \frac{\text{Force} \times \text{Time}}{(\text{length})^2 \times \frac{\text{mass}}{(\text{length})^3}}$$

$$= \frac{\text{Force} \times \text{Time}}{\text{Mass}}$$

$$\text{length}$$

$$= \text{Mass} \times \frac{\text{length}}{(\text{Time})^2} \times \text{Time}$$

$$= \frac{\left(\frac{\text{mass}}{\text{length}} \right)}{\text{Time}^2}$$

$$\left. \begin{aligned} & \therefore \text{Force} \\ & = \text{Mass} \times \text{Acc} \\ & = \text{mass} \times \frac{\text{length}}{\text{time}^2} \end{aligned} \right\}$$

In MKS and SI, the unit of kinematic viscosity is $\text{meter}^2/\text{sec}$ or m^2/s while in cgs unit it is written as cm^2/s . In cgs unit, kinematic viscosity is also known as stoke.

$$\text{Thus, one stoke} = \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4} \text{m}^2/\text{s}$$

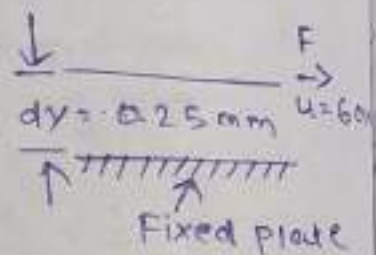
$$\text{Centistoke means} = \frac{1}{100} \text{ stoke}$$

Q1) A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e. 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.

Given,

Distance between plate,

$$dy = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}$$



velocity of upper plate, $u = 60 \text{ cm/s}$

$$= 0.6 \text{ m/s}$$

Force on upper plate,

$$F = 2.0 \frac{\text{N}}{\text{m}^2}$$

This is the value of shear stress, τ

Let the fluid viscosity between the plate is μ

using the equation (1.2), we have $\tau = \mu \frac{du}{dy}$

where, $du = \text{change of velocity} = u - 0 = u = 0.60 \text{ m/s}$

$dy = \text{change of distance} = 0.25 \times 10^{-3} \text{ m}$

$\tau = \text{Force per unit area} = 2.0 \text{ N/m}^2$

$$2.0 = \mu = \frac{0.60}{0.25 \times 10^{-3}} \quad \therefore \mu = \frac{2.0 \times 0.25 \times 10^3}{0.60} = 8.33 \times 10^{-5} \text{ poise}$$

$$= \boxed{8.33 \times 10^{-4} \text{ poise}}$$

2) A Flat plate of area $1.5 \times 10^6 \text{ mm}^2$ is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise .

Given,

Area of the plate, $A = 1.5 \times 10^6 \text{ mm}^2 = 1.5 \text{ m}^2$

Speed of plate relative to another plate,

$$du = 0.4 \text{ m/s}$$

Distance between the plates, $dy = 0.15 \text{ mm}$

$$= 0.15 \times 10^{-3} \text{ m}$$

viscosity $\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$

using eqn (1.2) we have $\tau = \mu \frac{du}{dy} = \frac{1}{10} \times \frac{0.4}{0.15 \times 10^{-3}}$

$$= 266.66 \frac{\text{N}}{\text{m}^2}$$

\therefore shear force, $F = \tau \times \text{area} = 266.66 \times 1.5$

$$= 400 \text{ N}$$

\therefore power required to move the plate at the speed

$$0.4 \text{ m/sec}$$

$$= F \times u = 400 \times 0.4 = 160$$

3) Determine the intensity of shear of an oil having velocity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotated at 150 r.p.m.

Given:

$$\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$$

$$\text{Dia of shaft } D = 10 \text{ cm} = 0.1 \text{ m}$$

Distance between shaft and journal bearing

$$dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$\text{Speed of shaft, } N = 150 \text{ r.p.m}$$

Tangential speed of shaft is given by

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.1 \times 150}{60}$$

$$= 0.785 \text{ m/s}$$

using eqn

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity between shaft

$$\text{and bearing} = u - 0 = u$$

$$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = 52.33 \text{ N/m}^2$$

4) calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size $0.8\text{ m} \times 0.8\text{ m}$ and an inclined plane with angle of inclination 30° as shown in fig. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s . The thickness of oil film is 1.5 mm .

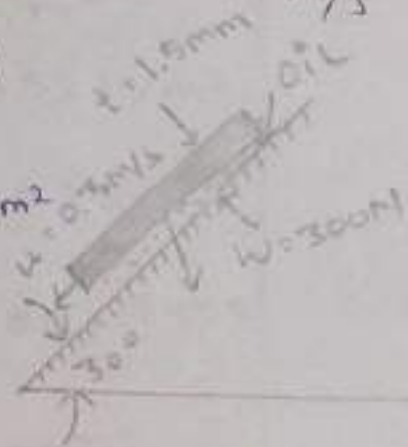
Given,

Area of plate, $A = 0.8 \times 0.8 = 0.64\text{ m}^2$

Angle of plane, $\theta = 30^\circ$

weight of plate, $W = 300\text{ N}$

velocity of plate, $u = 0.3\text{ m/s}$



$$\mu = \frac{W \sin \theta}{A \cdot u} = \frac{300 \times \sin 30^\circ}{0.64 \times 0.3} = 770.83\text{ Pa}\cdot\text{s}$$

5) Determine that two horizontal are placed 1.25 cm apart, the space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.

Given,

$$\text{Distance between plate, } dy = 1.25 \text{ cm} \\ = 0.0125 \text{ m}$$

$$\text{Viscosity, } \mu = 14 \text{ poise} = \frac{14}{10} \text{ N/m}^2$$

$$\text{Velocity of upper plate, } u = 2.5 \text{ m/sec}$$

Shear stress is given by eqn as

$$= \tau = \mu \frac{du}{dy}$$

where du = change of velocity between plate

$$= u - 0 = u = 2.5 \text{ m/sec}$$

$$dy = 0.0125 \text{ m}$$

$$\tau = \frac{14}{10} \times \frac{2.5}{0.0125} = \boxed{280 \text{ N/m}^2}$$

6) Find the kinematic viscosity of an oil having density 981 kg/m^3 . The shear stress at a point in oil is 0.2452 N/m^2 and velocity gradient at that point is 0.2 per second.

Given,

$$\text{Mass density, } \rho = 981 \text{ kg/m}^3$$

$$\text{Shear stress, } \tau = 0.2452 \text{ N/m}^2$$

$$\text{Velocity gradient } \frac{du}{dy} = 0.2 \text{ s}^{-1}$$

$$\text{using the eqn, } \tau = \mu \frac{du}{dy} \text{ or } 0.2452$$

$$= \mu \times 0.2$$

$$\mu = \frac{0.2452}{0.200} = 1.226 \text{ N s/m}^2$$

Kinematic viscosity ν is given by

$$\nu = \frac{\mu}{\rho} = \frac{1.226}{981}$$

$$= 125 \times 10^{-2} \text{ m}^2/\text{sec}$$

$$= 0.125 \times 10^2 \times 10^4 \text{ cm}^2/\text{s}$$

$$= 0.125 \times 10^2 \text{ cm}^2/\text{s}$$

$$= 12.5 \text{ cm}^2/\text{s}$$

$$= \boxed{12.5 \text{ stoke}}$$

⇒ Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes.

Given,

$$\text{viscosity, } \mu = 0.05 \text{ poise} = \frac{0.05}{10} \text{ N/s m}^2$$

$$\begin{aligned} \text{kinematic velocity, } \nu &= 0.035 \text{ Stokes} \\ &= 0.035 \text{ cm}^2/\text{s} \\ &= 0.035 \times 10^{-4} \text{ m}^2/\text{s} \end{aligned}$$

using the relation $\nu = \frac{\mu}{\rho}$,

$$\text{we get } 0.035 \times 10^{-4} = \frac{0.05}{10} \times \frac{1}{\rho}$$

$$\rho = \frac{0.05}{10} \times \frac{1}{0.035 \times 10^{-4}}$$

$$= 1428.5 \text{ kg/m}^3$$

$$\therefore \text{Sp gr on liquid} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$= \frac{1428.5}{1000}$$

$$= 1.4285$$

$$= 1.43$$

$$\boxed{= 1.43}$$

8) Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.

Given,

$$\text{Kinematic viscosity, } \nu = 6 \text{ stokes} = 6 \text{ cm}^2/\text{s} \\ = 6 \times 10^{-4} \text{ m}^2/\text{s}$$

$$\text{Sp. gr. of liquid} = 1.9$$

Let the viscosity of liquid = μ

$$\text{Now sp. gr. of a liquid} = \frac{\text{Density of the liquid}}{\text{Density of water}}$$

$$1.9 = \frac{\text{Density of liquid}}{1000}$$

$$\therefore \text{Density of liquid} = 1000 \times 1.9 = 1900 \frac{\text{kg}}{\text{m}^3}$$

\therefore - using the relation, $\nu = \frac{\mu}{\rho}$, we get

$$6 \times 10^{-4} = \frac{\mu}{1900}$$

$$\mu = 6 \times 10^{-4} \times 1900$$

$$= 1.14 \text{ N s/m}^2$$

$$= 1.14 \times 10 = \boxed{11.40 \text{ poise}}$$

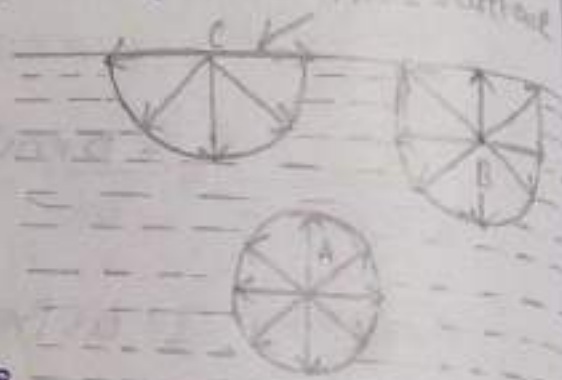
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Surface Tension and Capillarity:-

⇒ Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

⇒ The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kg f/m while in SI unit as N/m .

⇒ The phenomenon of surface tension is explained by Fig. Consider three molecules A, B, C of a liquid in a mass of liquid. The molecule A is attracted in all directions equally by the surrounding molecule of the liquid.



⇒ Thus the resultant force acting on the molecule A is zero. But the molecule B, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus a net resultant force on molecule B is acting in the downward direction.

⇒ The molecule C, situated on the free surface of liquid, does experience a resultant downward force.

experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid act as though it is an electric membrane under tension.

Surface Tension on Liquid Droplet :-

consider a small spherical droplet of a liquid of radius r on the entire surface of the droplet the tensile force due to surface tension will be acting.

Let σ = surface tension of the liquid

P = pressure intensity inside the

droplet (in excess of the outside pressure intensity)

d = Dia. of Droplet

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

→ tensile force due to surface tension acting around the circumference of the cut portion

(b) and this is equal to

$$= \sigma \times \text{circumference}$$

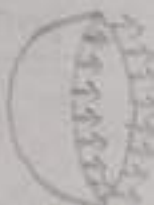
$$= \sigma \times \pi d$$

⇒ pressure force on the area

$$\frac{\pi}{4} d^2 = P \times \frac{\pi}{4} d^2$$



DROPLET



SURFA

These two forces will be equal and opposite under equilibrium conditions

$$P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$P = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d}$$



that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.

Surface Tension On a hollow Bubble:-

A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$P \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

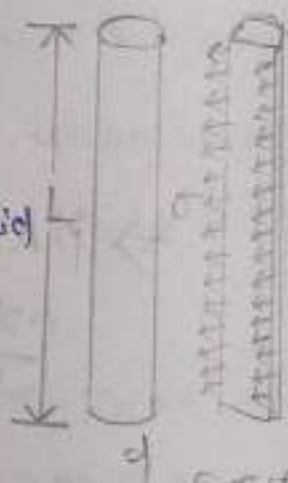
$$P = \frac{2\sigma \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$

Surface Tension on a liquid jet:-

Consider a liquid jet of diameter 'd' and length 'L' as

Let p = pressure intensity inside the liquid jet above the outside pressure

σ = Surface tension of the liquid



Force on liquid jet

Consider the equilibrium of the semi jet, we have

$$\begin{aligned}\text{Force due to pressure} &= p \times \text{area of semi jet} \\ &= p \times L \times d\end{aligned}$$

$$\text{Force due to surface tension} = \sigma \times 2L$$

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$p = \frac{\sigma \times 2L}{L \times d}$$

9) The surface tension of water in contact with air at 20°C is 0.0725 N/m . The pressure inside a droplet of water is to be 0.02 N/cm^2 greater than the outside pressure. Calculate the diameter of the droplet of water.

Given,

$$\text{Surface tension, } \sigma = 0.0725 \text{ N/m}$$

Pressure intensity, p in excess of outside pressure is

$$p = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \text{ N/m}^2$$

d = dia of the droplet

~~Using~~ we get $p = \frac{4\sigma}{d}$ or $0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$

$$\begin{aligned}d &= \frac{4 \times 0.0725}{0.02 \times (10)^4} = 0.00145 \text{ m} \\ &= 0.00145 \times 1000 \\ &= \boxed{1.45 \text{ mm}}\end{aligned}$$

10) Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Given,

Dia of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$

pressure in excess of outside, $p = 2.5 \text{ N/m}^2$

for a soap bubble,

$$\text{we get } p = \frac{8\sigma}{d} \quad \text{or } 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m}$$

$$= \boxed{0.0125 \text{ N/m}}$$

11) The pressure outside the droplet of water diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure) calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Given,

Dia of droplet, $d = 0.04 \text{ mm} = 0.04 \times 10^{-3} \text{ m}$

pressure outside the droplet = $10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside

$$\begin{aligned} \text{pressure} &= p = \frac{4\sigma}{d} \\ &= \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250 \text{ N/m}^2 \end{aligned}$$

$$= \frac{7250 \text{ N}}{104 \text{ cm}^2}$$

$$= \boxed{0.725 \text{ N/cm}^2}$$

Pressure inside the droplet = p + pressure outside the droplet

$$= 0.725 + 10.32$$

$$= \boxed{11.045 \text{ N/cm}^2}$$

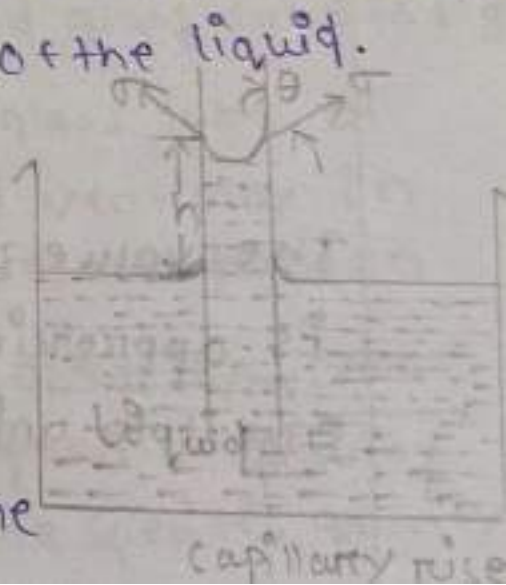
Capillarity:-

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as ~~capillarity~~ capillary rise while the fall of the liquid surface is known as capillary depression.

It is expressed in terms of cm or mm of liquid its value depends upon the specific weight of the liquid diameter of the tube and surface tension of the liquid.

Expression for capillary Rise:-

Consider a glass tube of small diameter 'd' opened at both ends and it is inserted in a liquid. Say water the liquid will rise in the tube above the level of the liquid.



Let h = height of the liquid in the tube. Under a state of equilibrium the weight of liquid ~~at rise~~ ^{in the} of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Let σ = surface tension of liquid

θ = Angle of contact between liquid and glass tube

The weight of liquid of height h in the tube

$$= (\text{Area of tube} \times h) \times \rho \times g$$

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{circumference}) \times \cos \theta$$

$$= \sigma \times \pi d \times \cos \theta$$

We get,

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos \theta$$

$$h = \frac{\sigma \times \pi d \times \cos \theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma \cos \theta}{\rho \times g \times d}$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos \theta$ is equal to unity. Then rise of water is given by

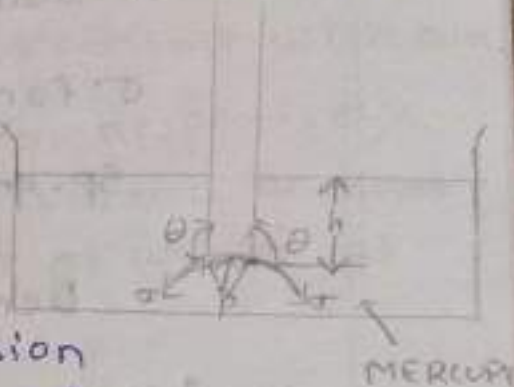
$$h = \frac{4\sigma}{\rho \times g \times d}$$

Expression For capillary Fall :-

If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown as fig.

Let h = Height of depression in tube

Then in equilibrium, two forces are acting on the mercury inside the tube. first one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos \theta$.



Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth ' h ' \times Area

$$= P \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \quad (\because P = \rho g h)$$

Equating the two, we get

$$\sigma \times \pi d \times \cos \theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore h = \frac{4 \sigma \cos \theta}{\rho g d}$$

value of θ for mercury and glass tube is 128°

10 Calculate the capillary rise h in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. take surface tension $\sigma = 0.0725 \text{ N/m}$ for water and $\sigma = 0.52 \text{ N/m}$ for mercury in contact with air. The specific gravity for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact =

Given:

$$\text{Dia. of tube, } d = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$$

$$\text{Surface tension, } \sigma_{\text{For water}} = 0.0725 \text{ N/m}$$

$$\sigma_{\text{For mercury}} = 0.52 \text{ N/m}$$

$$\text{Sp. gr. of mercury} = 13.6$$

$$\text{Density} = 13.6 \times 1000 \text{ Kg/m}^3$$

Capillary rise for water ($\theta = 0^\circ$)

using

$$h = \frac{4\sigma}{\rho \times g \times d}$$

$$= \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$$= 0.0118 \text{ m}$$

$$= 1.18 \text{ cm}$$

For mercury :-

Angle of contact between mercury and glass tube, $\theta = 130^\circ$

$$h = \frac{4\sigma \cos \theta}{\rho \times g \times d}$$

$$= \frac{4 \times 0.52 \times \cos 130^\circ}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$$= -0.004 \text{ m} = \boxed{-0.4 \text{ cm}}$$

The negative sign indicates the capillary depression.

- (11) Calculate the capillary effect in millimeters in a glass tube of 4mm diameter, when immersed in (i) water and (ii) mercury. The temperature of the liquid is 20°C and the values of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury is 130° . Take a density of water at 20°C as equal to 998 Kg/m^3 .

Given:

Dia of tube, $d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

The capillary effect (i.e. capillary rise or depression) is given by

$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

where σ = surface tension in N/m

θ = angle of contact and

ρ = density

Capillary effect for water

$$\sigma = 0.073575 \text{ N/m}, \theta = 0^{\circ}$$

$$\rho = 998 \text{ kg/m}^3 \text{ at } 20^{\circ}\text{C}$$

$$h = \frac{4 \times 0.073575 \times \cos 130^{\circ}}{13600 \times 9.81 \times 4 \times 10^{-3}}$$

$$= -246 \times 10^{-3} \text{ m}$$

$$= \boxed{-2.46 \text{ mm}}$$

The negative sign indicates the capillary depression.

(12) The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size given that surface tension for water in contact with air = 0.0725 N/m.

Given,
capillary rise, $h = 0.2 \text{ mm} = 0.2 \times 10^{-3} \text{ m}$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

Let dia. of tube = d

The angle θ for water = 0°

Density (ρ) for water = 1000 kg/m^3

Using equation $h = \frac{4\sigma}{\rho \times g \times d}$ or 0.2×10^{-3}

$$= \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$d = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2 \times 10^{-3}}$$

$$= 0.148 \text{ m} = \boxed{14.8 \text{ cm}}$$

\therefore Thus minimum diameter of the tube should be 14.8 cm.

(13) Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Given,
capillary rise $h = 2.0 \text{ mm} = 2.0 \times 10^{-3} \text{ m}$

Surface tension, $\sigma = 0.073575 \text{ N/m}$

Let dia of tube = d

The angle θ for water = 0°

using eqn $h = \frac{4\sigma}{\rho \times g \times d}$ or 2.0×10^{-3}

$$= \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

$$d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}}$$

$$= 0.015 \text{ m}$$

$$= \boxed{1.5 \text{ cm}}$$

\therefore Thus minimum diameter of the tube should be 1.5 cm.

12/7/21

- (13) The pressure intensity at a point in a fluid is given 3.924 N/cm^2 . find the corresponding height of fluid when the fluid is (a) water and (b) oil of sp. gr. 0.9

Given,

Pressure intensity, $P = 3.924 \frac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \frac{\text{N}}{\text{m}^2}$

The corresponding height, Z of the fluid is given by

$$Z = \frac{P}{\rho \times g}$$

(a) For water, $\rho = 1000 \text{ kg/m}^3$

$$\therefore Z = \frac{P}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = 4 \text{ m of water}$$

(b) For oil, sp. gr. = 0.9

\therefore Density of oil $\rho_o = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$$\therefore Z = \frac{P}{\rho_o \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = 4.44 \text{ m of oil}$$

(14)

An open tank contains water upto a depth of 2m and above it an oil of sp. grc 0.9 for a depth of 1m. find the pressure intensity (i) at the interface of the two liquids and (ii) at the bottom of the tank

Given,

Height of water, $z_1 = 2\text{m}$

Height of oil, $z_2 = 1\text{m}$

Sp. grc of oil, $S_o = 0.9$

Density of water, $P_1 = 1000\text{Kg/m}^3$

Density of oil, $P_2 = \text{sp. grc of oil} \times$
Density of water

$$= 0.9 \times 1000 = 900\text{Kg/m}^3$$

Pressure intensity at any point is given by

$$P = P \times g \times z$$

(i) At interface, i.e. at A

$$P = P_2 \times g \times 1.0$$

$$= 900 \times 9.81 \times 1.0$$

$$= 8829 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{8829}{10^4} = 0.8829 \text{ N/cm}^2$$

(ii) At the bottom, i.e. at B

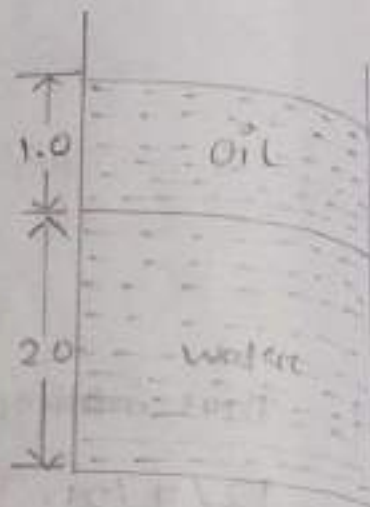
$$P = P_2 \times g \times z_2 + P_1 \times g \times z_1$$

$$= 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620$$

$$= 28449 \text{ N/m}^2$$

$$= \frac{28449}{10^4} \text{ N/cm}^2 = 2.8449 \text{ N/cm}^2$$



ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus:-

Absolute pressure:-

is defined as the pressure which is measured with reference to absolute vacuum pressure.

Gauge pressure:-

is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

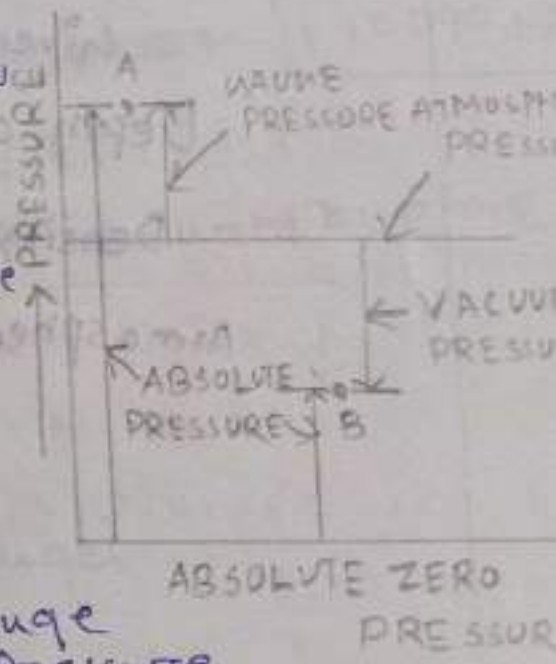
Vacuum pressure:-

is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure as shown as fig.

mathematically!

(i) Absolute pressure
= Atmospheric pressure + Gauge Pressure



$$\text{Or } P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

(ii) Vacuum pressure

= Atmospheric pressure - Absolute pressure

Note!—

⇒ The atmospheric pressure at sea level at 15°C or 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case of MKS unit, it is equal to 1.033 kgf/cm^2

⇒ The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

(15) What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of $1.53 \times 10^3 \text{ kg/m}^3$ if the atmospheric pressure is equivalent to 750 mm of mercury? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3 .

Given,

Depth of liquid, $z_1 = 3 \text{ m}$

Density of liquid, $\rho_1 = 1.53 \times 10^3 \text{ kg/m}^3$

Atmospheric pressure head, $z_0 = 750 \text{ mm of Hg}$
 $= \frac{750}{1000} = 0.75 \text{ m Hg}$

∴ Atmospheric pressure, $P_{atm} = \rho_0 \times g \times z_0$

where $\rho_0 =$ Density of Hg
 $=$ sp. gr. of mercury \times Density of water
 $= 13.6 \times 1000 \text{ Kg/m}^3$

and $z_0 =$ pressure head in term of mercury

$$P_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2$$
$$= 100062 \text{ N/m}^2 \quad (\because z_0 = 0.75)$$

Pressure at a point, which is at a depth of 3m from the free surface of the liquid is given by,

$$P = \rho \times g \times z_1$$
$$= (1.53 \times 1000) \times 9.81 \times 3$$
$$= 45028 \text{ N/m}^2$$

∴ Gauge pressure, $p = 45028 \text{ N/m}^2$

Now absolute pressure = Gauge pressure + Atmospheric pressure

$$= 45028 + 100062$$

$$= ~~1409~~ \boxed{145090 \text{ N/m}^2}$$

MEASUREMENT OF PRESSURE:-

The pressure of a fluid is measured by the following devices:-

→ Manometers

→ Mechanical Gauges

Manometers:-

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as follows:-

- Simple Manometers
- Differential manometers

Mechanical Gauges:-

Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The common types of mechanical pressure gauges are:-

- Diaphragm pressure gauge
- Bourdon tube pressure gauge
- Dead-weight pressure gauge and
- Bellows pressure gauge

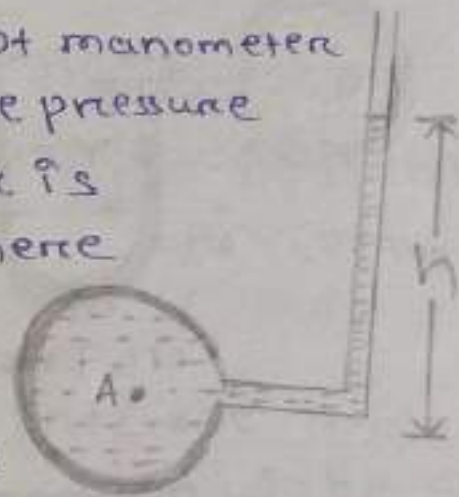
Simple Manometers:-

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmospheric pressure. Common type of simple manometers are:-

- ⇒ piezometer
- ⇒ U-tube Manometer and
- ⇒ single column Manometer

Piezometer :-

It is the simplest form of manometer used for measuring gauge pressure. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown as fig. The rise of liquid gives the pressure head at that point.



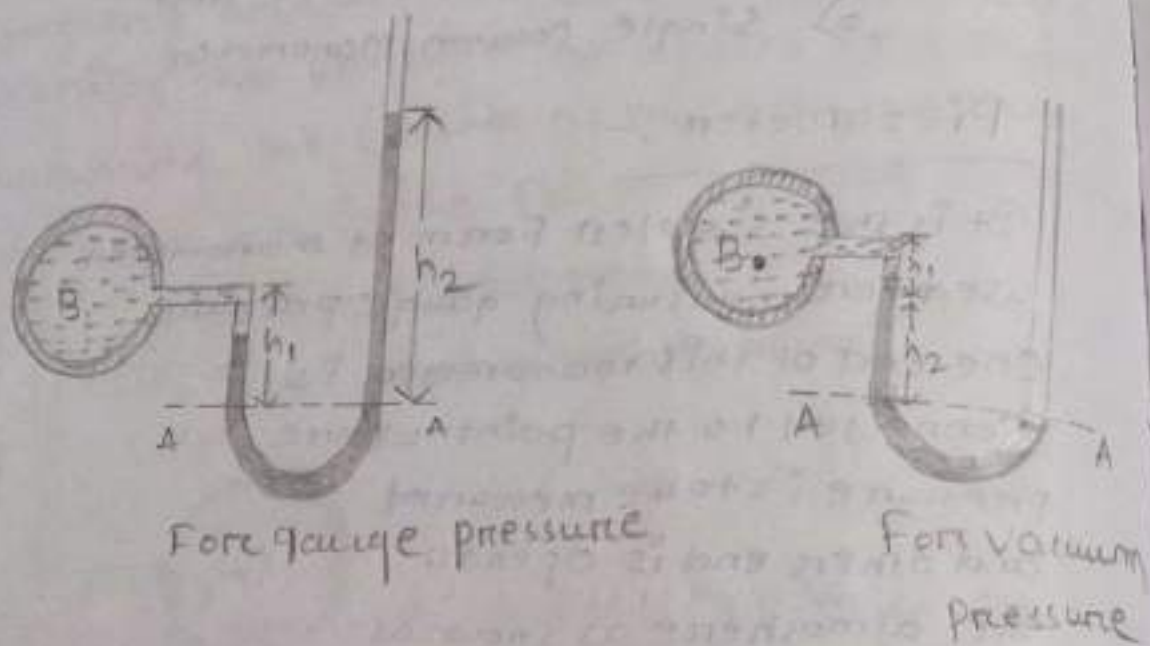
PIEZOMETER

It at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$P = \rho \times g \times h \quad \frac{N}{m^2}$$

U-tube Manometer :-

It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown as fig. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



U-TUBE MANOMETER

FOR GAUGE PRESSURE:-

Let B is the point at which pressure is to be measured, whose value is p the datum line is A-A

Let, h_1 = Height of light liquid above the datum line

h_2 = Height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

P_1 = Density of light liquid = $1000 \times S_1$

S_2 = Sp. gr. of heavy liquid

P_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface
Hence pressure above the horizontal datum line A-A
in the left column and in the right column at

U-tube manometer should be same.

Pressure above A-A in the left column

$$= P + \rho_1 \times g \times h_1$$

Pressure above A-A in the right column

$$= P_2 \times g \times h_2$$

Hence equating the two pressures $P_1 + \rho_1 g h_1 = P_2 g h_2$

$$P = (P_2 g h_2 - \rho_1 \times g \times h_1)$$

For vacuum pressure:-

For measuring vacuum pressure, the level of the heavy liquid in the manometer will be shown as fig.

Then, pressure above A-A in the left column

$$= P_2 g h_2 + \rho_1 g h_1 + P$$

Pressure head in the right column above A-A = 0

$$= P_2 g h_2 + \rho_1 g h_1 + P = 0$$

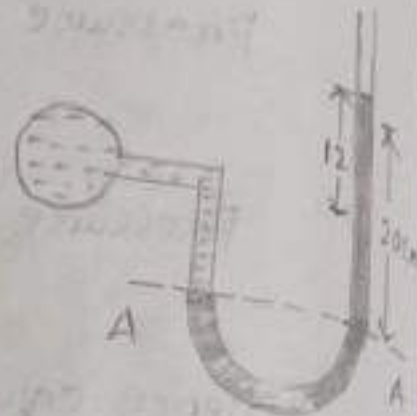
$$P = -(P_2 g h_2 + \rho_1 g h_1)$$

16) The right limb of simple U-tube manometer contains mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Given:

Sp. gr of fluid, $S_1 = 0.9$

$$\begin{aligned} \therefore \text{Density of fluid } \rho_1 &= S_1 \times 1000 \\ &= 0.9 \times 1000 \\ &= 900 \text{ kg/m}^3 \end{aligned}$$



Sp. gr of mercury, $S_2 = 13.6$

\therefore Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from A-A $h_1 = 20 - 12 = 8 \text{ cm}$

$= 0.08 \text{ m}$

$P + \rho$ = pressure of fluid in pipe

Equating the pressure above A-A, we get

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$P + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times 0.2$$

$$P = 13.6 \times 1000 \times 9.81 \times 0.2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706$$

$$= 25977 \text{ N/m}^2$$

$$= \boxed{2.5977 \text{ N/cm}^2}$$

17) A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe. If the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the center of pipe is 15 cm below.

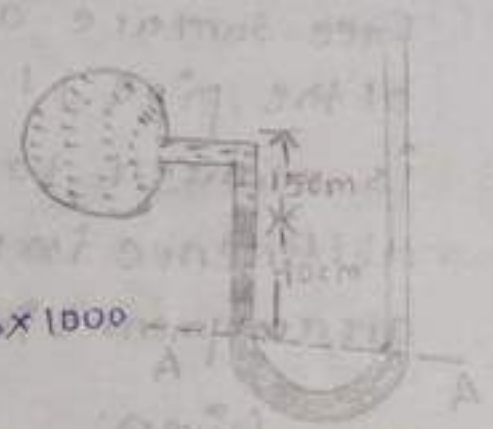
Given;

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $P_1 = 800$

Density of mercury, $P_2 = 13.6 \times 1000$



Difference of mercury level $h_2 = 40 \text{ cm} = 0.4 \text{ m}$

height of liquid in left limb $h_1 = 15 \text{ cm} = 0.15 \text{ m}$

Let the pressure in pipe = P ,

Equating pressure above datum line A-A

we get, $P_2 \rho h_2 + P_1 \rho h_1 + P = 0$

$$P = [P_2 \rho h_2 + P_1 \rho h_1]$$

$$= [13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= - [53366.4 + 1177.2]$$

$$= - 54543.6 \text{ N/m}^2$$

$$= \boxed{-5.454 \text{ N/cm}^2}$$

18) A U-Tube manometer is used to measure the pressure of water in pipe line which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe is reduced to 9810 N/m^2 , calculate the new difference in the level of mercury. Sketch the arrangements in both cases.

Given:

Difference of mercury = $10 \text{ cm} = 0.1 \text{ m}$

The arrangement is shown as fig

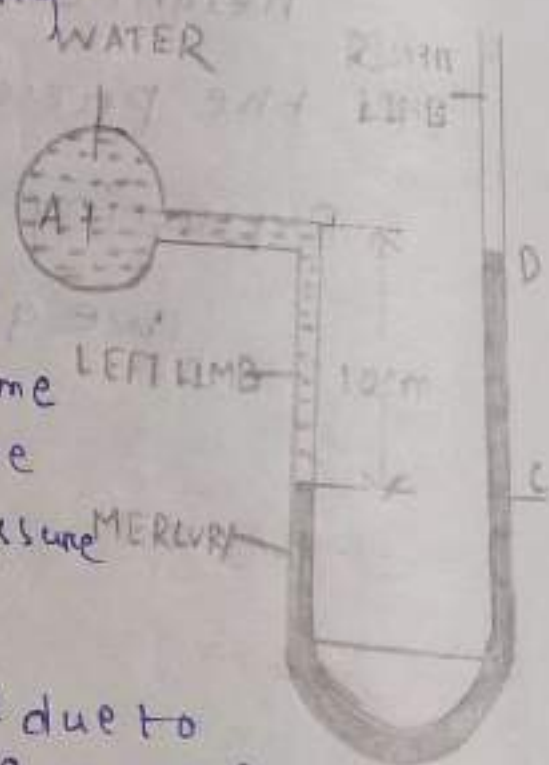
1st part

Let P_A = (pressure of water in pipe line at point A)

The point B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C. But pressure at B

$$= \text{pressure at A} + \text{pressure due to water } 10 \text{ cm (or } 0.1 \text{ m)}$$

$$= P_A + \rho \times g \times h$$



where $\rho = 1000 \text{ kg/m}^3$ and $h = 0.1 \text{ m}$

$$P_C = P_A + 1000 \times 9.81 \times 0.1$$

$$P_C = P_A + 981 \text{ N/m}^2 \quad \text{--- (1)}$$

Pressure at C = pressure at D + pressure due to 10 cm of mercury

$$P_C = 0 + \rho_0 \times g \times h_0$$

where ρ_0 for mercury = $13.6 \times 1000 \text{ kg/m}^3$

and $h_0 = 10 \text{ cm} = 0.1 \text{ m}$

$$\therefore \text{pressure at C} = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$$

$$= 13341.6 \text{ N} \quad \text{--- (2)}$$

But pressure at B is equal to pressure at C. Hence equating the equation (1) and (2) we get

$$P_A + 981 = 13341.6$$

$$P_A = 13341.6 - 981$$

$$= \boxed{12360.6 \text{ N/m}^2}$$

2nd part

Given, $P_A = 9810 \text{ N/m}^2$

find new difference of mercury level. The arrangement is shown as fig. In this case the pressure at A is 9810 N/m^2 which is less than the 12360.6 N/m^2 . Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x = Rise of mercury in left limb in cm

Then fall of mercury in right limb = x cm

The points B^* , C^* and D^* show the initial condition whereas point B , C and S show the final condition.

The pressure at B^* = pressure at C^*

Pressure at A + pressure due to $(10-x)$ cm of water = pressure at D^* +

pressure due to $(10-2x)$ cm of mercury

$$P_A + P_1 \times g \times h_1 = P_{D^*} + P_2 \times g \times h_2$$

$$1010 + 1000 \times 9.81 \times \left(\frac{10-x}{100}\right)$$

$$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10-2x}{100}\right)$$

Dividing by 9.81, we get

$$1010 + 1000 - 10x = 1360 - 272x$$

$$272x - 10x = 1360 - 1010$$

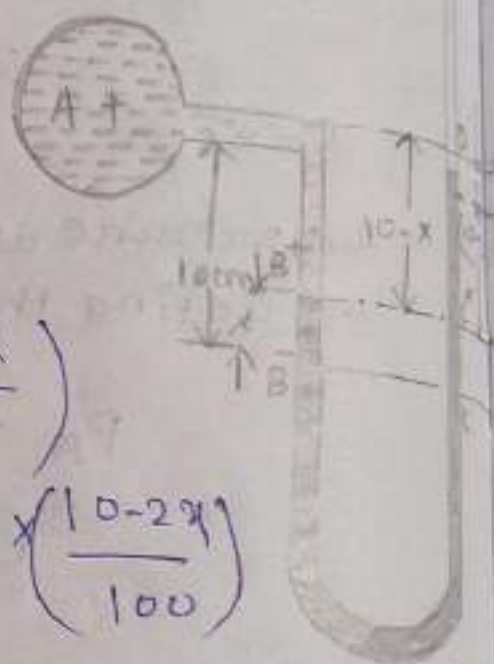
$$262x = 260$$

$$x = \frac{260}{262} = 0.992 \text{ cm}$$

New difference of mercury = $10 - 2x$ cm

$$= 10 - 2 \times 0.992$$

$$= \boxed{8.016 \text{ cm}}$$



Single Column Manometer:-

Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large ~~area~~ cross-sectional area (about 100 times) as compared to due to cross-sectional area of the reservoir, for any variation in pressure the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

→ Vertical single column Manometer

→ Inclined single column Manometer.

Vertical single column Manometer:-

The vertical single column manometer, let $x-x$ be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid right limb

h_1 = Height of center of pipe above $x-x$

P_A = pressure at A, which is to be measured

A = cross-sectional area of the reservoir

a = cross-sectional area of the right limb Vertical single column manometer

S_1 = sp. gr of liquid in pipe

S_2 = sp. gr of heavy liquid in reservoir and right limb

P_1 = Density of liquid in pipe

P_2 = Density of liquid reservoir

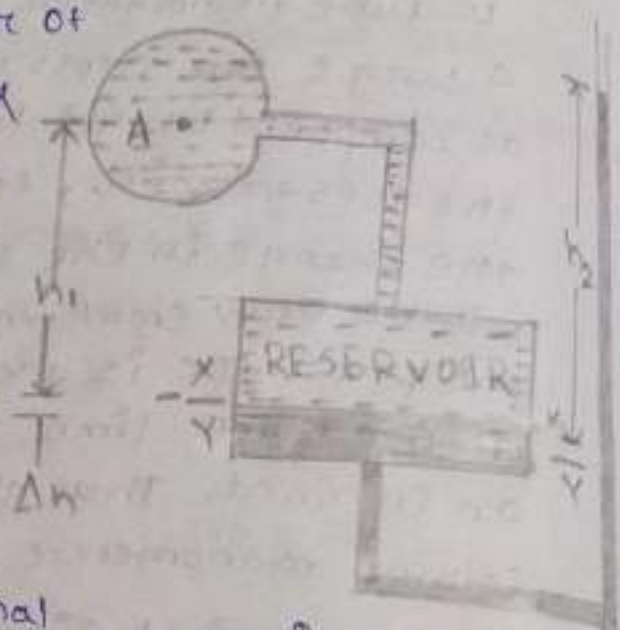
Fall of heavy liquid reservoir will cause a rise of heavy liquid level in the right limb.

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

Now consider the datum line $Y-Y$ as shown in fig. Then pressure in the right limb above $Y-Y$.

$$= P_2 \times g \times (\Delta h + h_2)$$



Pressure in the left limb above Y-Y
 $= P_1 \times g \times (\Delta h + h_2) + P_A$

Equating these pressure, we have

$$P_2 \times g \times (\Delta h + h_2) = P_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = P_2 g (\Delta h + h_2) - P_1 g (\Delta h + h_1)$$

$$= \Delta h [P_2 g - P_1 g] + h_2 P_2 g - h_1 P_1 g$$

$$\Delta h = \frac{a \times h_2}{A}$$

$$P_A = \frac{a \times h_2}{A} [P_2 g - P_1 g] + h_2 P_2 g - h_1 P_1 g$$

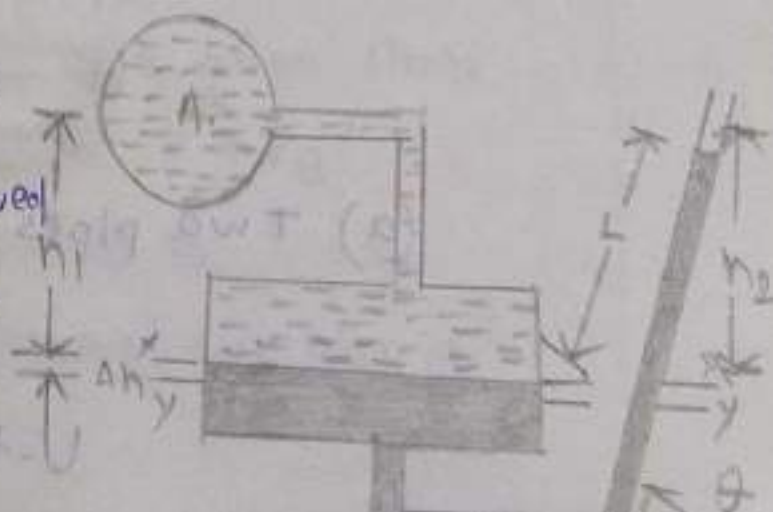
As the area A is very large as compared to a hence ratio $\frac{a}{A}$ becomes very small and can be neglected

$$\text{then } P_A = h_2 P_2 g - h_1 P_1 g$$

h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

Inclined Single column Manometer :-

The inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.



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Differential Manometers:-

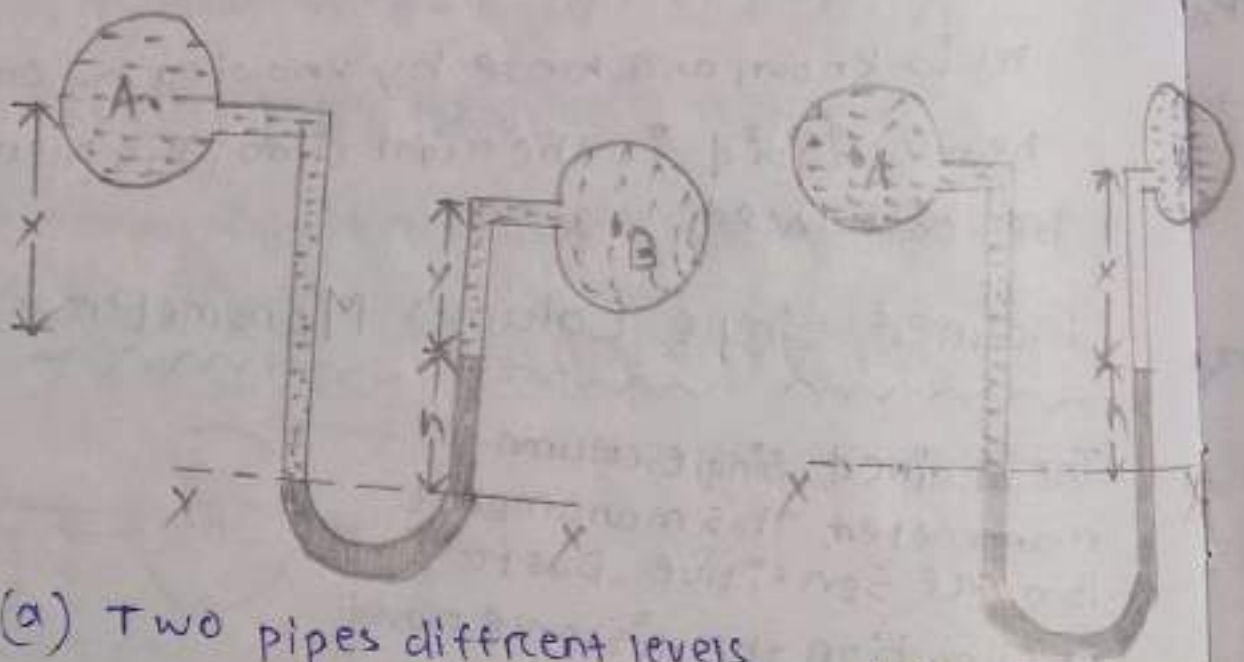
Differential manometers are the device used for measuring the difference of pressure between two points in a pipe or in two different pipes.

A differential manometer consists of U-tubes containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are:-

→ U-tube differential manometer

→ Inverted U-tube differential manometer

U-tube differential manometer:-



(a) Two pipes different levels

(b) A and B are at the same level

U-tube differential manometers

Fig (a), the two points A and B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B were P_A and P_B

Let, h = Difference of mercury level in the U-tube

y = Distance of the center of B, From the mercury level in the right limb

x = Distance of the center of A From the mercury level in the right limb

P_1 = ~~Dens~~ Density of liquid at A

P_2 = Density of liquid at B

P_g = Density of heavy liquid mercury

Taking datum line at $x-x$

Pressure above $x-x$ in the left limb = $P_1 g (h+x) + P_A$

where P_A = pressure at A

Pressure above $x-x$ in the right limb

$$= P_g \times g \times h + P_2 \times g \times y + P_B$$

where P_B = pressure at B.

Equating the two pressure, we have

$$P_1 \rho (h+x) + P_A = \rho g \times h + P_2 \rho y + P_B$$

$$P_A - P_B = \rho g \times h + P_2 \rho y - P_1 \rho (h+x)$$

$$= h \times \rho (P_2 - P_1) + P_2 \rho y - P_1 \rho x$$

∴ Difference of pressure at A and B

$$= h \times \rho (P_2 - P_1) + P_2 \rho y - P_1 \rho x$$

Fig (b), the two points A and B are at the same level and contains the same liquid of density P_1 Then

Pressure above $x-x$ in right limb

$$= \rho g \times h + P_1 \times \rho \times x + P_B$$

Pressure above $x-x$ in left limb = $P_1 \times \rho \times (h+x) + P_A$

Equating the two pressure,

$$\rho g \times h + P_1 \rho x + P_B = P_1 \rho (h+x) + P_A$$

$$P_A - P_B = \rho g \times h + P_1 \rho x - P_1 \rho (h+x)$$

$$= -\rho g \times h (P_2 - P_1)$$

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A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15cm. find the difference of pressure at the two points.

Given:

Sp. gr. of oil $S_1 = 0.9$ \therefore Density P_1
 $= 0.9 \times 1000$
 $= 900 \text{ kg/m}^2$

Difference in mercury level, $h = 15 \text{ cm} = 0.15 \text{ m}$

Sp. gr. of mercury, $S_2 = 13.6$

\therefore Density P_2
 $= 13.6 \times 1000$

The difference of pressure is given by

$$P_A - P_B = \rho \times h (P_2 - P_1)$$

$$= 9.81 \times 0.15 (13600 - 900)$$

$$= \boxed{18688 \text{ N/m}^2}$$

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A differential manometer is connected at the two points A and B of two pipes as shown in fig. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressure at A and B are 1 kgf/cm^2 and 1.80 kgf/cm^2 respectively. find the difference in mercury level in the differential manometer.

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Given:

Sp. gr of liquid at A, $S_1 = 1.5 \therefore P_1 = 1500$

Sp. gr of liquid at B, $S_2 = 0.9$

$$\therefore P_2 = 900$$

Pressure at A, $P_A = 1 \text{ kgf/cm}^2$

$$= 1 \times 10^4 \text{ kgf/cm}^2$$

$$= 10^4 \times 9.81 \text{ N/m}^2$$

$$(\because 1 \text{ kgf} = 9.81 \text{ N})$$

Pressure at B, $P_B = 1.8 \text{ kgf/cm}^2$

$$= 1.8 \times 10^4 \text{ kgf/cm}^2$$

$$= 1.8 \times 10^4 \times 9.81 \text{ N/m}^2 (\because 1 \text{ kgf} = 9.81 \text{ N})$$

Density of mercury = $13.6 \times 1000 \text{ kg/m}^3$

Taking $x-x$ as datum line. left

Pressure above $x-x$ in the ~~right~~ limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + P_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above $x-x$ in the right limb

$$= 900 \times 9.81 \times (h+2) + P_B$$

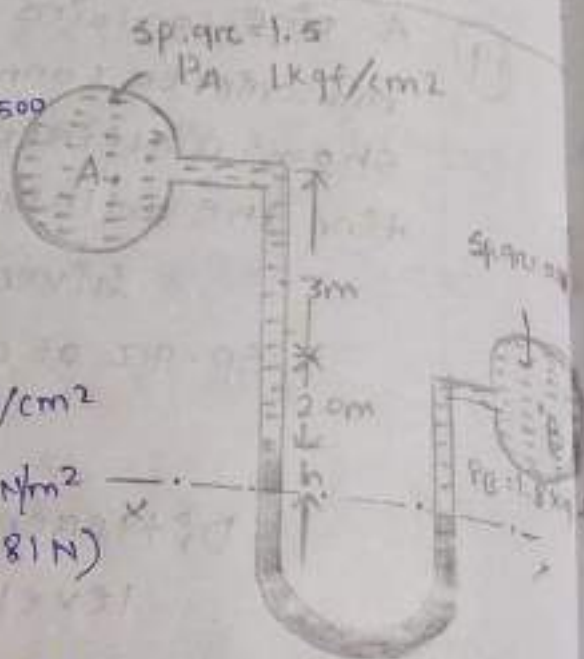
$$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$

Equating the 2 pressure

Dividing by ~~1000 x 9.81~~, we get

$$13.6 \times 1000 \times 9.81 h + 7500 \times 9.81 + 9.81 \times 10^4$$

$$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$



Dividing by 1000×9.81 , we get

$$13.6h + 7.5 + 10 \cdot (h + 2.0) \times 9 + 18$$

$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

$$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$$h = \frac{2.3}{12.7} = 0.181 \text{ m}$$

$$= \boxed{18.1 \text{ cm}}$$

21) A differential manometer is connected at the two points A and B as Fig. At B air pressure is 9.81 N/cm^2 (abs). Find the absolute pressure at A.

Given:

Air pressure at

$$B = 9.81 \text{ N/cm}^2$$

$$P_B = 9.81 \times 10^4 \text{ N/m}^2$$

$$\text{Density of oil} = 0.9 \times 1000$$

$$= 900 \text{ kg/m}^3$$

$$\text{Density of mercury} = 13.6 \times 1000 \text{ kg/m}^3$$

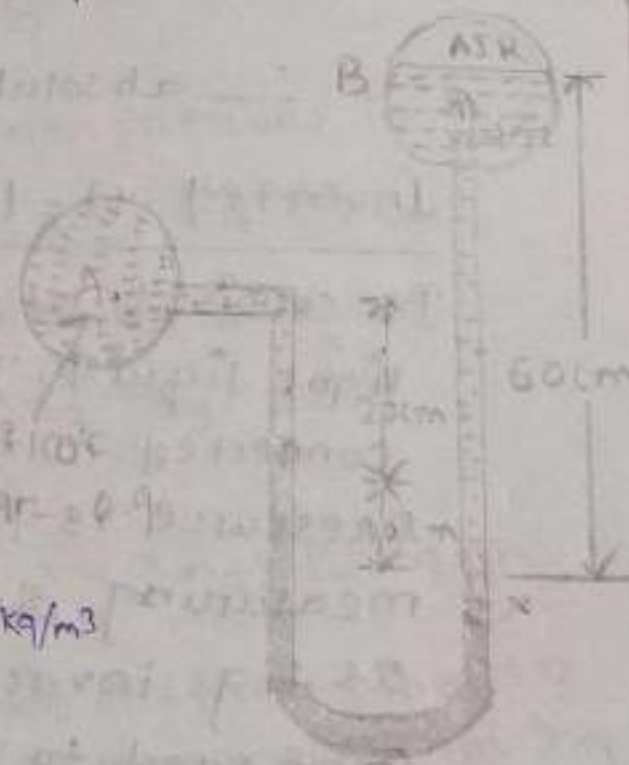
let the pressure =
at A is P_A

taking datum line at $x-x$

$$\text{Pressure above } x-x \text{ in the right limb}$$

$$= 1000 \times 9.81 \times 0.6 + P_B$$

$$= 5886 + 98100 = 103986$$



pressure above x-x in the left limb

$$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2$$

$$= 13341.6 + 1765.8 + P_A$$

Equating the two pressure heads

$$103986 = 13341.6 + 1765.8 + P_A$$

$$P_A = 103986 - 15107.4$$

$$= 88876.8$$

$$P_A = 88876.8 \text{ N/m}^2$$

$$= \frac{88876.8 \text{ N}}{10000 \text{ cm}^2} = 8.887 \frac{\text{N}}{\text{cm}^2}$$

\therefore - absolute pressure at $A = 8.887 \text{ N/cm}^2$

Inverted U-tube Differential Manometer:-

It consists of an inverted U-tube containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressure. as shown as fig inverted U-tube differential manometer connected to the two points A and B. let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line $x-x$

h_2 = Height of liquid of right limb

h = Difference of light liquid

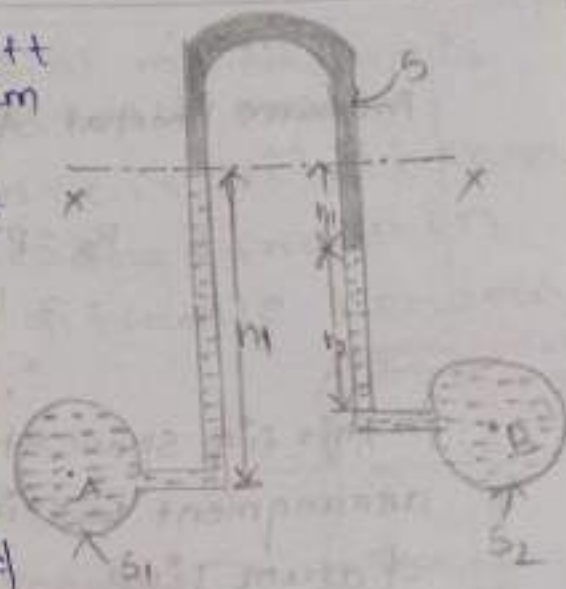
ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_3 = Density of liquid

P_A = pressure at A

P_B = pressure at B



taking $x-x$ as datum line. Then pressure in the left limb below $x-x$ = $P_A - \rho_1 \times g \times h_1$

pressure in the right limb below $x-x$
 = $P_B - \rho_2 \times g \times h_2 - \rho_3 \times g \times h$

Equating the two pressure

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_3 \times g \times h$$

(22)

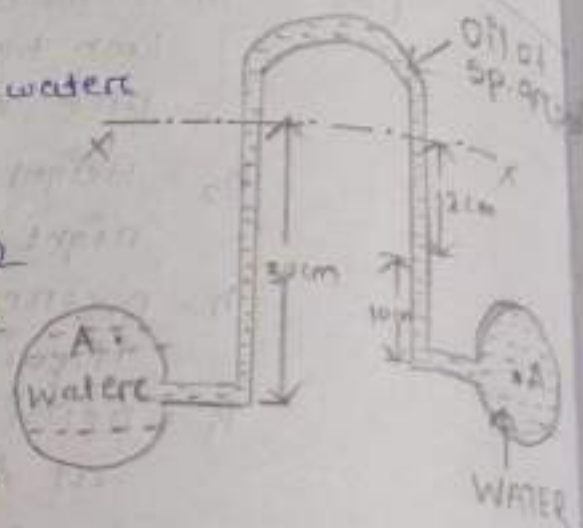
Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2m of water. Find the pressure in the pipe B for the manometer reading as shown in fig.

Given:

Pressure head at A = $\frac{P_A}{\rho g} = 2 \text{ m of water}$

$$\begin{aligned} P_A &= \rho \times g \times 2 \\ &= 1000 \times 9.81 \times 2 \\ &= 19620 \text{ N/m}^2 \end{aligned}$$

In this fig show the arrangement taking $x-x$ as datum line



Pressure below $x-x$ in the left limb

$$\begin{aligned} &= P_A - \rho \times g \times h_1 \\ &= 19620 - 1000 \times 9.81 \times 0.3 \\ &= 16677 \text{ N/m}^2 \end{aligned}$$

Pressure below $x-x$ in the right limb

$$\begin{aligned} &= P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12 \\ &= P_B - 981 - 941.76 = P_B - 1922.76 \end{aligned}$$

Equating the two pressure, we get

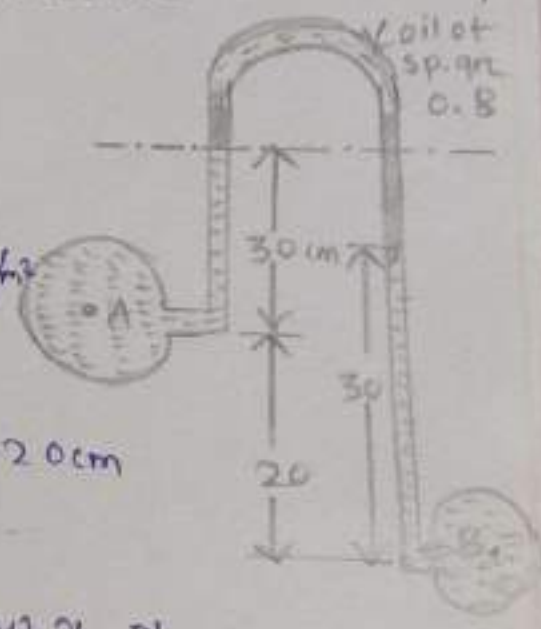
$$16677 = P_B - 1922.76$$

$$P_B = 16677 + 1922.76$$

$$= 18599.76 \text{ N/m}^2$$

$$P_B = \boxed{1.8599 \text{ N/m}^2}$$

3) In fig an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the fig. Find the pressure difference between A and B.



Given,

Sp. gr. of oil = 0.8 $\therefore \rho_o = 800 \text{ kg/m}^3$

Difference of oil in the two limbs = $(30+20) - 30 = 20 \text{ cm}$

taking datum line at $x-x$

Pressure in the left limb below $x-x$
 $= P_A - 1000 \times 9.81 \times 0.2$
 $= P_A - 2943$

Pressure in the right limb below $x-x$
 $= P_B + 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$
 $= P_B - 2943 - 1569.6 = P_B - 4512.6$

Equating the two pressure $P_A - 2943 = P_B - 4512.6$

$P_B - P_A = 4512.6 - 2943$
 $= 1569.6 \text{ N/m}^2$