LECTURE NOTES
ON
ENGINEERING MECHANICS

1ST SEMESTER

(MECHANICAL ENGINEERING)

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Chapter-1

< Fundamentals of Engg. Mechanics >

1.1 Mechanics may be defined as the branch of science which deals with the study of effect of forces on material bodies at state of rest or ~~motion~~ state of motion.

Engg. Mechanics
- mechanics of solid
  - mechanics of rigid body
    - statics
    - dynamics
      - Kinematics
      - Kinetics
  - mechanics of deformable bodies
    - Theory of elasticity
    - Theory of plasticity
- mechanics of fluids
  1. Ideal fluid
  2. viscous "
  3. Incompressible "

<u>Statics</u> → It is the branch of mechanics which deals with the forces & their effects on the body at rest.

<u>Dynamics</u> → It is the branch of mechanics which deals with the forces & their effects on the body when it is in motion.

<u>Rigid body</u> → A body is said to be rigid if relative positions of any two particles in it do not change after application of forces.

<u>deformable body</u> → If relative positions of particles changes after application of forces.

<u>Kinematics</u> → Dynamics dealing with the problems without considering the forces causing the motion of the body is called Kinematics.

<u>Kinetics</u> → Dynamics dealing with the problems considering the forces causing the motion of the body is called Kinetics.

body → An object is an identifiable collection of matter constrained by boundary is called body.
(Something which has a definite shape & consist of no. of particles)

Units →

MKS → meter - Kilogram - sec
CGS → cm - gram - sec
FPS → Foot - pound - sec
SI → International system of units

Fundamental Units	Derived units	
1. Mass → amount of matter contained in a body.	velocity	weight → force with which body is attracted towards the centre of earth due to gravity.
2. Length	acc^n	
3. Time	area	
4. Electric current	Volume	
5. Luminous Intensity	pressure	
6. Thermodynamic tempr		
7. Amount of substance		

1.2 Force :→ (F)

→ Force is defined as an external agent, which changes or tends to change the state of rest or state of uniform motion of the body, when it is applied on it.
→ Force is a vector quantity (have both magnitude & dir^n)
→ from Newton's 2^{nd} law, $F = ma$

unit of F = Newton

$1N = 1kg \times 1 m/s^2$ $\quad 1N = 10^5$ dyne
$= 1 \frac{kg\text{-}m}{s^2}$

Characteristics of a force →

- magnitude
- point of application
- Line of action
- Direction

Effect of force →

→ may change the state of the body
→ " " " dir^n of motion of the body
→ may retard the forces (de-acc^n)

→ may give rise to internal stresses
→ may produce turning effect

1.5 System of forces →

When several forces act simultaneously on a body, it is called system of forces.

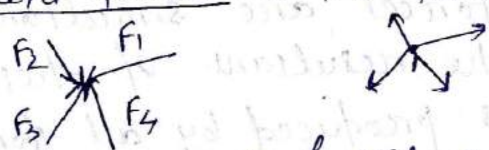
Acc^n to posi^n of lines of action of forces,

- system of forces
  - Coplanar forces
    - concurrent
    - parallel
      - like
      - unlike
    - Non-concurrent Non-parallel
  - Non-coplanar or spatial forces
    - concurrent
    - parallel
    - Non-concurrent none-parallel

1> Coplanar forces → The forces, whose lines of action lie on the same plane.

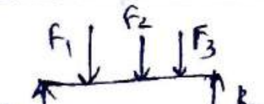
2> Collinear forces → The forces, whose lines of action lie on the same line.

3> Concurrent forces → The forces, which meet on one point.

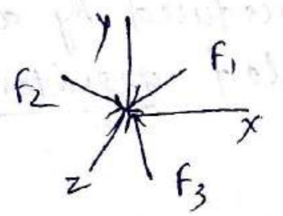


4> Coplanar concurrent forces → The forces, which meet at one point & their lines of action also lie on the same plane.

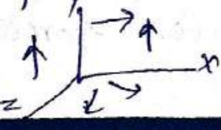
5> Coplanar non-concurrent forces → The forces, which do not meet at one point, but their lines of action lie on the same plane.



6> Non-coplanar concurrent forces → The forces, which meet at one point, but their line of action do not lie on the same plane.



7> Non-coplanar Non-concurrent forces → The forces which do not meet at one point & their lines of action do not lie on the same plane.

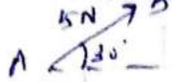


Characteristic & representation of force →

1> magnitude (50 N, 20 N ...)
2> Dirn of the force / line of action of the force (30°, 45° ...)
3> Nature of the force (push or pull)
4> point of application.

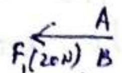
— point of application of force
dirn of sense of force.

It can be represented by 1> Vector representation



$\overrightarrow{AB}$

2> Bow's notation
. writing 2 capital Letters one on either side of force.



* **Particle** :→ Defined as a body ~~which can retain its shape & size, even if~~ of infinitely small volume & is considered to be concentrated point.

Principle of physical independence of forces →

States that "If a no. of forces are simultaneously acting on a particle, then the resultant of these forces will have the same effect as produced by all forces."

Principle of transmissibility of forces →

It states that "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action."

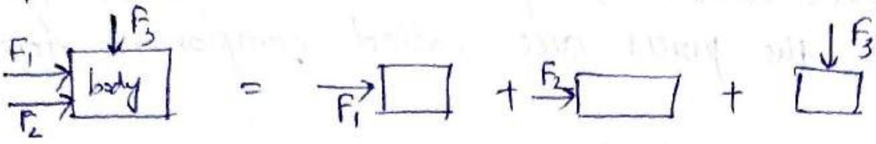
Resultant force →

If a no. of forces are acting simultaneously on a particle then a single force can be calculated which will replace them i.e. will produce same effect as produced by all the given forces. This single force is called resultant force.

~~Laws of forces & Methods to find resultant force~~ →

Principle of superposition of forces →

It states that the combined effect of force system acting on a particle or a rigid body is equal to the sum of effects of individual forces.



Action & reaction forces →

According to Newton's 3rd law of motion, "for every action there is an equal & opposite reaction."

Ex - hitting of ball on bat

During catching ball from a height

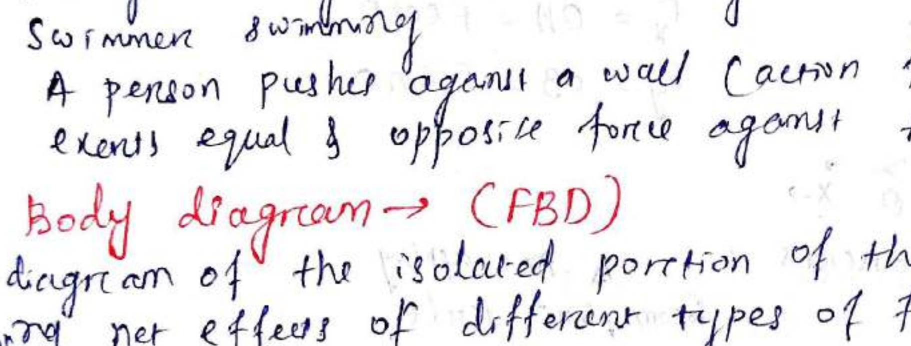
Swimmer swimming

A person pushes against a wall (action force) & wall exerts equal & opposite force against the person (rea^n force)

Free Body diagram → (FBD)

The diagram of the isolated portion of the structure, showing net effects of different types of forces on that portion is called FBD of the portion.

Structure consists of more than one elements, portions & supports. In FBD each portion can be isolated individually from the structure & effect of all supports, rea^n can be shown on that particular portion of the structure.



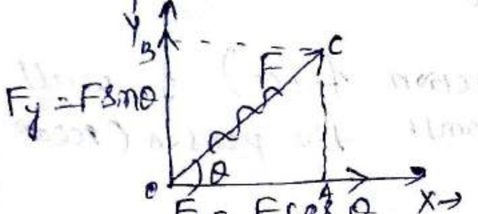
1.3 Resolution of a force :→

The process of spliting up of the given force into two components, without changing its effect on the body is called resolution of a force.

The force which is split into two parts is called as resolved force & the parts are called component forces or resolutes.

* Generally force is resolved into
1) mutually perpendicular components
2) Non " "

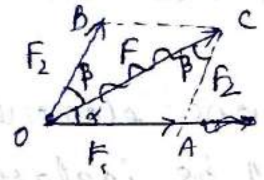
1) Mutually ⊥ components →



$F_x = OA = F\cos\theta$

$F_y = OB = F\sin\theta$

2) Non-⊥ components →



Δ AOC applying

from sine rule

$$\frac{OA}{\sin\beta} = \frac{OC}{\sin(180-(\alpha+\beta))} = \frac{AC}{\sin\alpha}$$

$$\Rightarrow \frac{F_1}{\sin\beta} = \frac{F}{\sin(\alpha+\beta)} = \frac{F_2}{\sin\alpha}$$

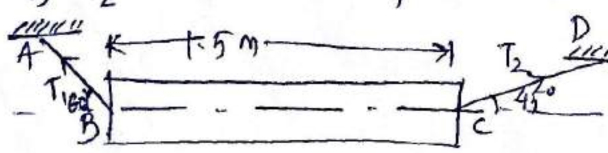
$\Rightarrow F_1 = ?$ & $F_2 = ?$

Principle of resolution of forces →

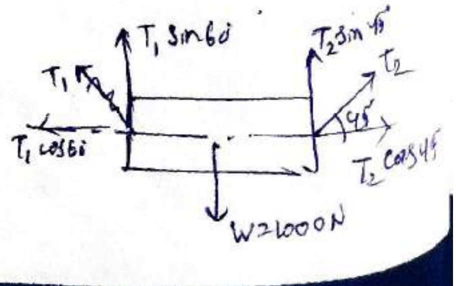
It states that "The algebraic sum of the resolved parts of a number of forces, in a given dirⁿ, is equal to the resolved parts of their resultant in the same dirⁿ."

* In general forces are ~~so~~ resolved in vertical & horizontal dirⁿ

Q. A m/c component 1.5m long & weight 1000N is supported by two ropes AB & CD as shown in fig. Calculate the tensions T_1 & T_2 in the ropes AB & CD



Solⁿ Weight (W) = 1000N



Resolving the forces T_1 & T_2 horizontally

$$T_1 \cos 60^\circ = T_2 \cos 45^\circ$$

$$\Rightarrow \frac{T_1}{2} = \frac{T_2}{\sqrt{2}} \Rightarrow \frac{T_1}{T_2} = \sqrt{2}$$

Method of resolution of forces :→ (to find resultant force) :→

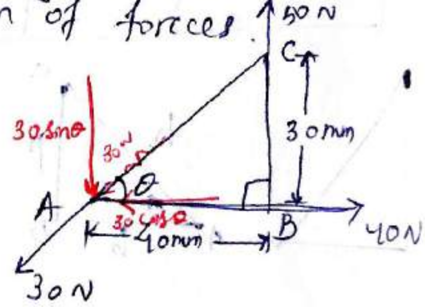
1. Resolve all the forces horizontally & find algebraic sum of all the horizontal components (i.e. ΣF_H)
2. Resolve all the forces vertically & find the algebraic sum of all the vertical components (i.e. ΣF_V)
3. Resultant force (R)

$$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$$

It will be inclined at an angle θ with the horizontal

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H}$$

Q.2 A triangle ABC has its side AB = 40 mm along +ve X-axis & side BC = 30 mm along +ve Y-axis, 3 forces of 40 N, 50 N & 30 N act along the sides AB, BC & CA respectively. Determine magnitude of the resultant of such system of forces.



Sol^n

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{40^2 + 30^2} = 50 \text{ mm}$$

$$\sin\theta = \frac{p}{h} = \frac{BC}{AC} = \frac{30}{50} = 0.6$$

$$\cos\theta = \frac{b}{h} = \frac{AB}{AC} = \frac{40}{50} = 0.8$$

Resolving the forces horizontally (along AB), & their sum is

$$\cancel{\tan\theta = \frac{p}{b} = \frac{30}{40} \Rightarrow \theta = \tan^{-1}\frac{30}{40} =}$$

$$\Sigma F_H = 40 - 30\cos\theta = 40 - 30(0.8) = 16 \text{ N}$$

&

$$\Sigma F_V = 50 - 30\sin\theta = 50 - 30(0.6) = 32 \text{ N}$$

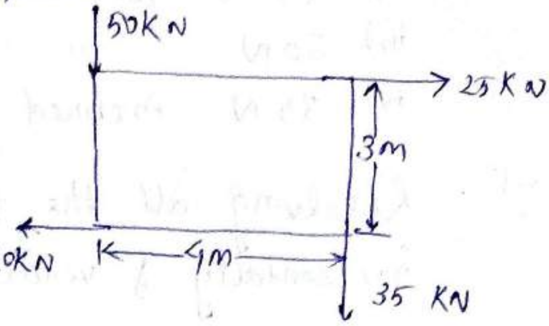
$$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$$
$$= \sqrt{16^2 + 32^2} = 35.8 \text{ N}$$

(x) $\tan\theta = \dfrac{\Sigma F_V}{\Sigma F_H} \Rightarrow \theta = \tan^{-1}\dfrac{32}{16} =$

or) $\theta = \tan^{-1}\dfrac{p}{b} = \tan^{-1}\left(\dfrac{30}{40}\right)$

then resolve

Q-3 A system of forces are acting at the corners of a rectangular block as shown in the fig. Determine the magnitude & dirn of the resultant force.



Soln $\Sigma F_H = 25 - 20 = 5\ KN$

$\Sigma F_V = -50 - 35 = -85\ KN$

$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$

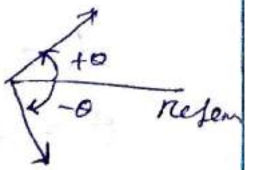
$= \sqrt{5^2 + (-85)^2} = 85.15\ KN$

Now, $\tan\theta = \dfrac{\Sigma F_V}{\Sigma F_H} = \dfrac{-85}{5} = -17$

$\Rightarrow \theta = \tan^{-1}(-17) = 86.6^\circ$

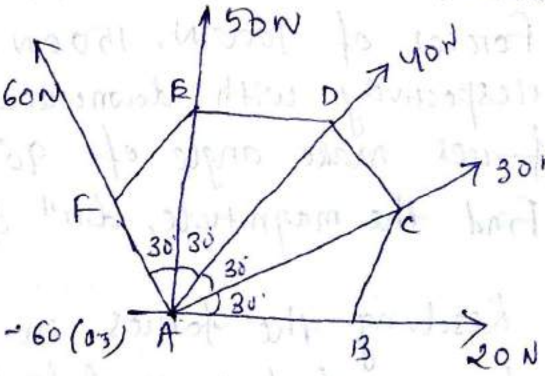
$\theta = 360 - 86.6 = 273.4^\circ$

As ΣF_V is -ve, R will be in 4th quadrant i.e. 270° to 360°.



Q-4 The forces 20N, 30N, 40N, 50N & 60N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude & dirn of the resultant force.

Soln Resolving all the forces horizontally along AB



$\Sigma F_H = 20 + 30\cos 30^\circ + 40\cos 60^\circ + 50\cos 90^\circ + 60\cos 120^\circ$

$= 20 + 30(0.866) + 40(0.5) + 0 - 60(0.5)$

$= 36N$

Resolving all the forces vertically along AE

$\Sigma F_V = 20\sin 0^\circ + 30\sin 30^\circ + 40\sin 60^\circ + 50\ \sin 90^\circ + 60\sin 120^\circ$

$= 0 + 30(0.5) + 40(0.866) + 50 + 60(0.866)$

$= 151.6\ N$

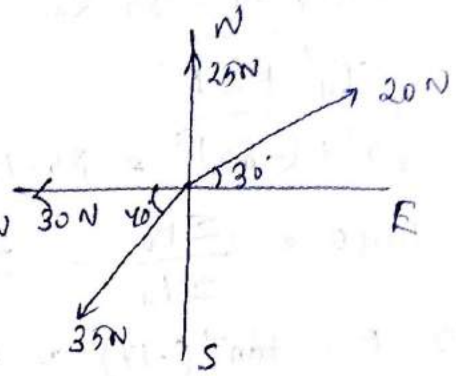
$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$

$= \sqrt{36^2 + 151.6^2} = 155.8\ N$

$\theta = \tan^{-1}\dfrac{\Sigma F_V}{\Sigma F_H} = \tan^{-1}\dfrac{151.6}{36} = 76.6^\circ$

Q-5 The following forces act at a point
i) 20N inclined at 30° towards North-East
ii) 25 N towards North
iii) 30N " " - West
iv) 35 N inclined at 40° towards South-West

Sol^n Resolving all the forces horizontally & vertically



$$\Sigma F_H = 20\cos 30° - 30 - 35\cos 40° = -30.7\,N$$

$$\Sigma F_V = 20\sin 30° + 25 - 35\sin 40° = 33.7\,N$$

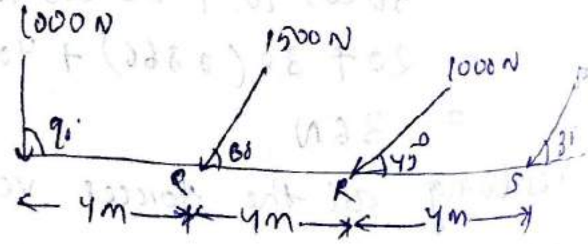
$$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2} = \sqrt{(-30.7)^2 + 33.7^2} = 45.6\,N$$

$$\theta = \tan^{-1}\frac{\Sigma F_V}{\Sigma F_H} = \tan^{-1}\frac{33.7}{-30.7} = 47.7°$$

As ΣF_H is -ve so, R will lie in 2nd quadrant. $\theta = 180 - 47.7 = 132.3°$

Q-6 A horizontal line PQRS is 12m long, where PQ = QR = RS = 4m. Forces of 1000 N, 1500 N, 1000 N & 500 N act at P, Q, R & S respectively with downward dir^n. The lines of action of these forces make angle of 90°, 60°, 45° & 30° respectively with PS. Find the magnitude, dir^n & ~~position of the resultant~~ force.

Sol^n Resolving the forces in horizontal & vertical dir^n



$$\Sigma F_H = -500\cos 30° - 1000\cos 45° - 1500\cos 60° = -1890\,N$$

$$\Sigma F_V = -1000 - 1500\sin 60° - 1000\sin 45° - 500\sin 30° = -3256\,N$$

$$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2} = \sqrt{(-1890)^2 + (-3256)^2} = 3765\,N$$

$$\theta = \tan^{-1}\frac{\Sigma F_V}{\Sigma F_H} = \tan^{-1}\frac{+3256}{+1890} = 59.8°$$

* position of the resultant force

Let x = Distance betn P & line of action of resultant force

$$3765x = (1000 \times 0) + (1500 \times 0.866)4 + (1000 \times 0.707)8 + (500 \times 0.5)12$$
$$= 13852$$
$$\Rightarrow x = 3.68\,m$$

1.6 Composition of forces :→

The process of finding the resultant force of a number of given forces is called composition of forces or compounding of forces.

Laws of forces (Methods used of composition of forces) :→

The resultant force of a given system of forces can be found by

1> Parallelogram law of force

2> Triangle " "

3> Polygon " "

Methods used to find resultant forces :→

<1> Analytical method

<2> Graphical method / Vector method

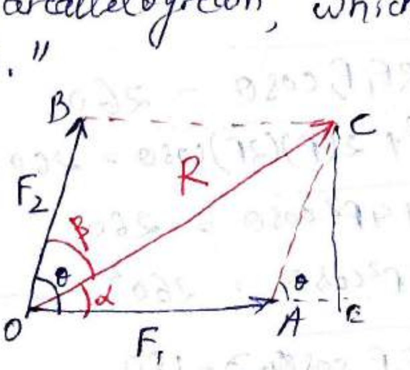
<1> Analytical method :→

i) Parallelogram law of forces

ii) method of resolution of forces.

i) Parallelogram law of forces →

It states that, "If two forces are acting simultaneously on a particle, are represented in magnitude & direction by the two adjacent sides of a parallelogram, then their resultant can be represented in magnitude & dirn by the diagonal of the parallelogram, which passes through their point of intersection."



$$R = \sqrt{OE^2 + CE^2}$$
$$= \sqrt{(OA + AE)^2 + CE^2}$$
$$= \sqrt{(F_1 + AC\cos\theta)^2 + (AC\sin\theta)^2}$$
$$= \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$
$$= \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2\sin^2\theta + F_2^2\cos^2\theta}$$
$$= \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

Consider two forces F_1 & F_2 acting at point O.

R = Resultant of F_1 & F_2

θ = Angle betn force F_1 & F_2

α = " " F_1 & R

β = " " F_2 & R

Now according to parallelogram law of forces

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

& $\tan\alpha = \dfrac{CE}{OE} = \dfrac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$

$\tan\beta = \dfrac{F_1\sin\theta}{F_2 + F_1\cos\theta}$

Particular cases

i) when $\theta = 0°$, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2} = \sqrt{(F_1+F_2)^2} = F_1 + F_2$

ii) when $\theta = 90°$, $R = \sqrt{F_1^2 + F_2^2 + 0} = \sqrt{F_1^2 + F_2^2}$

iii) when $\theta = 180°$ $R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2} = \sqrt{(F_1 - F_2)^2} = F_1 - F_2$

Q-1 Two forces of 100N & 150N are acting simultaneously at a point. Find the resultant, if angle betn two forces is 45°

Soln $F_1 = 100N$, $F_2 = 150N$, $\theta = 45°$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$
$$= \sqrt{100^2 + 150^2 + 2(100)(150)\cos 45°}$$
$$= 232N$$

Q-2 The resultant of two forces, one of which is double the other is 260N. If the dirn of the larger force is reversed & the other remains unaltered, the resultant reduces to 180N. Determine the magnitude of the forces & the angle betn the forces.

Soln Let $F_1 = F$

so, $F_2 = 2F$

Case-1

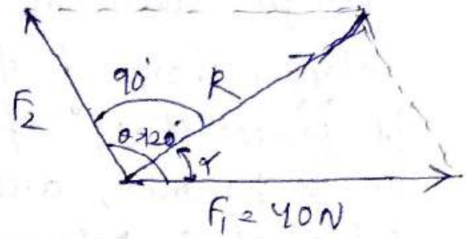
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} = 260$$
$$\Rightarrow \sqrt{F^2 + (2F)^2 + 2(F)(2F)\cos\theta} = 260$$
$$\Rightarrow \sqrt{F^2 + 4F^2 + 4F^2\cos\theta} = 260$$
$$\Rightarrow 5F^2 + 4F^2\cos\theta = 260^2 \quad \text{--- (1)}$$

Case-2

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} = 180$$
$$= \sqrt{F^2 + (-F)^2 + 2(F)(-F)\cos\theta} = 180$$
$$\Rightarrow F^2 + F^2 - 2F^2\cos\theta = 180^2$$
$$\Rightarrow 2F^2 - 2F^2\cos\theta = 180^2 \quad \text{--- (2)}$$

from equ^n ① & ② $F = 100N$ so, $F_1 = 100N$ & $F_2 = 2F = 200N$.
& $\cos\theta = 0.44 \Rightarrow \theta = 63.9°$.

Q.3 Two forces act at an angle of 120°. The bigger force is of 40N & the resultant is $\perp$ to the smaller one. Find the smaller force.



Sol^n

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$= \sqrt{40^2 + F_2^2 + 2(40) F_2 \cos 120°}$$

$\alpha = 120° - 90° = 30°$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta} \Rightarrow \tan 30° = \frac{F_2 \sin 120°}{40 + F_2 \cos 120°}$$

$$\Rightarrow F_2 = 20.$$

Q.4 Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.

Sol^n

$$\sqrt{F_1^2 + F_2^2} = \sqrt{10} \Rightarrow F_1^2 + F_2^2 = 10$$

$$\& \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60°} = \sqrt{13}$$

$$\Rightarrow F_1^2 + F_2^2 + F_1F_2 = 13$$

$$\Rightarrow 10 + F_1F_2 = 13 \Rightarrow F_1F_2 = 3$$

we know that $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2$

$$\Rightarrow (F_1 + F_2)^2 = 10 + (2\times3) = 16 \Rightarrow F_1 + F_2 = 4 \quad \text{—— (i)}$$

Again $(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2$

$$\Rightarrow (F_1 - F_2)^2 = 10 - (2\times3) = 4 \Rightarrow F_1 - F_2 = 2 \quad \text{—— (ii)}$$

from equ^n (i) & (ii) $2F_1 = 6 \Rightarrow F_1 = 3N$

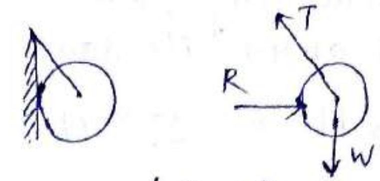
so, $F_2 = 4 - 3 = 1N$

FBD →

Diagram in which body under consideration is freed from all contact surfaces & is shown with all the forces on it (including self weight & reac's from other contact surfaces) is called FBD.

1) 

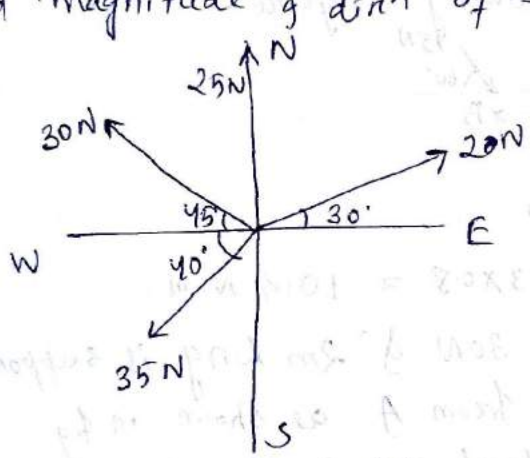
(Ex-1) FBD

2) 

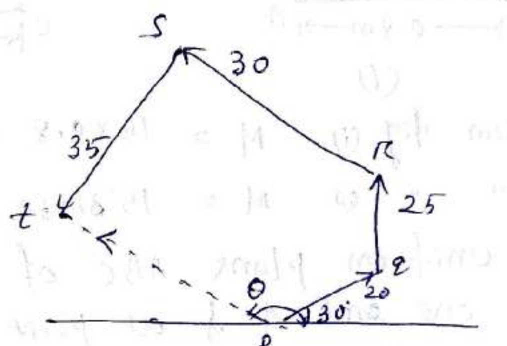
(Ex-2)

Q-2 The following forces act at a point.
i) 20N inclined at 30° towards North of East.
ii) 25N towards North
iii) 30N towards North west
iv) 35N inclined at 40° towards South of west.
Find magnitude & dirn of the resultant force.

Soln



(Space diagram)



(vector diagram)

1.4 Moment of Force :→

It is the turning effect produced by a force, on the body, on which it acts.

The moment of a force is equal to the product of the force & the ⊥ distance of the point, about which the moment is required.

Mathematically, $M = F \times l$

Unit of moment → N-m

Types of moment : → (Acc^n to dir^n of rotation)

1) Clockwise moment 

2) Anti-clockwise moment 

Sign convention

↻(+) & ↺(−) or ↻(−) & ↺(+)

Laws of moments →

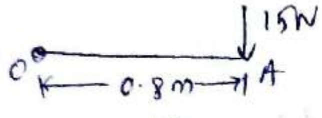
It states that, " If a body is in rotational equ^n under the action of a no. of forces, the sum of the clockwise moments of the force about any point is equal to the anti-clockwise moments of the forces about the same point."

$$\Sigma \circlearrowright M = \Sigma \circlearrowleft M$$

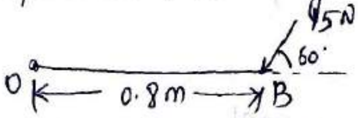
Varignon's theorem of moments →

It states that, " If a number of coplanar forces are acting simultaneously on a particle, the algebric sum of the moments of all the forces about any point is equal to their resultant force about the same point."

Q find moment about O of the following figure.



(i)



(ii)

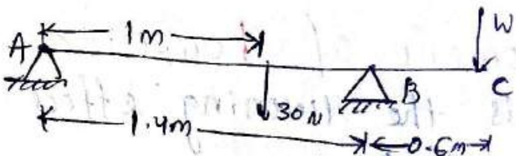
Sol^n

From fig (i) $M = 15 \times 0.8 = 12$ N-m

" " (ii) $M = 15 \sin 60 \times 0.8 = 13 \times 0.8 = 10.4$ N-m

Q-2 A uniform plank ABC of weight 30N & 2m long is supported at one end A & at point B 1.4m from A as shown in fig. find max^m weight W, that can be placed at C so that the plank does not topple.

Sol^n

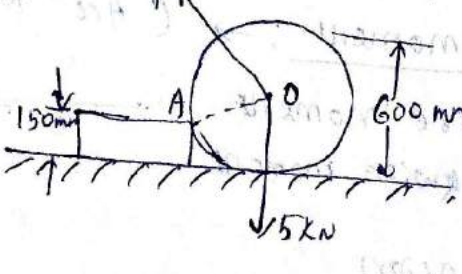


$\Sigma M_B = 0$

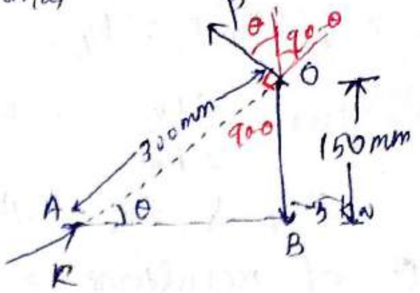
$\Rightarrow 30 \times 0.4 = W \times 0.6$

$\Rightarrow W = 20N$

Q-3 A uniform wheel of 600mm dia, weighing 5KN rests against a rigid rectangular block of 150mm height as shown in fig. Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the rea^n on the block. Take all the surfaces to be smooth.



For the least pull, it must be applied normal to AO.



From the fig, $\sin\theta = \frac{150}{300}$

$\Rightarrow \theta = 30^\circ$

$AB = \sqrt{300^2 - 150^2} = 260$ mm

$\Sigma M_A = 0$

$\Rightarrow 5(260) = P \times 300 \Rightarrow P = \frac{1300}{300} = 4.33$ kN

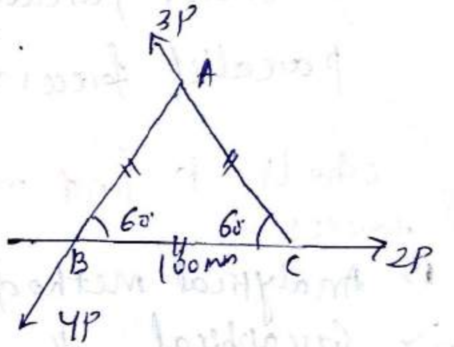
R = Reaⁿ on the block.

Resolving the forces horizontally,

$R\cos 30^\circ = P\sin 30^\circ$

$\Rightarrow R = 2.5$ kN

Q-4 Three forces of 2P, 3P & 4P act along the 3 sides of an equilateral Δ of side 100mm taken in order. Find the magnitude & position of the resultant force.



Solⁿ Resolving all forces

$\Sigma F_H = 2P + 3P\cos 120^\circ - 4P\cos 60^\circ$

$= 2P - \frac{3}{2}P - 2P = -1.5P$

$\Sigma F_V = 3P\sin 60^\circ - 4P\sin 60^\circ$

$= -0.866P$

$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2} = \sqrt{(-1.5P)^2 + (-0.866P)^2} = 1.732P$

Posⁿ of resultant

Let x = ⊥ distance betⁿ B & line of action of resultant.

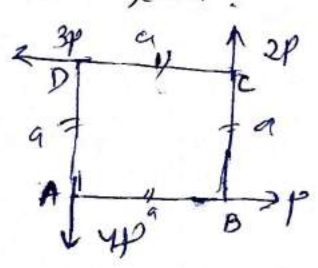
Using Varignon's theorem

$\Sigma M_B = 0$

$1.732P\,x = 3P \times 100\sin 60^\circ$

$\Rightarrow x = 150$ mm

Q-5 Four forces equal to P, 2P, 3P & 4P are respectively acting along the four sides of square ABCD taken in order. Find the magnitude, dirⁿ & posⁿ of the resultant force.



Solⁿ Resolving all forces horizontally & vertically

$\Sigma F_H = P - 3P = -2P$

$\Sigma F_V = 2P - 4P = -2P$

$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2} = \sqrt{(-2P)^2 + (-2P)^2} = 2\sqrt{2}P$

$\theta = \tan^{-1} \frac{\Sigma F_V}{\Sigma F_H} = \tan^{-1} \frac{-2P}{-2P} = 45^\circ$

As ΣF_H & ΣF_V both are $-ve$. so lies in 3rd quadrant

θ of resultant $= 180 + 45^\circ = 225^\circ$.

Posn

Let x = ⊥ distance betn A & line of action of resultant force

Applying varignon's law,

$2\sqrt{2}P \times x = (2P \times a) + (3P \times a)$

$2\sqrt{2}\cancel{P} \times x = 5\cancel{P} \times a \Rightarrow x = \frac{5a}{2\sqrt{2}}$

Parallel forces & ~~couples~~ :→

The forces whose lines of action are parallel to each other are called parallel forces.

parallel forces — like e.g. ↑↑↑↑ (line of actions same but dirn of all forces same)
— unlike e.g. ↑↓↑↓ (" " not same)

methods to find magnitude & posn of the resultant of parallel forces →

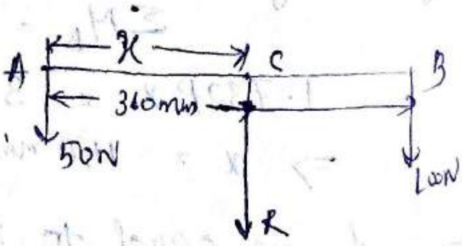
1) Analytical method
2) Graphical "

1) Analytical method

Here $\Sigma M_{\circlearrowright} = \Sigma M_{\circlearrowleft}$ about a point.

Q Two like parallel forces of 50N & 100N act at the ends of a rod 360 mm long. find magnitude of resultant force & the point where it acts.

Soln As forces are like & parallel,



$R = 50 + 100 = 150$ N

Let x = distance betn line of action of R & point A.

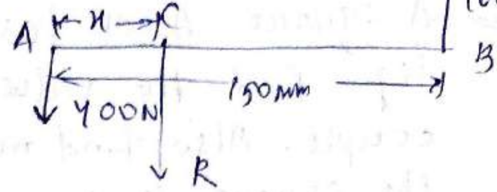
Now, $\Sigma M_C = 0$ or $\Sigma M_{\circlearrowright} = \Sigma M_{\circlearrowleft}$ about C

$\Rightarrow 50x = 100(360 - x)$

$\Rightarrow x = 240$ mm

Q. Two unlike parallel forces of magnitude 400 N & 100 N are acting in such a way that their lines of action are 150 mm apart, Determine the magnitude of the resultant force & the point at which it acts.

Sol^n



$R = 100 - 400 = -300\,N$

$\Sigma M\circlearrowright = \Sigma M\circlearrowleft$ about A

$400x - Rx = 100 \times 150$

$\Rightarrow (300 \times x) = 100 \times 150$

$\Rightarrow x = \frac{-15000}{300} = -50\,mm$

couple :→

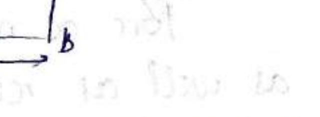
A pair of two equal & unlike parallel forces (i.e. forces equal in magnitude, with lines of action parallel to each other & acting in opposite dir^n's) is called couple.

Arm of couple

⊥ distance bet^n the lines of action of two equal & opposite parallel forces.

Moment of couple

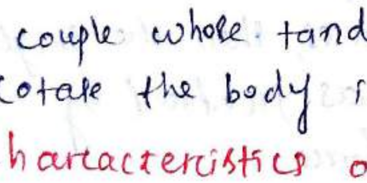
It is the product of either force & ⊥ distance or arm of couple.



$M = P \times a$

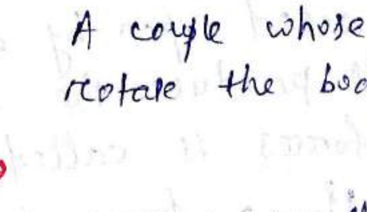
$= N\text{-}m$ (unit)

Types of couple →



(clockwise couple)

A couple whose tandancy is to rotate the body in ↻ dir^n.



(Anti-clockwise couple)

A couple whose tandancy is to rotate the body in ↺ dir^n.

Characteristics of a couple →

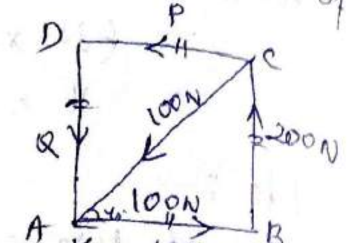
→ algebraic sum of the forces, constituting the couple is zero.

→ " " moments of the forces forming the couple about any point is same & equal to moment of couple itself.

→ A couple cann't be balanced by a single force but can be balanced by a couple of opposite sense.

→ Any no. of coplanar couples can be added or subtracted to a resultant single couple, whose magnitude will be equal to the algebraic sum of the moments of all the compound couples.

Q. A square ABCD has forces acting along its sides as shown in fig. Find the values of P & Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1m.



Soln. Values of P & Q

To reduce the system into a couple, the resultant force in horizontal & vertical dirn must be zero.

$\Sigma F_H = 0 \Rightarrow 100 - 100\cos 45° - P = 0 \Rightarrow P = 29.3N$

$\Sigma F_V = 0 \Rightarrow 200 - Q - 100\sin 45° = 0 \Rightarrow Q = 129.3N$

Magnitude of the couple

moment of the couple is equal to the algebraic sum of the moments about any point.

So, $M_A = 200 \times 1 + (P \times 1)$

$= 200 + 29.3 = 229.3$ N-m.

Chapter - 2
EQUILIBRIUM

If the resultant of all the forces, acting on a stationary rigid body is zero, the body is in equilibrium.

For a moving body to be in equm state, all resultant force as well as resultant moments should be zero.

Difference between resultant & equilibrant →

When body is acted upon by a no. of forces, then the single force which will produce the same effect as produced by the system of forces is called resultant force.

If a force equal, opposite & collinear with the resultant force is acting on the body, then the body will come to equm state. This force is called equilibrant.

Conditions & equ^{n}s of static equilibrium :→

If a no. of coplanar concurrent forces act on a body, the body is said to be at rest or in equ^n, if the resultant of all the forces acting on the body is zero.

Cond^n of equ^n are

$$\Sigma F_H = 0 \quad \& \quad \Sigma F_V = 0$$

Methods for equ^n of coplanar forces :→

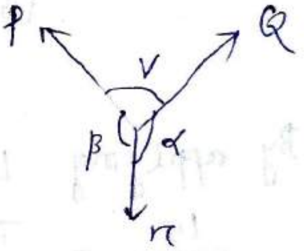
1) Analytical method
2) Graphical method

1) Analytical method :→

It is studied by Lami's theorem.

Lami's theorem →

It states that, "If three coplanar forces acting at a point are in equ^n, then each force is proportional to the sine of the angle between the other two."



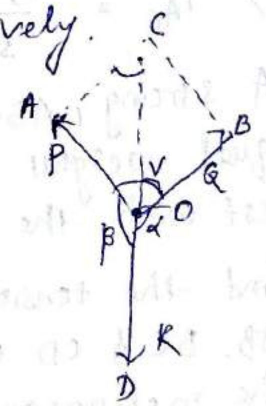
mathematically

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin v}$$

Proof

Consider 3 coplanar force P, Q & R acting at point O.

α, β & v are the opposite angles of 3 forces respectively.



1st, complete the parallelogram OACB.

Resultant of force P & Q can be given by diagonal OC.

As forces P, Q & R are in equ^n so, resultant of P & Q will be in line with OD & equal to R, but in opposite dir^n.

From the geometry BC = P & AC = Q

$\angle AOC = 180^\circ - \beta$, $\angle ACO = \angle BOC = 180^\circ - \alpha$

$\angle CAO = 180^\circ - (\angle AOC + \angle ACO)$

$= 180^\circ - (180^\circ - \beta + 180^\circ - \alpha)$

$= \cancel{180} - \cancel{180} + \beta - 180 + \alpha$

$= \alpha + \beta - 180^\circ$

Again $\alpha + \beta + v = 360^\circ$

$\Rightarrow \alpha + \beta + v - 180^\circ = 360^\circ - 180^\circ$

$\Rightarrow \angle CAO + v = 180^\circ$

$\Rightarrow \angle CAO = 180^\circ - v$

from triangle AOC

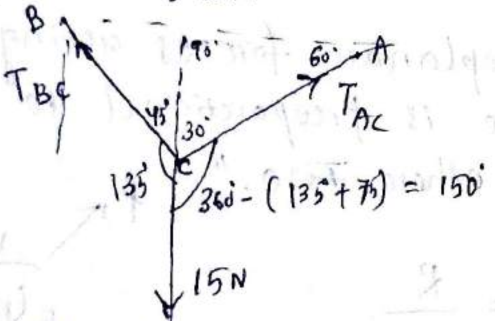
$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle CAO}$$

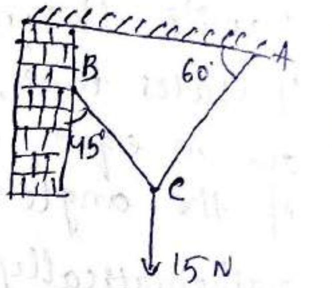
$$\Rightarrow \frac{OA}{\sin(180^\circ-\alpha)} = \frac{AC}{\sin(180^\circ-\beta)} = \frac{OC}{\sin(180^\circ-\gamma)}$$

$$\Rightarrow \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}} \quad \text{(proved)}$$

Q-1 An electric light fixture weighing 15N hangs from a point C, by two strings AC & BC. The string AC is inclined at 60° to the horizontal & BC at 45° to the horizontal as shown in fig. Using Lami's theorem, determine the forces in strings AC & BC.

Sol^n



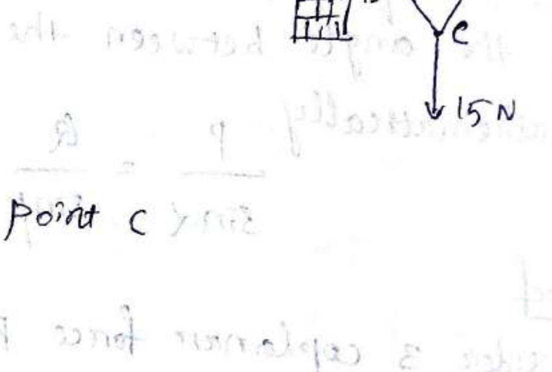


By applying Lami's theorem at point C

$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}.$$

$$\Rightarrow T_{AC} = \frac{15\sin 135^\circ}{\sin 75^\circ} = 10.98N \quad \& \quad T_{BC} = \frac{15\sin 150^\circ}{\sin 75^\circ} = 7.76N$$

Q-2 A string ABCD, attached to fixed points A & D has two equal weights of 1000N attached to it at B & C. The weights rest with the portions AB & CD inclined at angles as shown in fig. Find the tensions in the portions AB, BC & CD of the string, if the inclination of the portion BC with the vertical is 120°.

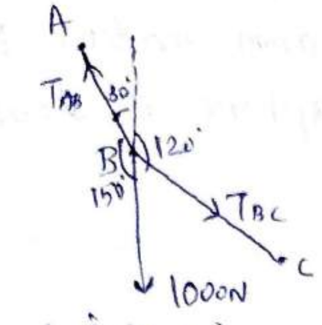


T_{AB} = Tension in string AB

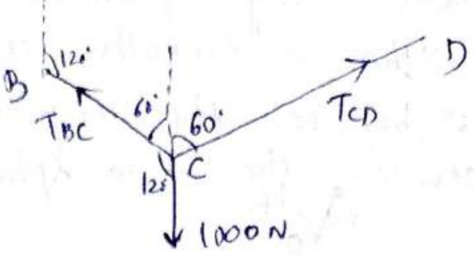
T_{BC} = " " BC

T_{CD} = " " CD

Sol^n



(joint B)



(Joint C)

Applying Lami's theorem at point B

$$\frac{1000}{\sin 150^\circ} = \frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\Rightarrow T_{AB} = \frac{1000 \sin 60^\circ}{\sin 150^\circ} = 1732\,N$$

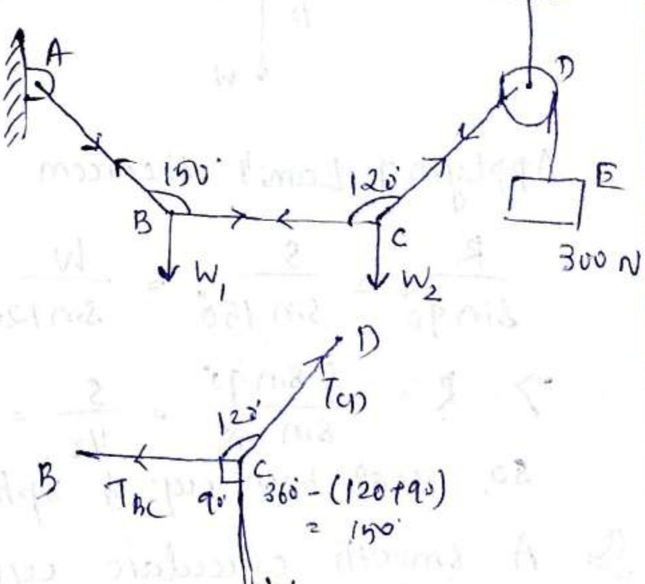
$$\& \; T_{BC} = \frac{1000 \sin 150^\circ}{\sin 150^\circ} = 1000\,N$$

Applying Lami's theorem at point C

$$\frac{1000}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

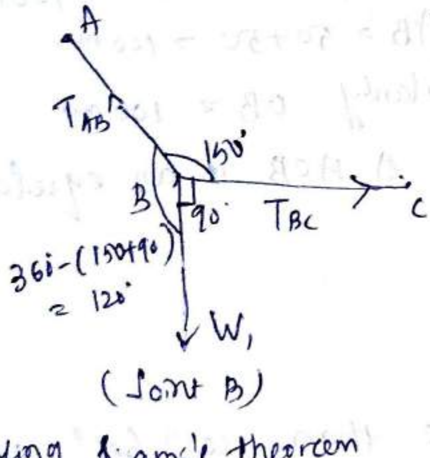
$$\Rightarrow T_{CD} = \frac{1000 \sin 120^\circ}{\sin 120^\circ} = 1000\,N$$

Q-3 A light string ABCDE whose extremity A is fixed, has weights W_1 & W_2 attached to it at B & C. It passes round a small smooth peg at D carrying a weight of 300N at the free end E as shown in fig. If in the equ^m pos^n, BC is horizontal & AB & CD make 150° & 120° with BC, find
i) Tension in the portion AB, BC & CD of the string
ii) magnitudes of W_1 & W_2.



Sol^n

②



(Joint B)

Applying Lami's theorem

$$\frac{W_1}{\sin 150^\circ} = \frac{T_{BC}}{\sin 120^\circ} = \frac{T_{AB}}{\sin 90^\circ}$$

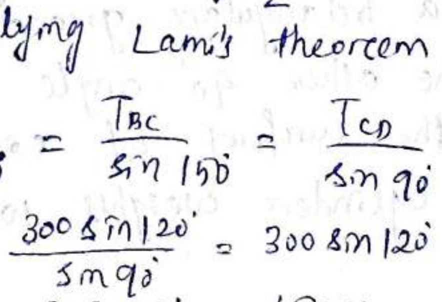
$$\Rightarrow \frac{W_1}{\sin 150^\circ} = \frac{150}{\sin 120^\circ}$$

$$\Rightarrow W_1 = 86.6\,N$$

& ② Again $\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{AB}}{\sin 90^\circ} = \frac{150}{\sin 120^\circ}$

$\Rightarrow T_{AB} =$

①



Applying Lami's theorem

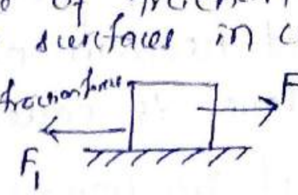
$$\frac{W_2}{\sin 120^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{T_{CD}}{\sin 90^\circ} = \frac{300}{\sin 90^\circ}$$

$$\Rightarrow W_2 = \frac{300 \sin 120^\circ}{\sin 90^\circ} = 300 \sin 120^\circ = 259.8\,N$$

$$\& \; T_{BC} = 300 \sin 150^\circ = 150\,N$$

Chapter-3
FRICTION

The opposing force which act in opposite direction of the movement of the body is called force of friction or simply friction. It always acts between two surfaces in contact.



Types of friction →

friction
- Static
- dynamic / Kinetic
  - Sliding
  - Rolling

Static friction → It is the friction experienced by a body when it is at rest or when body tends to move.

Dynamic friction → It is the friction experienced by a body when it is in motion.

Sliding friction → It is the friction experienced by a body when it slides over another body.

Rolling friction → It is the friction experienced by a body when it rolls over another body.

Limiting friction →

When a small force is applied to a body, it does not move due to frictional force as the applied force is balanced by frictional force. But when the applied force exceeds a limit (beyond which frictional force cann't increase), the force of friction cann't balance the applied force & the body moves in the dirn of applied force.

The maxm value of frictional force, when the body begins to slide over another body/surface when external force is applied is called limiting friction.

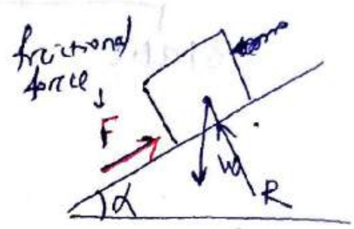
When applied force < limiting friction (body remains rest)
" " > " (body moves)

Noremal reaction → (R)

When a body is placed on a horizontal or inclined surface is in equ^n, its weight acts vertically downwards through its centre of gravity. The surface in return, exerts an upward reaction on the body, which act perpendicular to the plane is called normal reaction.

Angle of friction → (α)

i) It is the max^m angle of inclination when the body just starts sliding down the plane.



on) Angle of inclined plane, at which a body just begins to slide down the plane is called angle of friction.

α = angle made by normal rea^n with vertical.

Coefficient of friction → (μ)

It is defined as the ratio between limiting friction to the normal reaction between the two bodies.

mathematically $\mu = \dfrac{F}{R} = \tan\phi$

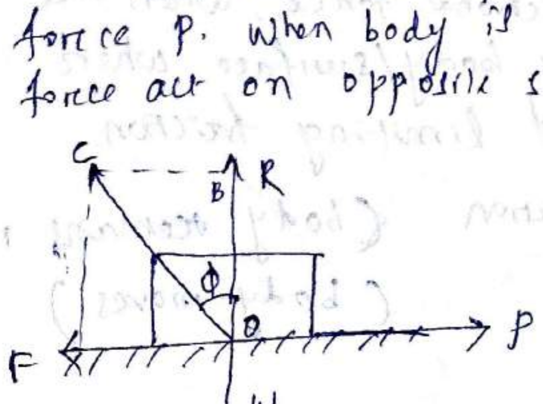
$\Rightarrow \boxed{F = \mu R}$

where ϕ = Angle of friction
F = limiting friction
R = Normal rea^n between two bodies

~~Cone of friction~~ →

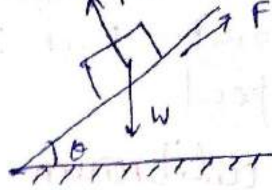
Angle of friction is the angle which the resultant of force of limiting friction & normal reaction makes with the normal rea^n.

Let mass m kept on horizontal plane pulled by a force P. When body is just about to slide, limiting frictional force act on opposite side. R be the normal rea^n of weight.



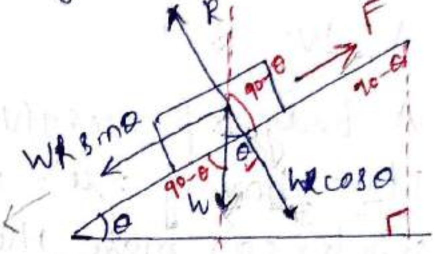
Let OC = Resultant betn R & F.
makes angle ϕ with R.
From ΔOBC, $\tan\phi = \dfrac{BC}{BO} = \dfrac{F}{R}$

Angle of repose →



Consider a block of weight W resting on an inclined plane which makes an angle θ with the horizontal.

When θ is very small, the block will rest on the plane. If θ increases gradually, a stage will come when the block starts to slide. That angle is called angle of repose.



$\Sigma F_V = 0$

$\Rightarrow R = W\cos\theta$ —— (1)

$\Sigma F_H = 0 \Rightarrow F = W\sin\theta$ —— (2)

equⁿ (2) ÷ (1) $\Rightarrow \frac{F}{R} = \frac{\cancel{W}\sin\theta}{\cancel{W}\cos\theta} \Rightarrow \tan\theta = \frac{F}{R}$

$\Rightarrow \tan\theta = \tan\phi$

$\Rightarrow \boxed{\theta = \phi}$

$\boxed{\text{Angle of friction} = \text{Angle of repose}}$

Laws of friction: →

Laws of static friction →

i) The force of friction always act opposite to the dirⁿ of applied force.

ii) The magnitude of force of friction is exactly equal to the applied force, which tend to move the body.

iii) The magnitude of limiting friction maintains a constant ratio to the normal reaⁿ between the two surfaces

$\frac{F}{R} = \text{const.}$ (F ← Limiting friction, R ← Normal Reaⁿ)

iv) The force of friction is independent of the area of contact between the two surfaces.

v) The force of friction depends upon the surface roughness.

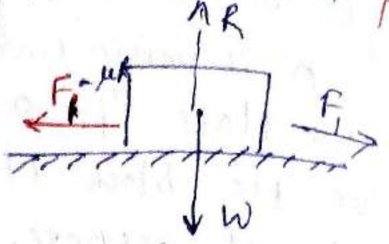
Laws of dynamic friction →

i) The force of friction always act opposite to the dirⁿ of motion of the body.

ii) The magnitude of kinetic friction maintains a constant ratio with normal reaⁿ betⁿ two surfaces.

iii) For moderate speeds, the force of friction remains const. But it decreases slightly with the increase of speed.

Equilibrium of a body on a rough horizontal plane

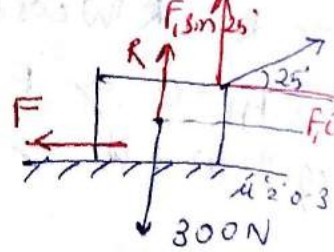


for equ^m of the body,

$F = F_1 \Rightarrow F = \mu R$

& $W = R$

Q-1 A body of weight 300N is lying on a rough horizontal plane having $\mu = 0.3$. find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.



Sol^n Given $W = 300N$ $\quad \theta = 25°$

$\mu = 0.3$

$F_1 = ?$

for equ^m $\quad \Sigma F_H = 0 \Rightarrow F = F_1 \cos 25°$

& $\Sigma F_V = 0 \Rightarrow R + F_1 \sin 25° = 300 \Rightarrow R = 300 - F_1 \sin 25°$

we know that $\quad F = \mu R$

$\Rightarrow F_1 \cos 25° = 0.3 (300 - F_1 \sin 25°)$

$\Rightarrow F_1 = 87.1N$

Q-2 A body, resting on a rough horizontal plane, required a pull of 180N inclined at 30° to the plane just to move it. It was found that a push of 220N inclined at 30° to the plane just moved the body. Determine the weight of the body & coefficient of friction.

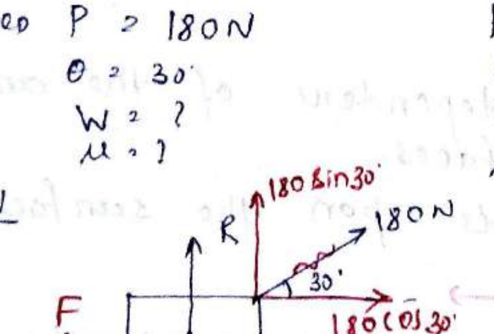
Sol^n Given $P = 180N$ $\qquad P = -220N$

$\theta = 30°$ $\qquad \theta = 30°$

$W = ?$ $\qquad W = ?$

$\mu = ?$ $\qquad \mu = ?$

case-1



(FBD of th for case-1)

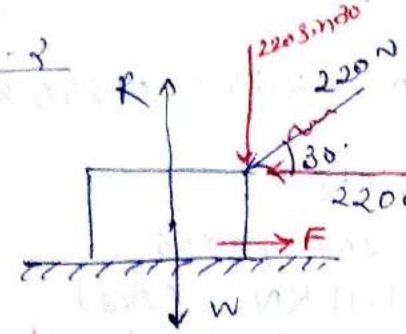
For equ^n $\Sigma F_H = 0 \Rightarrow F = 180\cos 30°$

& $\Sigma F_V = 0 \Rightarrow R + 180\sin 30° = W \Rightarrow R = W - 180\sin 30°$

We know that $F = \mu R$

$\Rightarrow 180\cos 30° = \mu(W - 180\sin 30°)$ ——— (i)

case - 2



For equ^n $\Sigma F_H = 0 \Rightarrow F = 220\cos 30°$

$\Sigma F_V = 0 \Rightarrow R = W + 220\sin 30°$

Again we know that $F = \mu R$

$\Rightarrow 220\cos 30° = \mu(W + 220\sin 30°)$ ——— (ii)

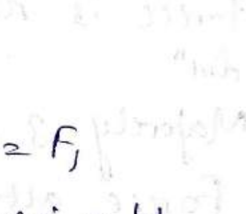
Now equ^n (i) ÷ equ^n (ii) $\Rightarrow \frac{180}{220} = \frac{W - 180\sin 30°}{W + 220\sin 30°}$

$\Rightarrow W = 991.2\ N$

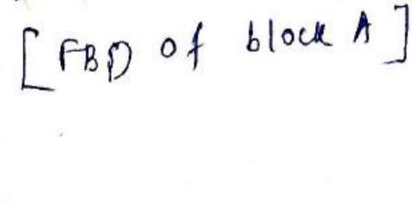
Putting the value of W in equ^n (i)

$180\cos 30° = \mu(991.2 - 180\sin 30°) \Rightarrow \mu = 0.173$ (Ans)

Q-3 Two blocks A & B of weights 1 KN & 2 KN respectively are in equ^m position as shown in the figure. If coeff. of friction between the two blocks as well as block B & floor is 0.3, find the force (P) required to move the block B.



Sol^n



[FBD of block A]

For equ^m

$\Sigma F_H = 0 \Rightarrow T\cos 30° = F_1$

$\Sigma F_V = 0 \Rightarrow R_1 + T\sin 30° = W$

Again $F_1 = \mu R_1$

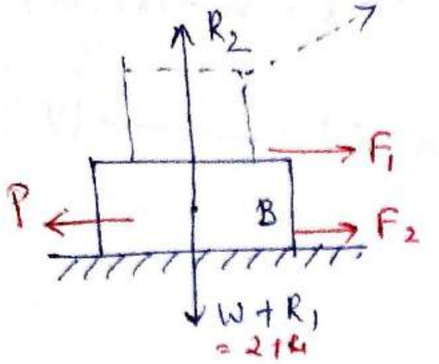
$\Rightarrow T\cos 30° = 0.3(1 - T\sin 30°)$

$\Rightarrow T = $ ____

Now, $R_1 = W - T\sin 30°$

$= 1 - \_\ \sin 30° = 0.85\ KN$

So, $F_1 = \mu R_1 = 0.3 \times 0.85 = 0.255$ KN.



[FBD of block B]

Force equ^m

$\Sigma F_H = 0 \Rightarrow P = F_1 + F_2$

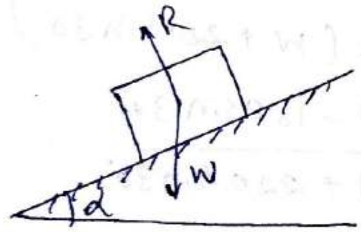
& $\Sigma F_V = 0 \Rightarrow R_2 = 2 + R_1 \Rightarrow R_2 = 2 + 0.85$
$= 2.85$

Again $F_2 = \mu R_2 = 0.3 \times 2.85 = 0.855$ KN

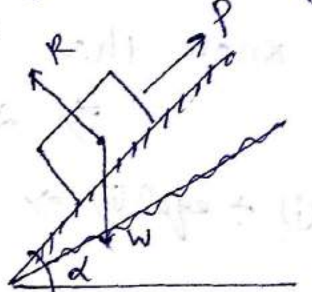
So, $P = F_1 + F_2$
$= 0.255 + 0.855$
$= 1.11$ KN (Ans)

Equilibrium of a body on a rough inclined plane.

consider a body of weight W, lying on a rough plane inclined at α with the horizontal.



Angle of inclination (α) less than angle of friction (ϕ)



Angle of inclination more than angle of friction.

If inclination of plane with horizontal < angle of friction, the body will be automatically in equ^n.
So to move the body either in upward or downward dir^n, force is required to be applied.

If the inclination of plane with horizontal > angle of friction, the body will move down. So an upward force is required to oppose motion of the body in downward dir^n.

Majorly 3 cases for movement of body is there

a) Force acting along the inclined plane
b) Force acting horizontally
c) Force acting at some angle with the inclined plane.

Equilibrium of a body on a rough inclined plane subjected to a force acting along the inclined plane →

Consider a body resting on a rough horizontal plane subjected to force acting along the inclined plane.

Let W = weight of the body

α = Angle of inclined plane with horizontal

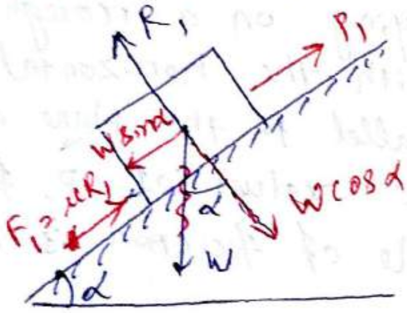
R = Normal Reaⁿ

μ = coeff. of friction

ϕ = Angle of friction $\mu = \tan\phi$

If force is not applied, the body will move downward. There will be two cases

Case-1: Minᵐ force (P_1) which will keep the body in equⁿ, when it starts sliding downwards :→



F_1 will act in upward dirⁿ as body will always move in ↓ dirⁿ if P_1 is not applied to it.

For equᵐ $\Sigma F_H = 0 \Rightarrow P_1 + F_1 = W\sin\alpha$

$\Rightarrow P_1 = W\sin\alpha - \mu R_1$ ——— (i)

& $\Sigma F_V = 0 \Rightarrow R_1 = W\cos\alpha$ ——— (ii)

Now equⁿ (i) $\Rightarrow P_1 = W\sin\alpha - \mu(W\cos\alpha)$

$= W(\sin\alpha - \mu\cos\alpha)$

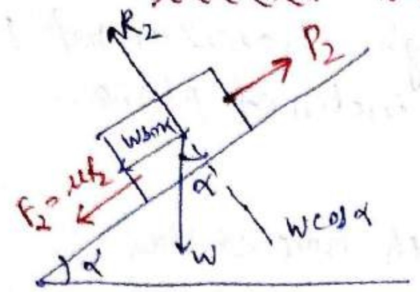
$= W(\sin\alpha - \tan\phi\cos\alpha)$ (∵ multiplying $\cos\phi$ both sides)

$\Rightarrow P_1\cos\phi = W(\sin\alpha\cos\phi - \sin\phi\cos\alpha)$

$\Rightarrow P_1\cos\phi = W\sin(\alpha - \phi)$

$$\Rightarrow P_1 = \frac{W\sin(\alpha-\phi)}{\cos\phi}$$

Case-2 : Max^m force (P_2) which will keep the body in equ^m when it is about to slide upward.



For equ^m $\Sigma F_V = 0 \Rightarrow R_2 = W\cos\alpha$

& $\Sigma F_H = 0 \Rightarrow P_2 = W\sin\alpha + \mu R_2$

$\Rightarrow P_2 = W\sin\alpha + \mu W\cos\alpha$

$\Rightarrow P_2 = W(\sin\alpha + \tan\phi \cos\alpha)$

By multiplying both sides of equ^n by $\cos\phi$

$P_2\cos\phi = W(\sin\alpha\cos\phi + \sin\phi\cos\alpha)$

$\Rightarrow P_2\cos\phi = W\sin(\alpha+\phi)$

$$\Rightarrow \boxed{P_2 = W\frac{\sin(\alpha+\phi)}{\cos\phi}}$$

Q A body of weight 500N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in fig. Determine the min^m & max^m values of P for which the equ^m can exist, if the angle of friction is 20°.

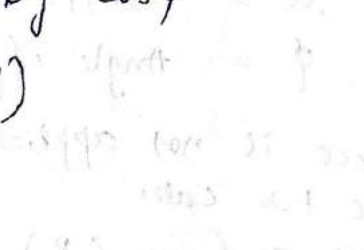
Sol^n Given $W = 500N$

$\alpha = 25°$

$P_{max} = ?$

$P_{min} = ?$

$\phi = 20°$



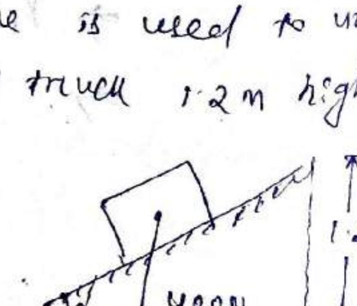
$$P_{min} = \frac{W\sin(\alpha-\phi)}{\cos\phi} = \frac{500\sin(25°-20°)}{\cos 20°} = 46.4N$$

$$P_{max} = \frac{W\sin(\alpha+\phi)}{\cos\phi} = \frac{500\sin(25°+20°)}{\cos 20°} = 376.2N$$

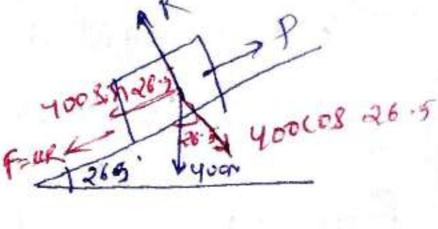
Q An inclined plane as shown in figure is used to unload slowly a body weighing 400N from a truck 1.2m high into the ground.

μ bet^n underside of body & plank is 0.3

State whether it is necessary to push the body down the plane or hold it back from sliding down. What min^m force is required parallel to the plane for this purpose.



Sol^n $\tan\alpha = \frac{1.2}{2.4} \Rightarrow \alpha = 26.5^\circ$



For equ^n $\Sigma F_H = 0 \Rightarrow P = 400\sin 26.5^\circ + F$

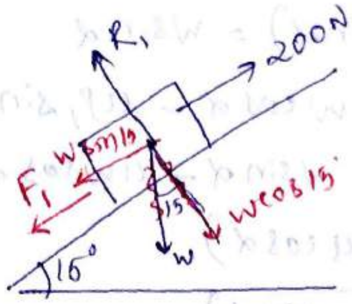
& $\Sigma F_V = 0 \Rightarrow R = 400\cos 26.5^\circ$
$= 357.9\ N$

Now $P = 400\sin 26.5^\circ + \mu R$
$= 400\sin 26.5^\circ + (0.3 \times 357.9) = 71.2\ N$

Q An effort of 200N is required just to move a certain body up an inclined plane of angle 15° the force acting parallel to the plane. If the angle of inclination of the plane is made 20° the effort required, again applied parallel to the plane, is found to be 230 N. Find the weight of the body & μ.

Given $P_1 = 200N$ $P_2 = 230N$
$\alpha_1 = 15^\circ$ $\alpha_2 = 20^\circ$

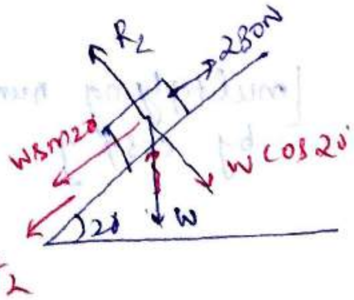
Sol^n



For equ^n $\Sigma F_H = 0$
$\Rightarrow F_1 + W\sin 15^\circ = 200 \Rightarrow \mu R_1 + W\sin 15^\circ = 200 \Rightarrow \mu W\cos 15^\circ + W\sin 15^\circ = 200$

& $\Sigma F_V = 0 \Rightarrow R_1 = W\cos 15$

$\Rightarrow W(\mu\cos 15^\circ + \sin 15^\circ) = 200$ —— (i)



For equ^n $\Sigma F_H = 0$
$\Rightarrow F_2 + W\sin 20^\circ = 230$

& $\Sigma F_V = 0$
$\Rightarrow R_2 = W\cos 20^\circ$

$\Rightarrow \mu W\cos 20^\circ + W\sin 20^\circ = 230$
$\Rightarrow W(\mu\cos 20^\circ + \sin 20^\circ) = 230$ —— (ii)

$\frac{\text{equ}^n\text{ (ii)}}{\text{equ}^n\text{ (i)}} \Rightarrow \frac{\mu\cos 20^\circ + \sin 20^\circ}{\mu\cos 15^\circ + \sin 15^\circ} = \frac{230}{200}$

$\Rightarrow \mu = 0.259$

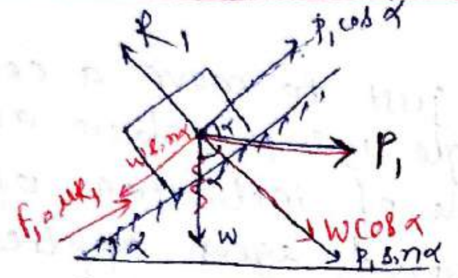
& Now eqn (i) becomes

$$W(0.259 \cos 15^\circ + \sin 15^\circ) = 200$$

$$\Rightarrow W = 392 \cdot N \text{ (Ans)}$$

Equilibrium of a body on a rough inclined plane subjected to a force acting horizontally :→

1> Minm force (P_1) which will keep the body in eqn, when it is at the point of sliding downwards →



body at the point of sliding downwards

For eqn $\Sigma F_H = 0 \Rightarrow P_1 \cos\alpha + \mu R_1 = W\sin\alpha$

& $\Sigma F_V = 0 \Rightarrow R_1 = W\cos\alpha + P_1 \sin\alpha$

Now, $P_1 \cos\alpha + \mu(W\cos\alpha + P_1 \sin\alpha) = W\sin\alpha$

$$\Rightarrow P_1 \cos\alpha = W\sin\alpha - \mu W\cos\alpha - \mu P_1 \sin\alpha$$

$$\Rightarrow P_1(\cos\alpha + \mu\sin\alpha) = W\sin\alpha - \mu W\cos\alpha$$

$$\Rightarrow P_1 = \frac{W(\sin\alpha - \mu\cos\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

Again $\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$

$$\text{So, } P_1 = \frac{W\left(\sin\alpha - \frac{\sin\phi}{\cos\phi}\cos\alpha\right)}{\left(\cos\alpha + \frac{\sin\phi}{\cos\phi}\sin\alpha\right)}$$

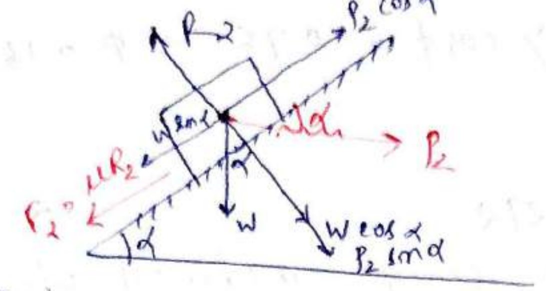
$$= W\frac{\sin\alpha\cos\phi - \sin\phi\cos\alpha}{\cos\alpha\cos\phi + \sin\alpha\sin\phi}$$ [multiplying num & deno by $\cos\phi$]

$$= W\frac{\sin(\alpha - \phi)}{\cos(\alpha - \phi)}$$

$$= W\tan(\alpha - \phi)$$

$$\boxed{P_1 = W\tan(\phi - \alpha)}$$

2) Max^m force (P_2) which will keep the body in equ^n, when it is at the point of sliding upwards →



for equ^m $\Sigma F_V = 0 \Rightarrow R_2 = W\cos\alpha + P_2 \sin\alpha$

& $\Sigma F_H = 0 \Rightarrow P_2 \cos\alpha = \mu R_2 + W\sin\alpha$

$\Rightarrow P_2 \cos\alpha = \mu(W\cos\alpha + P_2 \sin\alpha) + W\sin\alpha$

$\Rightarrow P_2(\cos\alpha - \mu\sin\alpha) = W\sin\alpha + \mu W\cos\alpha$

$$\Rightarrow P_2 = \frac{W\sin\alpha + \mu W\cos\alpha}{\cos\alpha - \mu\sin\alpha}$$

Again $\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$

$$\text{so, } P_2 = \frac{W(\sin\alpha \cdot \cos\phi + \sin\phi\cos\alpha)}{\cos\alpha\cos\phi - \sin\phi\sin\alpha}$$

$$= W\frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)} = W\tan(\alpha+\phi)$$

$$\Rightarrow \boxed{P_2 = W\tan(\alpha+\phi)}$$

Q.1 An object of weight 100N is kept in position on a plane inclined at 30° to the horizontal by a horizontally applied force (F). If $\mu = 0.25$, determine min^m force (F).

Sol^n Given $W = 100N$, $\alpha = 30°$, $\mu = 0.25$ & $\tan\phi = .25 \Rightarrow \phi = 14°$

$F = W\tan(\alpha - \phi)$

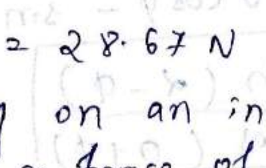
$= 100\tan(30° - 14) = 28.67\ N$

Q.2 A load of 1.5KN, resting on an inclined rough plane, can be moved up the plane by a force of 2KN applied horizontally or by a force of 1.25 KN applied parallel to the plane. Find the inclination of the plane & μ.

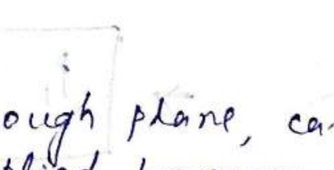
from ① $P = W\tan(\alpha+\phi)$

$\Rightarrow 2 = 1.5\tan(\alpha+\phi)$

$\Rightarrow \alpha+\phi = 53.1°$



②



①

from ⑤ $P = W \dfrac{\sin(\alpha+\phi)}{\cos\phi}$

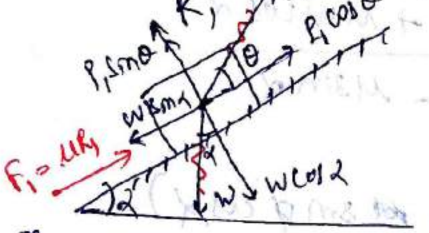
$\Rightarrow 1.25 = 1.5 \dfrac{\sin 53.1}{\cos\phi} \Rightarrow \cos\phi = 0.96 \Rightarrow \phi = 16.3^\circ$

$\alpha = 53.1^\circ - 16.3^\circ = 36.8^\circ$

Again $\mu = \tan\phi = \tan 16.3^\circ = 0.292$

Equilibrium of a body on a rough inclined plane subjected to a force acting at some angle with the inclined plane →

1) Min^m force (P_1) which will keep the body in equ^m when it is at the point of sliding downwards



For equ^m $\Sigma F_V = 0 \Rightarrow R_1 + P_1\sin\theta = W\cos\alpha \Rightarrow R_1 = W\cos\alpha - P_1\sin\theta$

& $\Sigma F_H = 0 \Rightarrow \mu R_1 + P_1\cos\theta = W\sin\alpha$

$\Rightarrow \mu(W\cos\alpha - P_1\sin\theta) + P_1\cos\theta = W\sin\alpha$

$\Rightarrow P_1(\cos\theta - \mu\sin\theta) = W\sin\alpha - \mu W\cos\alpha$

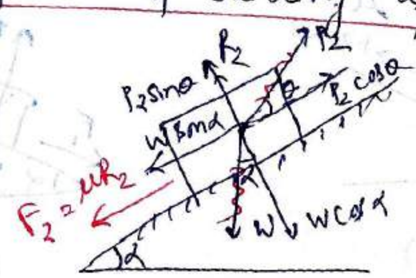
$\Rightarrow P_1 = \dfrac{W(\sin\alpha - \mu\cos\alpha)}{(\cos\theta - \mu\sin\theta)}$

Again $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

Now, $P_1 = \dfrac{W(\sin\alpha\cos\phi - \sin\phi\cos\alpha)}{\cos\theta\cos\phi - \sin\phi\sin\theta}$

$\Rightarrow \boxed{P_1 = W\dfrac{\sin(\alpha-\phi)}{\cos(\theta+\phi)}}$

2) Max^m force (P_2) which will keep the body in equ^m when it is at the point of sliding upwards



For equ^m $\Sigma F_V = 0 \Rightarrow R_2 + P_2 \sin\theta = W\cos\alpha$

& $\Sigma F_H = 0 \Rightarrow P_2\cos\theta = W\sin\alpha + \mu R_2$

$\Rightarrow P_2\cos\theta = W\sin\alpha + \mu(W\cos\alpha - P_2\sin\theta)$

$\Rightarrow P_2(\cos\theta + \mu\sin\theta) = W\sin\alpha + \mu W\cos\alpha$

$$\Rightarrow P_2 = \frac{W(\sin\alpha + \mu\cos\alpha)}{(\cos\theta + \mu\sin\theta)}$$

Again $\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$

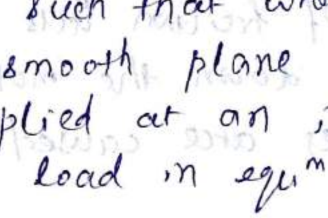
$$P_2 = \frac{W(\sin\alpha\cos\phi + \sin\phi\cos\alpha)}{\cos\theta\cos\phi + \sin\phi\sin\theta}$$

$$= W\frac{\sin(\alpha+\phi)}{\cancel{\cos(\phi-\theta)}\ \cos(\theta-\phi)} \Rightarrow \boxed{P_2 = \frac{W\sin(\alpha+\phi)}{\cos(\theta-\phi)}}$$

Q Find the force required to move a load of 300N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at the same angle, a force of 60N applied at an inclination of 30° to the plane, keeps the same load in equ^m. μ between rough plane & load is 0.3

Sol^n

(Smooth plane)



(Rough plane)

For smooth plane $\mu = 0 \Rightarrow \phi = 0$

$$P = W\frac{\sin(\alpha+\phi)}{\cos(\theta-\phi)} \Rightarrow 60 = \frac{300\sin\alpha}{\cos 30°} \Rightarrow \alpha = 10°$$

For rough plane $\mu = 0.3 \Rightarrow \phi = 16.7°$ & $\alpha = 10$

$$P = \frac{W\sin(\alpha+\phi)}{\cos\phi\cancel{(\theta-\phi)}} = \frac{300\sin(10+16.7)}{\cos 16.7°} = 140.7\text{ N}$$

Advantages of friction →

→ It is responsible for many types of motion
→ It helps us to walk on the ground/floor.
→ Breakes are applied in automobile to stop or reduce its motion by using friction principle.
→ Asteroids are burnt in the atmosphere before reaching earth due to friction.
→ It helps in generation of heat when we rub our hands
→ we can write on paper or board.
→ we can not fix nail in the wall or wood if there is no friction. (as it holds the nail)
→ Dragging of atmosphere with earth is possible.

Disadvantages of friction →

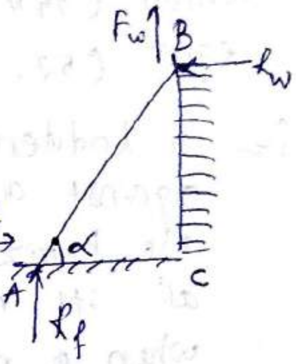
→ It produces unnecessary heat which causes wastage of energy
→ The force of friction acts in opposite dirⁿ of motion. So it slows down the motion of moving objects.
→ Forest fires are caused due to friction between tree branches.
→ A lot of money are used to prevent friction & usual wear & tear caused by it by using techniques like greasing & oiling.
→ Due to friction have to exert more power in machines.
→ " noise is produced in machine.
→ " engine of automobile consume more fuel.
→ m/c $\eta \downarrow$.
→

Applications of friction →

It is used in
1> Ladder friction
2> wedge "
3> Screw "

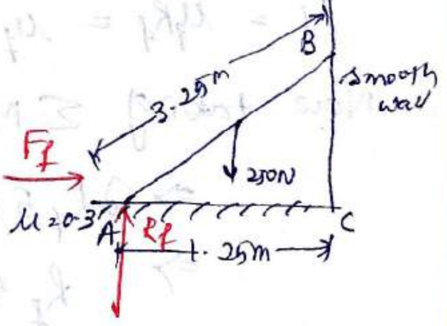
Ladder friction →

It is a device used for climbing on roofs or walls.
It consists of 2 long uprights of wood, iron or rope connected by a number of cross pieces called rungs used as or steps.



consider a ladder AB resting on the rough ground & leaning against a wall.
As upper end of the ladder tend to slip downwards, so frictional force (F_w) will act in upward dir^n.
As lower end of the ladder tends to move left, so frictional force (F_f) will act along right.
The system to be in equ^m, algebraic sum of all horizontal & vertical components of forces will be zero
i.e. $\Sigma F_H = 0$ & $\Sigma F_V = 0$.

Q A uniform ladder of length 3.25 m & weighing 250N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. μ betn ladder & floor is 0.3. What is the frictional force acting on the ladder at the point of contact bet^n ladder & the floor? Show that the ladder will remain in equ^m in this position.



Sol^n
$l = 3.25$ m
$W = 250$ N
$AC = 1.25$ m
$\mu_f = 0.3$

F_f → frictional force acting betn ladder & floor
R_f → normal rean at the floor.
As wall is smooth. So no frictional force acting on wall.

$R_f = 250$ N

from the figure $BC = \sqrt{AB^2 - AC^2} = \sqrt{3.25^2 - 1.25^2} = 3m$

Now $\Sigma M_B = 0$

$\Rightarrow R_f(1.25) - F_f \, 3 - 250\left(\frac{1.25}{2}\right) = 0$

$\Rightarrow 1.25 R_f = 3F_f + (0.625 \times 250)$

$\Rightarrow 1.25 \times 250 = 3F_f + (0.625 \times 250)$

$\Rightarrow F_f = 52.1\,N$

For equ^m of the ladder,

max^m force of friction available at the point of contact bet^n ladder & floor is $= \mu R_f = 0.3 \times 250 = 75\,N$

So the amount of force of friction available at the point of contact (75 N) is more than the force of friction required for equ^m (52.1N). So ladder will remain in equ^m cond^n.

Q A ladder 5m long rests on a horizontal ground & leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900N & acts at its middle. The ladder is at the point of sliding when a man weighing 750N stands on a rung 1.5 m from the bottom of the ladder. Find μ bet^n ladder & floor.

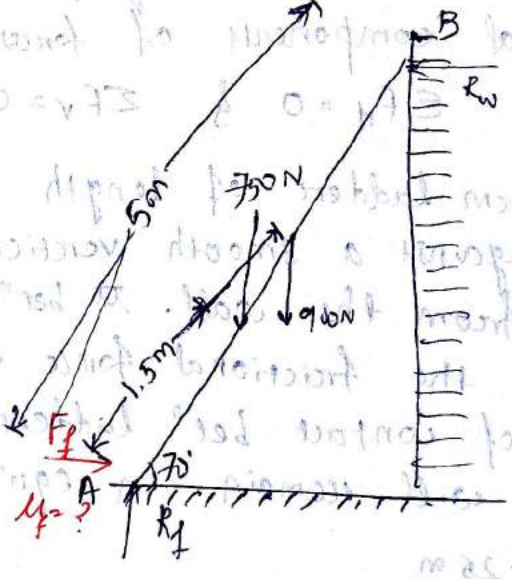
Sol^n Given $l = 5m$

$\alpha = 70°$

$W_1 = 900N$

$W_2 = 750N$

$= 1.5m$



For equ^m $\Sigma F_v = 0$

$\Rightarrow R_f = 750 + 900$

$= 1650\,N$

$F_f = \mu_f R_f = \mu_f (1650)$

Now taking $\Sigma M_B = 0$

$\Rightarrow R_f\, 5\cos 70° = 900(2.5\cos 70°) + 750(3.5\cos 70°) + F_f 5\sin 70°$

$\Rightarrow R_f\, 5\cos 70 = (900 \times 2.5\cos 70°) + (750 \times 3.5\cos 70°) + (F_f 5\sin 70)$

$\Rightarrow 1650 \times 5\cos 70° = \quad " \quad + \quad " \quad + (\mu_f 1650 \times 5\sin 70)$

$\Rightarrow \mu_f = 0.15$

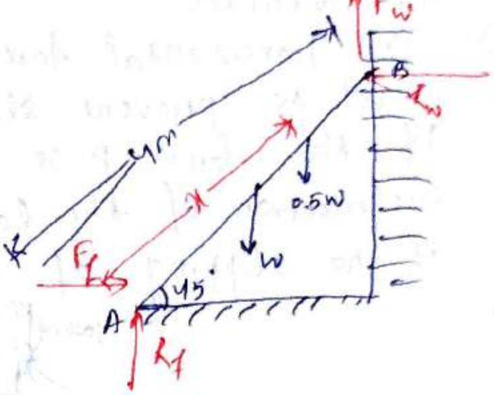
Q. A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45°. μ betn ladder & wall is 0.4 & that betn ladder & floor is 0.5. If a man whose weight is $\frac{1}{2}$ that of ladder, ascends it, how high will it be when the ladder slips.

Sol' Given $l = 4m$

$\alpha = 45°$

$\mu_w = 0.4$

$\mu_f = 0.5$



when ladder is at its point of slipping,

W = weight of ladder

R_f = Normal reacn at floor

weight of man $= \frac{W}{2} = 0.5W$

frictional force at the floor $= F_f = \mu_f R_f = 0.5 R_f$

" " " wall $= F_W = \mu_w R_w = 0.4 R_w$

For equm $\Sigma F_V = 0$

$\Rightarrow R_f + F_W = W + 0.5W$

$\Rightarrow R_f + F_W = 1.5W$ ——— ①

$\Sigma F_H = 0$

$\Rightarrow F_f = R_W = 0.5 R_f \Rightarrow R_f = 2R_W$

Now equn ① becomes $2R_W + F_W = 1.5W$

$\Rightarrow 2R_W + 0.4R_W = 1.5W \Rightarrow R_W = 0.625W$

$F_W = 0.4 \times 0.625W = 0.25W$

Now taking $\Sigma M_A = 0$

$\Rightarrow F_W(4\cos 45°) + R_W(4\sin 45°) - W(2\cos 45°) - 0.5W(x\cos 45°) = 0$

$\Rightarrow 0.5R_W 4\cos 45° + (0.625W \times 4\sin 45°) = 2W\cos 45° + 0.5W x \cos 45°$

$\Rightarrow (0.5 \times 0.625W \times 4\cos 45°) + (0.625W \times 4\sin 45°) =$ "

$\Rightarrow x = 3m$

Chapter-4

Centroid & Moment of Inertia

Centre of gravity →

The point, through which the total weight of the body acts is called centre of gravity or C.G.
Every body has only one C.G. It depends on the shape of the body. It is the mid point of 3D object.

Centroid →

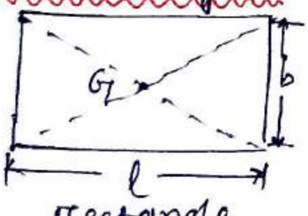
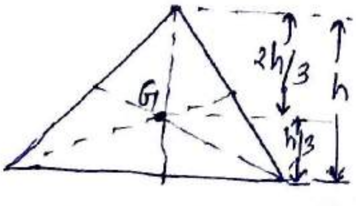
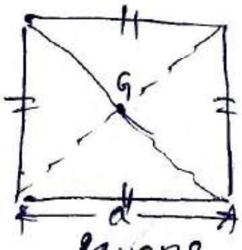
The centre of area of plane figures like triangle, rectangle, circle, trapezium etc is called centroid.
It is the middle or centre point of two dimensional object.

Methods for finding centroid of an object →

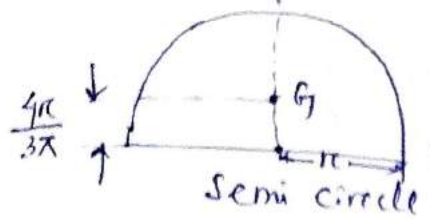
It can by found by following ways

i) from the geometry of the object
ii) By moments
iii) By graphical method.

i) C.G from geometry of the object :→

SL NO	Figure	C.G	Where C.G lies
	Plane Area figures →		
1.	 rectangle	$\frac{l}{2}, \frac{b}{2}$	point where two diagonal meets or at middle point of length & breadth
2.	 Triangle		At intersection point of 3 medians. ↓ line connecting vertex & middle point of opposite side.
3	 Square	$\frac{a}{2}, \frac{a}{2}$	at Intersection point of two diagonals.

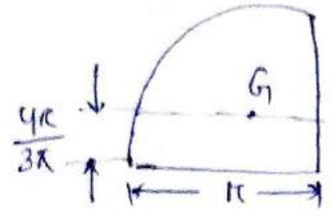
4.



Semi circle

At a distance of $\frac{4r}{3\pi}$ from the base

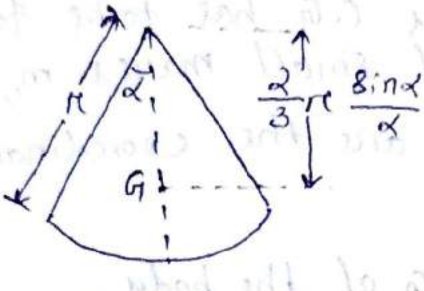
5.



Quarter-circle

At a distance of $\frac{4r}{3\pi}$ from its base.

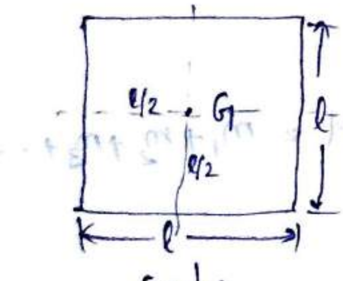
6.



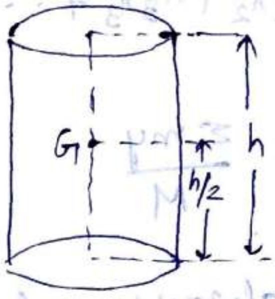
circular sector

At a distance of $\frac{2r}{3}\frac{\sin\alpha}{\alpha}$ from centre.

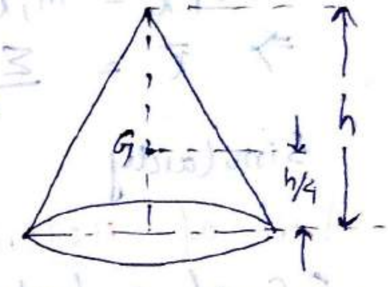
Solid figures →



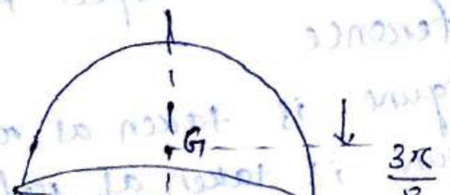
Cube



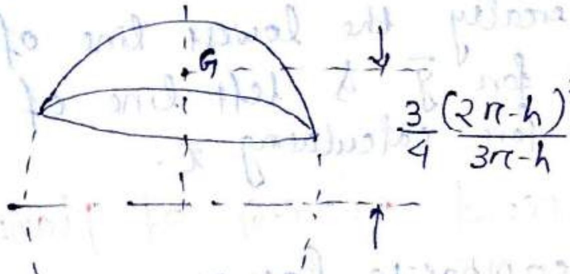
cylinder



Right circular cone

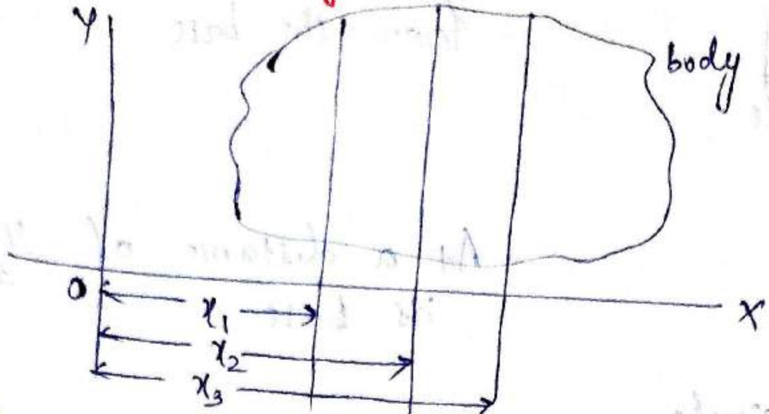


Hemisphere



Segment of a sphere

ii) C.G or centroid by moments →



Consider a body of mass M & its C.G has to be found out. Now dividing the body into nos. of small masses $m_1, m_2, m_3 \ldots$ & $(x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots$ are the coordinates of the C.G.s from the fixed point O.

Let $\bar{x}$ = X-coordinate of C.G of the body

$\bar{y}$ = Y- " "

Now from the principle of moments

$$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + \ldots$$

$$\Rightarrow \bar{x} = \frac{\Sigma mx}{M}$$

Similarly $\bar{y} = \frac{\Sigma my}{M}$ where $M = m_1 + m_2 + m_3 + \ldots$

Axis of reference →

C.G of a body is always calculated with respect to some assumed axis called as axis of reference. Generally the lowest line of the figure is taken as reference line for $\bar{y}$ & left line of the figure is taken as reference line for calculating $\bar{x}$.

Centroid or C.G of plane figure.

In composite figures

Let $\bar{x}$ & $\bar{y}$ are the coordinates of C.G w.r.t some reference axis then

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

$$\& \ \bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

where $a_1, a_2, a_3 \ldots$ → Areas of divided figures from main figure

$x_1, x_2, x_3 \cdots \rightarrow$ x coordinates of divided figures from reference axis

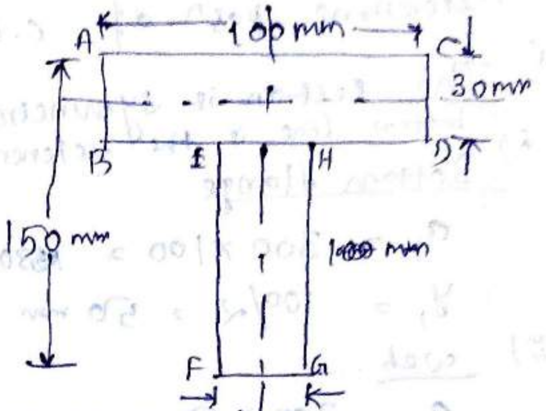
$y_1, y_2, y_3, \cdots \rightarrow$ y coordinates " "

C.G of Symmetrical Sections →

As the figure is symmetrical about one axis, so we have to calculate either $\bar{x}$ or $\bar{y}$ here. Here C.G will lie on the axis of symmetry.

Q. Find the C.G of a 100mm x 150mm x 30mm T-section

Sol^n The section is symmetrical about Y-axis. (bisecting the web)



Now Dividing the section into ▭ ABCD & ▯ EFGH & taking FG line as reference.

▭ ABCD

$a_1 = 100 \times 30 = 3000 \text{ mm}^2$

$y_1 = 150 - \frac{30}{2} = 135 \text{ mm}$

▭ EFGH

$a_2 = 120 \times 30 = 3600 \text{ mm}^2$

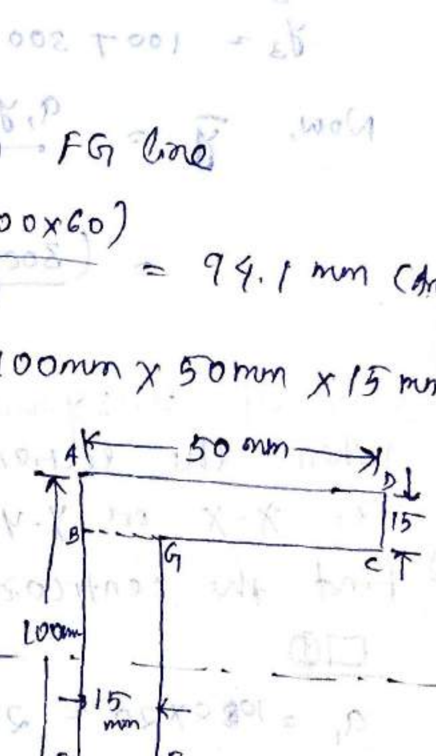
$y_2 = \frac{120}{2} = 60 \text{ mm}$

distance of CG of the T-section from FG line

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} = 94.1 \text{ mm (Ans)}$$

Q-2 Find the CG of a channel section 100mm x 50mm x 15 mm

Sol^n The section is symmetrical about X-axis.



▭ ABCD

$a_1 = 50 \times 15 = 750 \text{ mm}^2$

$x_1 = 50/2 = 25 \text{ mm}$

▯ BGFE

$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$

$x_2 = 15/2 = 7.5 \text{ mm}$

▭ EJIH

$a_3 = 50 \times 15 = 750 \text{ mm}^2$

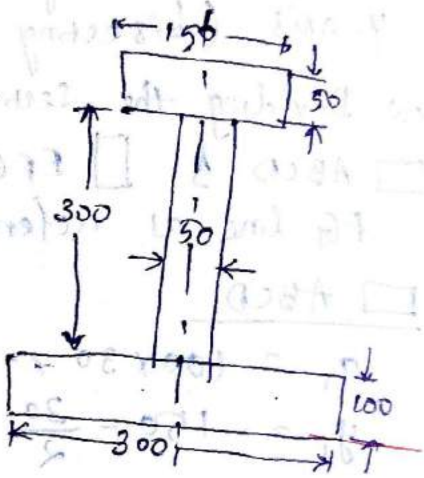
$x_3 = 50/2 = 25 \text{ mm}$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm}$$

Q. An I section has the following dimensions in mm
bottom flange = 300 × 100
Top flange = 150 × 50
web = 300 × 50
Determine posn of C.G of the section.

Sol. The section is symmetrical about y axis.



i) bottom line is the reference line
Bottom flange

$a_1 = 300 \times 100 = 30000 \text{ mm}^2$

$y_1 = 100/2 = 50 \text{ mm}$

ii) web

$a_2 = 300 \times 50 = 15000 \text{ mm}^2$

$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$

iii) Top flange

$a_3 = 150 \times 50 = 7500 \text{ mm}^2$

$y_3 = 100 + 300 + 50/2 = 425 \text{ mm}$

Now, $\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$

$$= \frac{(30000 \times 50) + (15000 \times 250) + (7500 \times 425)}{30000 + 15000 + 7500} = 160.7 \text{ mm}$$

C.G of unsymmetrical sections :→

When the section is not symmetric about any axis i.e. X-X or Y-Y, then both $\bar{x}$ & $\bar{y}$ have to be found out

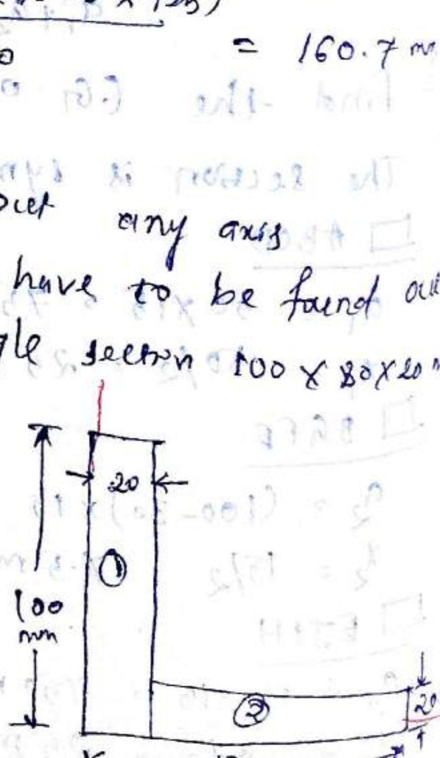
Q. Find the centroid of an unequal angle section 100 × 80 × 20 mm

Sol. □①

$a_1 = 100 \times 20 = 2000 \text{ mm}^2$

$x_1 = 20/2 = 10 \text{ mm}$

$y_1 = 100/2 = 50 \text{ mm}$



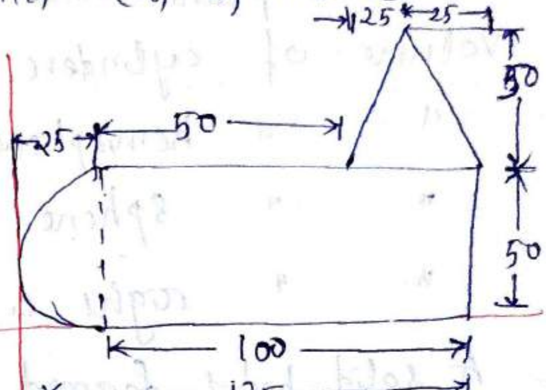
$a_2 = (80 - 20) \times 20 = 1200\, mm^2$

$y_2 = 20/2 = 10\, mm \qquad x_2 = 80/2 = 40\, mm$

Now, $\bar{x} = \dfrac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \dfrac{(2000 \times 10) + (1200 \times 40)}{2000 + 1200} = 25\, mm$

$\bar{y} = \dfrac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \dfrac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35\, mm$

Q.2 A uniform lamina as shown in the fig. consists of a rectangle, a circle & a triangle. Find C.G of the lamina. All dimensions are in mm.



Sol Section is not symmetrical about any axis.

a) Rectangular portion

$a_1 = 100 \times 50 = 5000\, mm^2$

$x_1 = 25 + 100/2 = 75\, mm$

$y_1 = 50/2 = 25\, mm$

b) Semi-circular portion

$a_2 = \dfrac{\pi r^2}{2} = \dfrac{\pi \times 25^2}{2} = 982\, mm^2$

$x_2 = 25 - \dfrac{4r}{3\pi} = 25 - \dfrac{4 \times 25}{3\pi} = 14.4\, mm$

$y_2 = 50/2 = 25\, mm$

c) Triangular portion

$a_3 = \dfrac{1}{2} \times 50 \times 50 = 1250\, mm^2$

$x_3 = 25 + 50 + 25 = 100\, mm$

$y_3 = 50 + \dfrac{50}{3} = 66.7\, mm$

Now, $\bar{x} = \dfrac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \dfrac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1280}$

$= 71.7\, mm$

$\bar{y} = \dfrac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \dfrac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250}$

$= 32.2\, mm$

Moment of Inertia :→

MI of an object is defined by the distribution of mass around an axis.

moment = force × ⊥ distance.

If this moment is again multiplied by ⊥ distance between the point & line of action of the force then it is called moment of the moment of a force or 2nd moment of area force or moment of inertia.

M.I is of 2 types

		Unit
1) mass moment of Inertia		$Kg\text{-}m^2$
2) Area " "		m^4 or mm^4

MI of a plane area →

consider a plane area, whose MI has to be found out.
Dividing the whole area into a no. of small elements.

Let a_1, a_2, a_3 = Areas of small elements

r_1, r_2, r_3 = distance of elements from the line about which MI will be calculated.

Now $I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \cdots$

$= \Sigma a r^2$

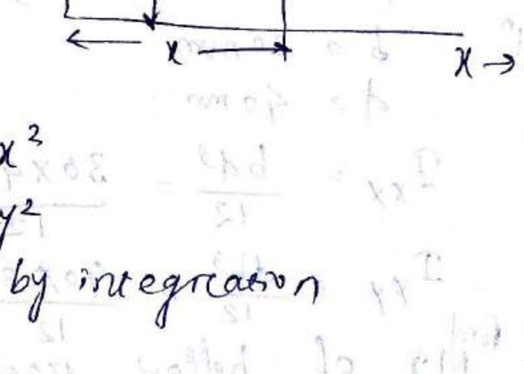
Methods for finding MI :→

1) By Routh's rule.
2) " Integration.

MI by integration →

Dividing the body into small no. of strips & considering 1 strip.

Let dA = Area of the strip



x = distance of C.G of the strip on X-X axis

y = " " " Y-Y axis.

MI of the strip about Y-Y axis $= dA \cdot x^2$

" " " XX " $= dA \cdot y^2$

MI of the whole body can be found by integration

$I_{YY} = \Sigma dA \cdot x^2$

& $I_{XX} = \Sigma dA \cdot y^2$

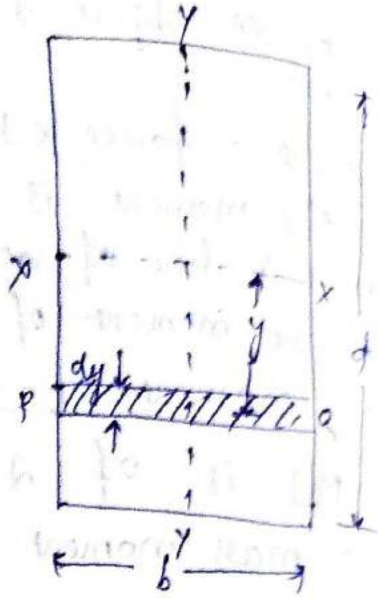
MI of a rectangular section :→

consider a rectangular section

Let b = width of the section

d = depth " "

Now considering a small strip PQ of thickness dy at a distance of y from X-X axis.



Area of strip = b dy

MI of the strip about X-X axis = $(b\,dy) \times y^2$

$= b y^2 dy$

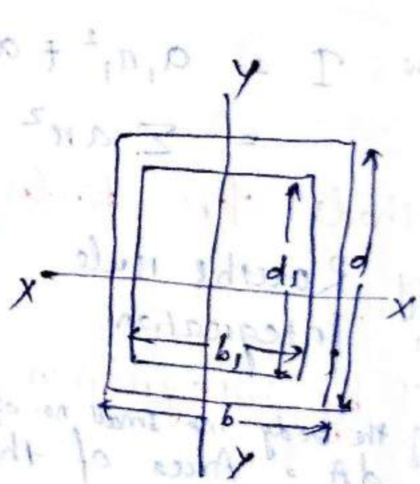
Now MI of the whole section will be

$$I_{XX} = \int_{-d/2}^{d/2} b y^2 dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2} = \frac{b}{3}\left(\frac{d^3}{8} + \frac{d^3}{8}\right) = \frac{b\,\cancel{2}d^3}{\cancel{24}_{12}} = \frac{bd^3}{12}$$

Similarly $\boxed{I_{YY} = \frac{db^3}{12}}$ $\qquad$ $\boxed{I_{XX} = \frac{bd^3}{12}}$

MI of hollow rectangular section →

MI of hollow rectangular section about XX axis



$$I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

& $$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$$

Q find MI of a rectangular section 30 mm wide & 40 mm deep about X-X & Y-Y axis.

Solⁿ $b = 30$ mm

$d = 40$ mm

$$I_{XX} = \frac{bd^3}{12} = \frac{30 \times 40^3}{12} = 160 \times 10^3 \text{ mm}^4$$

$$I_{YY} = \frac{db^3}{12} = \frac{40 \times 30^3}{12} = 90 \times 10^3 \text{ mm}^4$$

Q find MI of hollow rectangular section about its C.G external b & d are 50 mm & 80 mm respectively & internal " 30 mm & 40 mm "

Given $b = 60$ mm $\quad b_1 = 30$ mm

$d = 80$ mm $\quad d_1 = 40$ mm

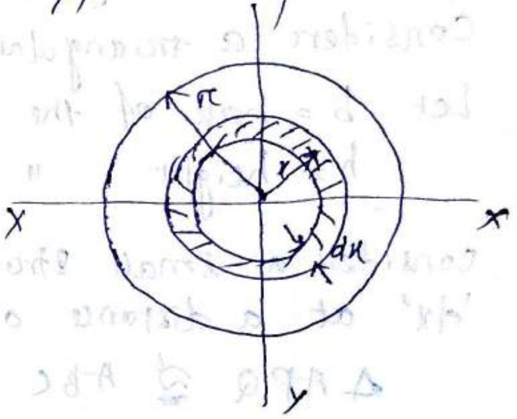
$$I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12} = \frac{60\times80^3}{12} - \frac{30\times40^3}{12} = 2400\times10^3 \text{ mm}^4$$

$$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12} = \frac{80\times60^3}{12} - \frac{40\times30^3}{12} = 1350\times10^3 \text{ mm}^4$$

Perpendicular axis theorem →

States that "If I_{XX} & I_{YY} be the MI of a plane section about two ⊥ axis meeting at O, MI I_{ZZ} about the axis Z-Z, ⊥ to the plane & passing through the intersection of X-X & Y-Y is given by $I_{ZZ} = I_{XX} + I_{YY}$.

MI of a circular section →



Consider a circle of radius r with centre as O.

Consider an elementary ring of radius x at and of thickness dx.

$dA = 2\pi x\,dx$

MI of the ring about X-X or Y-Y axis.

$= \text{Area} \times (\text{distance})^2$

$= 2\pi x\,dx \times x^2$

$= 2\pi x^3\,dx$

Now MI of the whole section $= I_{ZZ} = \int_0^r 2\pi x^3\,dx$

$$= 2\pi\left[\frac{x^4}{4}\right]_0^r = \frac{2\pi}{4}(r^4 - 0^4) = \frac{\pi}{2}r^4$$

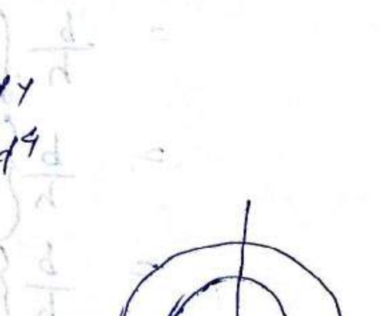
As $r = \frac{d}{2}$

$$I_{ZZ} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{2\times16} = \frac{\pi}{32}d^4$$

from ⊥ axis theorem $\quad I_{ZZ} = I_{XX} + I_{YY}$

so, $I_{XX} = I_{YY} = \frac{\pi}{64}d^4$

MI of hollow circular section →



$$I_{XX} = \frac{\pi}{64}(D^4 - d^4)$$

$$I_{YY} = \frac{\pi}{64}(D^4 - d^4)$$

Parallel axis theorem →

It states that "If the MI of a plane area about an axis through its CG is denoted by I_G the MI of area about any other axis AB, parallel to the first & at a distance of h from the CG is given by $I_{AB} = I_G + ah^2$

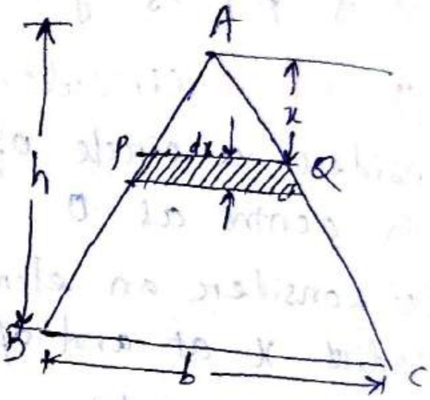
where $I_{AB} \rightarrow$ MI of the area about axis AB

$I_G \rightarrow$ " " its C.G

$a \rightarrow$ area of the section

$h \rightarrow$ distance betn C.G of setion & axis AB.

MI of triangular section →



Consider a triangular section ABC.

Let b = base of the Δ section

h = height " "

Consider a small strip of thickness 'dx' at a distance of x from vertex A.

$\Delta APQ \cong ABC$ (similar)

so, $\frac{PQ}{BC} = \frac{x}{h} \Rightarrow PQ = \frac{BC\, x}{h} = \frac{bx}{h}$

Area of strip PQ $= \frac{bx}{h} \times dx$

MI of the strip about base BC = Area × distance^2

$= \frac{bx}{h} dx\, (h-x)^2$

$= \frac{bx}{h} (h-x)^2 dx$

MI of the whole section

$$I_{BC} = \int_0^h \frac{bx}{h} (h-x)^2 dx$$

$$= \frac{b}{h} \int_0^h x(h^2 + x^2 - 2hx)\, dx$$

$$= \frac{b}{h} \left\{ \left[\frac{x^2}{2} h^2\right]_0^h + \left[\frac{x^4}{4}\right]_0^h - 2h\left[\frac{x^3}{3}\right]_0^h \right\}$$

$$= \frac{b}{h} \left\{ \frac{h^4}{2} + \frac{h^4}{4} - \frac{2h^4}{3} \right\}$$

$$= \frac{b}{h} \left\{ \frac{2h^4}{4} - \frac{2h^4}{3} \right\} = \frac{2h^4 b}{h} \left(\frac{1}{4} - \frac{1}{3}\right) = \frac{2h^4 b}{h} \left(\frac{3-4}{12}\right)$$

$= \frac{b}{h}\left(\frac{6h^4+3h^4-8h^4}{12}\right) = \frac{b}{\cancel{h}}\frac{h^{\cancel{4}3}}{12} = \frac{bh^3}{12}$

distance between C.G of Δ & base is $= \frac{h}{3}$

So, M.I of the Δ section about an axis through its CG & parallel to X-X axis,

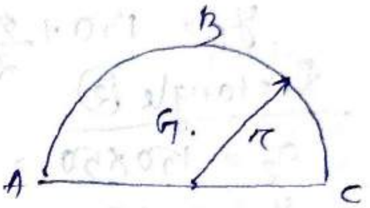
$I_{xx} = I_G + ah^2$

$\Rightarrow I_G = I_{xx} - ah^2$

$I_G = I_{BC} - ad^2$

$I_G = \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$

MI of Semi-circular section →



$I = \frac{1}{2} \times \frac{\pi}{64} d^4$

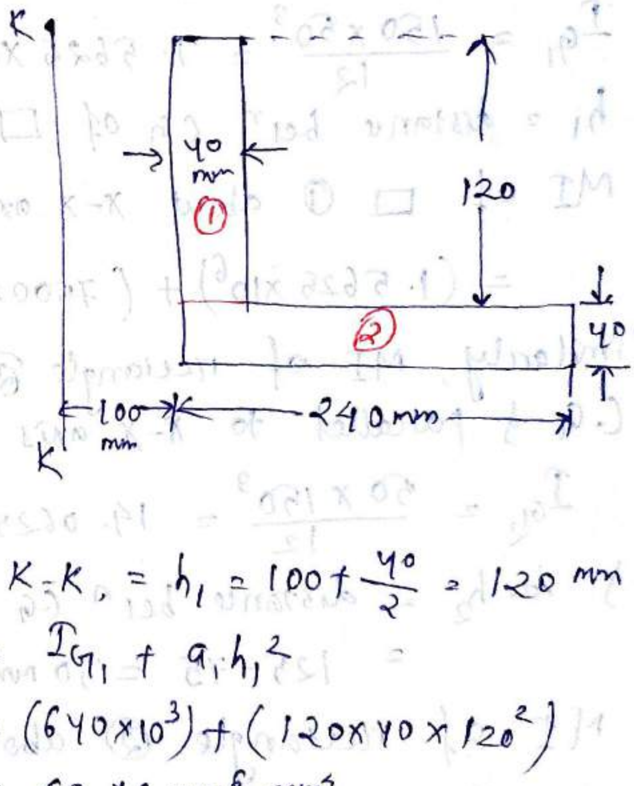
MI of ◠ about its base

$A = \frac{\pi r^2}{2} \qquad h = \frac{4r}{3\pi}$

MI of ◠ about C.G $= I_G = I_{AC} - ah^2$

$= \frac{\pi}{32} d^4 - \frac{\pi r^2}{2} \times \left(\frac{4r}{3\pi}\right)^2$

Q. Calculate MI of the given figure about K-K axis.



Sol: Dividing it into 2 figures

No need to find C.G of the 2 areas as MI will be found about K-K axis.

$I_{G_1} = \frac{40^3 \times 120}{12} = 640 \times 10^3 \text{ mm}^4$

Distance betn CG of section (1) & K-K $= h_1 = 100 + \frac{40}{2} = 120$ mm

MI of section (1) about K-K $= I_{G_1} + a_1 h_1^2$

$= (640 \times 10^3) + (120 \times 40 \times 120^2)$

$= 69.76 \times 10^6 \text{ mm}^4$

$I_{G_2} = \frac{40 \times 240^3}{12} = 46.08 \times 10^6 \text{ mm}^4$

Distance betn CG of section (2) & K-K $= h_2 = 100 + \frac{240}{2} = 220$ mm

MI of section (2) about K-K $= I_{G_2} + a_2 h_2^2$

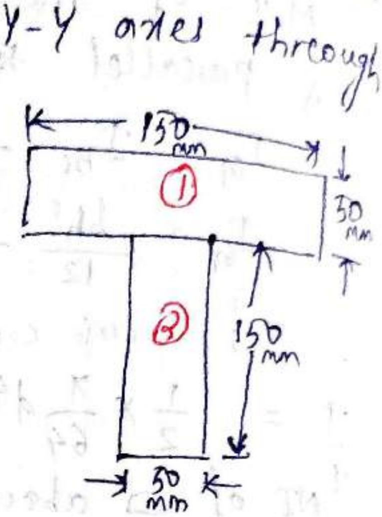
$= (46.08 \times 10^6) + (240 \times 40 \times 220^2)$

$= 510.72 \times 10^6 \text{ mm}^4$

MI of the whole area about K-K

$I_{KK} = (69.76\times10^6) + (510.72\times10^6) = 580.48\times10^6\ mm^4$

Q.2 Find MI of a T-section with flange as 150 mm × 50 mm & web as 150 mm × 50 mm about X-X & Y-Y axes through the C.G of the section.



Soln Symmetrical about Y-axis.

Rectangle ①

$a_1 = 150\times 50 = 7500\ mm^2$

$y_1 = 150 + \frac{50}{2} = 175\ mm$

Rectangle ②

$a_2 = 150\times50 = 7500\ mm^2$

$y_2 = \frac{150}{2} = 75\ mm$

So, $\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{(7500\times175)+(7500\times75)}{7500+7500} = 125\ mm$

M.I about X-X axis

$I_{G_1} = \frac{150\times50^3}{12} = 1.5625\times10^6\ mm^4$

h_1 = distance betn CG of □ ① & X-X axis = 175 − 125 = 50 mm

MI of □ ① about X-X axis = $I_{G_1} + a_1h_1^2$

$= (1.5625\times10^6) + (7500\times50^2) = 20.3125\times10^6\ mm^4$

Similarly, MI of rectangle ② about about an axis through C.G & parallel to X-X axis

$I_{G_2} = \frac{50\times150^3}{12} = 14.0625\times10^6\ mm^4$

& h_2 = distance betn CG of □ ② & X-X axis

$= 125 - 75 = 50\ mm$

MI of rectangle ② about X-X axis

$= I_{G_2} + a_2h_2^2$

$= (14.0625\times10^6) + (7500\times50^2) = 53.125\times10^6\ mm^4$

MI about Y-Y axis

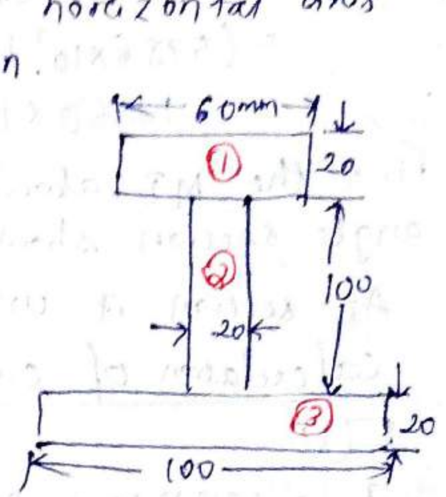
MI of □ ① about Y-Y axis $= \frac{50(150)^3}{12} = 14.0625\times10^6\ mm^4$

MI of □ ② " $= \frac{150\times50^3}{12} = 1.5625\times10^6\ mm^4$

MI of whole section about Y-Y axis

$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6\ mm^4$

Q 2017 An I-section is made up of 3 rectangles as shown in fig. Find the MI of the section about horizontal axis passing through the CG of the section.



Sol^n Given section is symmetrical about y axis.

□ ①

$a_1 = 60 \times 20 = 1200\ mm^2$

$y_1 = 20 + 100 + \frac{20}{2} = 130\ mm$

□ ②

$a_2 = 100 \times 20 = 2000\ mm^2$

$y_2 = 20 + \frac{100}{2} = 70\ mm$

□ ③

$a_3 = 100 \times 20 = 2000\ mm^2$

$y_3 = 20/2 = 10\ mm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} = 60.8\ mm$$

MI of □ ① about its CG & || to X-X axis $= I_{G_1} = \frac{60 \times 20^3}{12} = 40 \times 10^3\ mm^4$

h_1 = distance betn CG of □ ① & X-X axis $= 130 - 60.8 = 69.2\ mm$

MI of □ ① about X-X axis $= I_{G_1} + a_1 h_1^2$

$= (40 \times 10^3) + (1200 \times 69.2^2) = 5.786 \times 10^3\ mm^4$

Similarly for □ ② about C.G & || to axis

$I_{G_2} = \frac{20 \times 100^3}{12} = 1666.7 \times 10^3\ mm^4$

$h_2 = 70 - 60.8 = 9.2\ mm$

MI of □ ② about X-X axis $= I_{G_2} + a_2 h_2^2$

$= (1666.7 \times 10^3) + (2000 \times 9.2^2)$

$= 1836 \times 10^3\ mm^4$

MI of □ ② about axis through CG & || to X-X axis

$I_{G_3} = \frac{100 \times 20^3}{12} = 66.7 \times 10^3\ mm^4$

$h_3 = 60.8 - 10 = 50.8\ mm$

MI of □ ③ about CG & || to X-X axis $= I_{G_3} + a_3 h_3^2$

Chapter-5
SIMPLE MACHINES

Simple m/c can be defined as a device, which enables us to do some useful work at some point or to overcome some resistance when an effort or force is applied to it, at some other convenient point.

By use of simple m/c, heavy work can be lifted with the help of less effort.

Compound m/c →

It consists of a no. of simple m/cs which enables us to do some useful work at a faster speed or with less effort as compared to simple m/c.

Lifting m/c →

It is a device, which enables us to lift a heavy load(W) by applying a comparatively smaller effort (P).

Mechanical Advantage →

It is defined as the ratio between weight lifted (W) to the effort applied (P) & is always expressed in pure number.

mathematically, $M.A = \frac{W}{P}$

Input of a m/c →

It is the workdone on the m/c.
In a lifting m/c, it is measured by the product of effort & the distance through which it has moved.

Output of a m/c →

It is the actual work done by the m/c.
In a lifting m/c, it is measured by the product of the weight lifted & the distance through which it has been lifted.

Efficiency of a m/c →

Ratio bet^n output to input of a m/c.
Generally expressed in %.

$$\eta = \frac{o/p}{i/p} \times 100$$

Ideal m/c →

If efficiency of a m/c is 100%. i.e. o/p = i/p, the m/c is called as a perfect or an ideal m/c.

velocity ratio →

Defined as the ratio betn distance moved by effort (y) to the distance moved by the load (x) & is always expressed in pure number.

Mathematically $V.R = \frac{y}{x}$

Relation between η, M.A & V.R of a lifting m/c →

Let W = load lifted by the m/c

P = Effort required to lift the load

Y = Distance moved by the effort, in lifting the load in

X = " " load.

$$M.A = \frac{W}{P}$$

$$\& \; V.R = \frac{y}{x}$$

i/p of a m/c = effort applied × distance through which the effort has moved

$= P \times y$

o/p of a m/c = Load lifted × " " load has been lifted

$= W \times x$

$$\text{Now, } \eta = \frac{o/p}{i/p} = \frac{Wx}{Py} = \frac{W}{P} \Big/ \frac{y}{x} = \frac{M.A}{V.R}$$

when $M.A = V.R$, then $\eta = 100\%$

Q. In a certain weight lifting m/c, a weight of 1KN is lifted by an effort of 25N. While the weight moves up by 100 mm, the point of application of effort moves by 8m. Find M.A, V.R & η of the m/c.

Solⁿ W = 1KN = 1000 N

P = 25 N

x = 100 mm

y = 8 m

$$M.A = \frac{W}{P} = \frac{1000}{25} = 40$$

$$V.R = \frac{y}{x} = \frac{8}{0.100} = 80$$

$$\eta = \frac{M.A}{V.R} = \frac{40}{80} = 0.5 = 50\%$$

Reversibility of a machine →

Here machine can do some work in the reversed direction after effort is removed.

condition for reversibility of a m/c →

Let W = load lifted by the m/c

P = Effort required to lift the load

y = distance moved by the effort

x = distance moved by the load

i/p of the m/c = $P \times y$

o/p of " = $W \times x$

m/c friction = i/p − o/p = $(P\times y) - (W\times x)$

For reversibility, o/p > m/c friction

$$\Rightarrow W\times x > (P\times y) - (W\times x)$$

$$\Rightarrow 2(W\times x) > (P\times y)$$

$$\Rightarrow \frac{Wx}{Py} > \frac{1}{2}$$

$$\Rightarrow \frac{W/P}{y/x} > \frac{1}{2}$$

$$\Rightarrow \frac{M\cdot A}{V\cdot R} > \frac{1}{2}$$

$$\Rightarrow \eta > \frac{1}{2}$$

$$\Rightarrow \eta > 0.5$$

$$\Rightarrow \boxed{\eta > 50\%}$$

Self locking machine or Non-Reversible m/c →

The m/c which can not work in reversed direction when effort is removed.

Here $\boxed{\eta < 50\%}$

Q A certain weight lifting m/c of V.R 30 can lift a load of 1500N with the help of 125N effort. Determine if the m/c is reversible.

Solⁿ Given V.R = 30

W = 1500N

P = 125N

$M.A = \frac{W}{P} = \frac{1500}{125} = 12$

$\eta = \frac{M.A}{V.R} = \frac{12}{30} = 0.4 = 40\%$

→ m/c is non-reversible.

In a lifting m/c, whose V.R is 50, an effort of 100 N is required to lift a load of 4KN. Is the m/c reversible? If so, what effort should be applied, so that the m/c is at the point of reversing.

Sol^n Given data ~~$M.A = \frac{W}{P} = 40$~~

$V.R = 50$

$P = 100N$

$W = 4KN = 4000N$

i) $M.A = \frac{W}{P} = \frac{4000}{100} = 40$

$\eta = \frac{M.A}{V.R} = \frac{40}{50} = 0.8 = 80\% > 50\%$

→ Reversible machine.

ii) The m/c will be at the point of reversing at $\eta = 50\%$.

Let P_1 = Effort required to lift a load of 4000N when m/c is at the point of reversing.

$M.A = \frac{W}{P_1} = \frac{4000}{P_1}$

$\eta = \frac{M.A}{V.R} \Rightarrow 0.5 = \frac{4000/P_1}{50} = \frac{80}{P_1}$

$\Rightarrow P_1 = \frac{80}{0.5} = 160\ N$ (Ans)

Friction in a machine →

Let P = Actual effort required to lift load

P' = Ideal " "

W = Actual load to be lifted

W' = Ideal " "

$\eta = \frac{M.A}{V.R} = \frac{W/P}{V.R}$ & $V.R = \frac{W}{P}$ (for Ideal m/c $\eta = 1$)

For actual m/c $V.R = \frac{W}{P'} \Rightarrow P' = \frac{W}{V.R}$

$\underbrace{P - P'}_{\text{friction}} = P - \frac{W}{V.R}$ W' − W is the friction.

Finally $\boxed{F_{Load} = (P \times V.R) - W}$

Q. In a certain m/c, an effort of 100N is just able to lift a load of 840N. Calculate η & friction both on effort & load side, if V.R of m/c is 10.

Soln Given data $P = 100N$ $V.R = 10$

$W = 840N$

i) $M.A = \frac{W}{P} = \frac{840}{100} = 8.4$

$\eta = \frac{M.A}{V.R} = \frac{8.4}{10} = 0.84 = 84\%$.

ii) Friction of m/c

friction of the m/c in terms of effort

$F_{effort} = P - \frac{W}{V.R} = 100 - \frac{840}{10} = 16\ N$

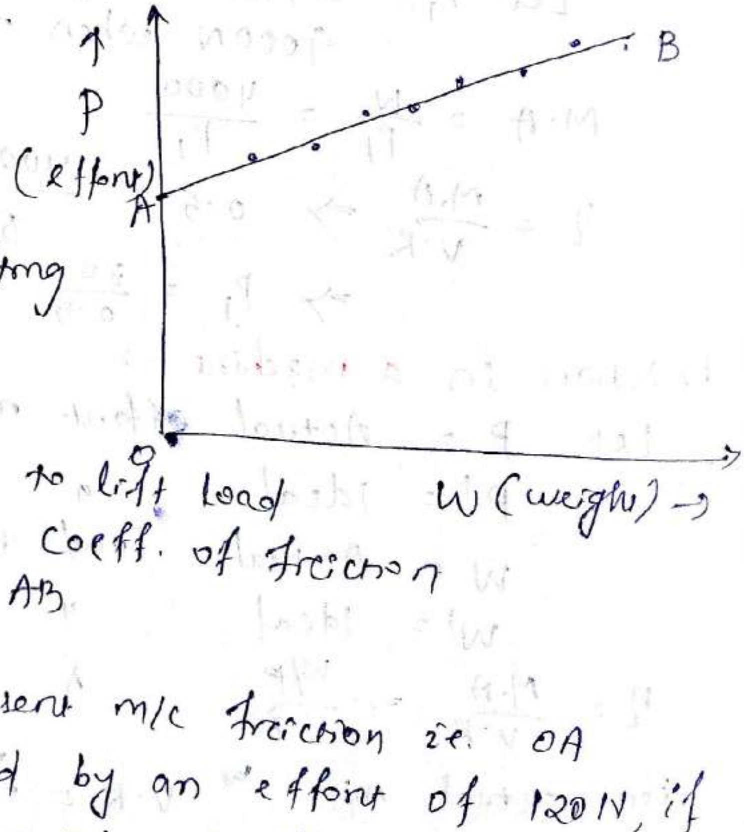
friction of the m/c in terms of load.

$F_{Load} = (P \times V.R) - W = (100 \times 10) - 840 = 160N$

Law of Machine :→

It is defined as the relationship between the effort applied and the load lifted.

OA → represent amount of friction offered by m/c.



mathematically, law of lifting m/c relation is

$P = mW + C$

where P = Effort required to lift load

m = const. called as coeff. of friction = slope of line AB

W = load lifted

C = const to represent m/c friction i.e. OA

Q. What load can be lifted by an effort of 120N, if the V.R is 18 & η of the m/c at this load is 60%.

Determine the law of m/c, if it is observed that an effort of 200N is required to lift a load of 2600N & find the effort required to run the m/c at a load of 3.5 KN

Sol^n Given data $P = 120N$ i) $W = ?$

$V.R = 18$ ii) Law of m/c ?

$\eta = 60\% = 0.6$ iii) P for $W = 3.5 KN$

i) $M.A = \frac{W}{P} = \frac{W}{120}$

$\eta = \frac{M.A}{V.R} \Rightarrow 0.6 = \frac{W}{120 \times 18} = \frac{W}{2160} \Rightarrow W = 1296N.$

ii) Here $P = 200N$, $W = 2600N$

Law of m/c is $P = mW + C$

$120 = m \times 1296 + C$ —— (i)

$200 = m \times 2600 + C$ —— (ii)

equ^n (ii) − (i) $\Rightarrow 80 = 1304m \Rightarrow m = \frac{80}{1304} = 0.06$

Now, equ^n (ii) becomes, $200 = (0.06 \times 2600) + C$

$\Rightarrow C = 44$

So, $P = mW + C$

$\Rightarrow P = 0.06W + 44$

iii) Putting $W = 3.5 KN$

$P = 0.06W + 44 = (0.06 \times 3.5 \times 1000) + 44 = 254N$

Q-2 In a lifting m/c, an effort of 40N raised a load of 1KN. If η of m/c is 0.5, what is its V.R ? If on this m/c, an effort of 74N raised a load of 2KN, what is now the η ? What will be the effort required to raise a load of 5KN?

Sol^n Given data i) $P = 40N$ ii) $P = 74N$ iii) $P = ?$

$W = 1KN = 1000N$ $W = 2KN = 2000N$ $W = 5KN$

$\eta = 0.5$ $\eta = ?$

$V.R = ?$

i) $M.A = \frac{W}{P} = \frac{1000}{40} = 25$

$\eta = \frac{MA}{VR} \Rightarrow 0.5 = \frac{25}{V.R} \Rightarrow V.R = \frac{25}{0.5} = 50$ Ans

ii) $M.A = \frac{W}{P} = \frac{2000}{74} = 27$

$\eta = \frac{M.A}{V.R} = \frac{27}{50} = 0.54 = 54\%$

iii) From law of m/c & from two above cond^n.s $40 = m \times 1000 + C$ —(i)

$74 = m \times 2000 + C$

equ^n ② - equ^n ① $\Rightarrow$ $34 = 1000m$ $\Rightarrow m = 0.034$
Now equ^n ① becomes $40 = (0.34 \times 1000) + c$
$\Rightarrow c = 6$

Now $P = 0.034W + 6$
for $W = 5KN = 5000N$, $P = (0.034 \times 5000) + 6$
$= 176N$

Q What load will be lifted by an effort of 12N, If the V.R is 18 & η of the m/c at this load is 60%.
If the m/c has a const. friction resistance. determine the law of m/c & find effort required to run this m/c at
i) no load
ii) a load of 900N

Sol^n Given data $P = 12N$
$V.R = 18$
$\eta = 60\% = 0.6$
$W = ?$

$M.A = \frac{W}{P} = \frac{W}{12}$

$\eta = \frac{M.A}{V.R} \Rightarrow 0.6 = \frac{W}{12 \times 18} = \frac{W}{216} \Rightarrow W = 0.6 \times 216 = 129.6N$ Ans

Law of m/c

Effort lost in friction = $F_{effort} = P - \frac{W}{V.R} = 12 - \frac{129.6}{18} = 4.8N$
4.8N is the const. frictional resistance offered by the m/c.
Now putting $P = 12$ & $C = 4.8$

$P = mW + C$
$\Rightarrow 12 = (m \times 129.6) + 4.8 \Rightarrow m = \frac{1}{18}$

So, Now $\boxed{P = \frac{1}{18}W + 4.8}$

i) when $W = 0$, $P = 4.8$
ii) when $W = 900N$, $P = \left(\frac{1}{18} \times 900\right) + 4.8 = 54.8N$

Max^m M.A of a lifting m/c →

$M.A = \frac{W}{P}$

for max^m M.A, $P = mW + C$

so, max^m M.A $= \frac{W}{mW + C} = \frac{1}{m + \frac{C}{W}} = \frac{1}{m}$ ($\because$ neglecting $\frac{C}{W}$)

Max^m η of a lifting m/c →

$$\eta = \frac{M.A}{V.R} = \frac{\frac{W}{P}}{V.R} = \frac{W}{P \times V.R}$$

$$\eta_{max} = \frac{W}{(m + \frac{c}{W}) \times V.R} = \frac{1}{m \times V.R} \quad (\because \text{neglecting } \frac{c}{W})$$

Transmission of power by gear drive :→

Gear can be defined as a pulley or wheel having projections on its rim called as teeth.

In gear drive the problem of slipping (in case of belt drive, rope drive, chain drive) is reduced. So it has max^m power transmission efficiency. So it is a positive drive. It is used when distance between driver & driven shaft is less.

Advantages of gear drive →

→ Transmits exact velocity ratio.
→ High η of power transmission.
→ Has compact structure
→ Can transmit large power
→ Has a reliable service

Disadvantages →

→ Manufacturing of toothed wheels requires special tools & equipments.
→ Any error in teeth machinery causes noise & vibration during operation.
→ Any defect in one wheel damages the whole set up

Pitch →

Centre to centre distance between two consecutive teeth is called pitch.

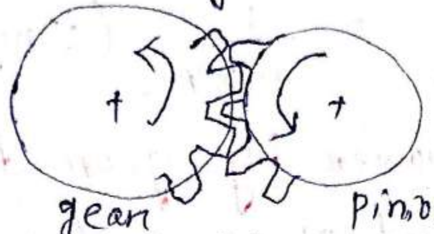
Mathematically, $P = \frac{\pi d}{T}$

where d = dia of pitch circle
T = No. of teeth

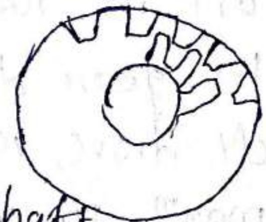
* For meshing, two gears or wheels should have same pitch.

Types of gear →

1> External gears → Here gears of two shafts mesh externally with each other.

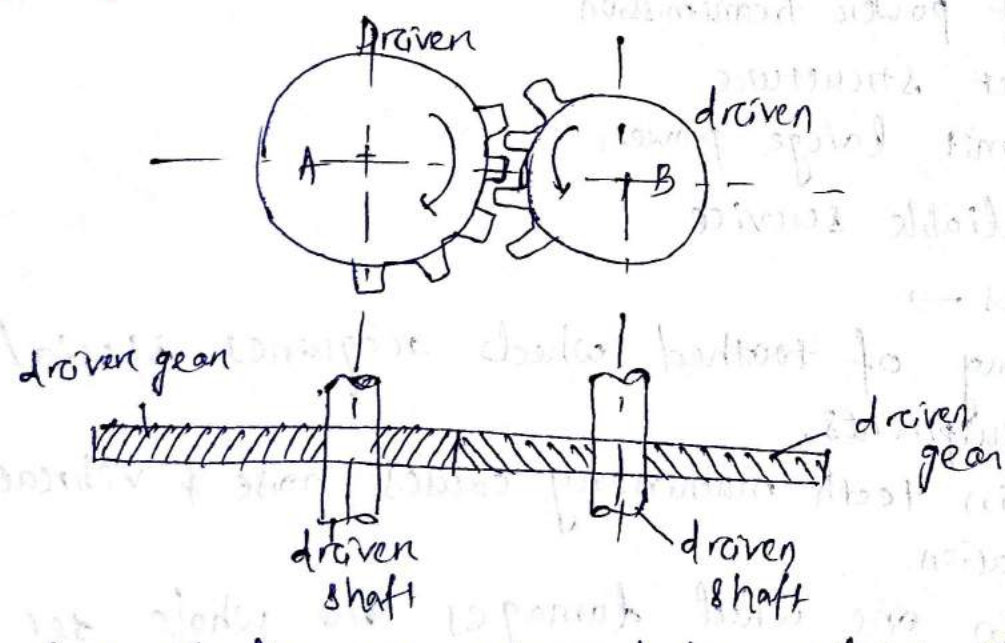


2> Internal gears → Here gears of two shafts mesh internally



3> Rack & pinion → Here gear of a shaft meshes internally or externally with the gears in a straight line

Simple gear drive →



When both shafts are connected by meshing of two gears power will be transmitted from driver ~~gea~~ shaft to driver gear then to driven gear & finally to driven shaft.

* Gears are always mounted on the shaft.

Velocity ratio of a simple gear drive →

It is the ratio between speed of driven to driver member.

Let N_1, N_2 → speed of driver & driven respectively.

T_1, T_2 → No. of teeth " "

d_1, d_2 → dia of " "

pitch = $p = \dfrac{\pi d_1}{T_1}$ for driver

$p = \dfrac{\pi d_2}{T_2}$ for driven

For meshing of two gears $\dfrac{\pi d_1}{T_1} = \dfrac{\pi d_2}{T_2} \Rightarrow \dfrac{d_1}{d_2} = \dfrac{T_1}{T_2}$

$$\boxed{V.R = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}}$$

Power transmitted by a simple gear →

Let F = Tangential force exerted by the driver (i.e. pressure betn the teeth)

v = linear or peripheral velocity of the driver at pitch point.

$P = F \times v$

Gear Train →

When two or more gears mesh with each other to operate as a single system, to transmit power from one shaft to another, this is called gear train or train of wheels.

Types of gear train →

1) Simple gear train
2) Compound " "
3) Epicyclic " "
4) Reversed " "

1) Simple gear train →

If one gear is located in each shaft then the gear train is called simple gear train.

~~Simple Wheel & axle →~~

Chapter-6 DYNAMICS

Dynamics is the study of body in motion. Dynamics is based on three fundamental laws of motion proposed by Sir Issac Newton in 1680.

Newton's laws of motion →

1) **Newton's 1st law of motion →**

It states that, "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external forces."

It is also called as law of inertia.

Ex - i) A body at rest want to be in rest until some external force is applied.

ii) A moving body want to move in uniform velocity along the same dirn if any external force is not used.

2) **Newton's 2nd Law of motion →**

It states that "The rate of change of momentum is directly proportional to the impressed force & takes place, in the same direction in which the force acts."

It is also called as law of dynamics.

It consist of two parts.

a) A body can posses acceleration only when some force is applied on it.

b) The force applied on the body is proportional to the product of mass of the body & the acceleration produced in it

$$\boxed{\text{ie } F = ma \quad \text{force} = \text{mass} \times \text{acc}^n}$$

$v = \frac{s}{t}$

$a = \frac{v}{t}$

Rate of change of momentum

$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma$

$u \to$ Initial velocity
$v \to$ final "
$m \to$ mass of the body
$t \to$ time taken

Accⁿ to Newton's 2nd law of motion

$\boxed{F = ma}$

* 1 Newton is defined as the force acting on 1 kg mass of a body & it produces acceleration of $1 m/s^2$.

Q-1 Determine the force, which can move a body of mass 100 kg with an accⁿ of $3.5 m/s^2$.

Solⁿ Given $m = 100 kg$

$a = 3.5 m/s^2$

So, $F = m \times a = 100 \times 3.5 = 350 N$

Q-2 A body has 50 kg mass on the earth. Find its weight on
a) on the earth where $g = 9.8 m/s^2$
b) on the moon, where $g = 1.7 m/s^2$
c) on the sun, where $g = 270 m/s^2$

Solⁿ Given $m = 50 kg$

a) $F = mg = 50 \times 9.8 = 490 N$

b) $F = mg = 50 \times 1.7 = 85 N$

c) $F = mg = 50 \times 270 = 13500 N$

Q-3 A body of mass 7.5 kg is moving with a velocity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.

Solⁿ Given $m = 7.5 kg$ $F = 15 N$ $t = 2 s$

$u = 1.2 m/s$ $v = ?$

Accⁿ $= a = \frac{F}{m}$ $\left[as \because F = ma\right]$

$= \frac{15}{7.5} = 2 m/s^2$

Again $v = u + at$

$= 1.2 + (2 \times 2) = 5.2 m/s$

Three important relations of laws of motion are

$v = u + at$ where $u \rightarrow$ Initial velocity

$S = ut + \frac{1}{2}at^2$ $v \rightarrow$ final velocity

$v^2 - u^2 = 2aS$ $S \rightarrow$ displacement of the body

$a \rightarrow$ acceleration of the body

$t \rightarrow$ time period

Q-4 A vehicle of mass 500kg is moving with a velocity of 25 m/s. A force of 200N acts on it for 2 minutes. find the velocity of the vehicle.

a) when the force acts in the dirn of motion

b) " " in the opposite dirn of the motion

Sol Given $m = 500\,kg$ $F = 200N$

$u = 25\,m/s$ $t = 2\,min = 120\,sec$

a) We know that $F = ma \Rightarrow a = \frac{F}{m} = \frac{200}{500} = 0.4\,m/s^2$

$V = u + at$

$= 25 + (0.4 \times 120) = 73\,m/s$

Here $a = -0.4\,m/s^2$

b) V at 120 sec will be $= V = u + at$

$= 25 + (-0.4 \times 120) = -23\,m/s$

acc^n is -ve as vehicle is moving in opposite dirn to the dirn of motion of the vehicle.

Q-5 A constant retarding force of 50N is applied to a body of mass 20kg moving initially with a velocity of 15 m/s. How long the body will take to stop!

Sol Given $F = 50N$ $u = 15\,m/s$ $V = 0$

$m = 20\,kg$ $t = ?$

Retardation is the opposite of acceleration

$F = ma \Rightarrow a = \frac{F}{m} = \frac{50}{20} = 2.5\,m/s^2$

again $V = u + at$

$\Rightarrow 0 = 15 + (-2.5 \times t) \Rightarrow t = \frac{15}{2.5} = 6\,sec.$

Q-6 A car of mass 2.5 tonnes moves on a level road under the action of 1KN force. find the time taken by the car to increase its velocity from 36 Kmph to 54 Kmph.

Sol Given $m = 2.5 \times 1000 = 2500\,kg$ $t = ?$

$F = 1\,KN = 1000N$

$u = 36\,Kmph = \frac{36 \times 1000}{60 \times 60}\,m/s$

$V = 54\,Kmph = \frac{54 \times 1000}{60 \times 60}\,m/s$

$F = ma \Rightarrow a = \frac{F}{m} \Rightarrow a = \frac{1000}{2500} = 0.4\,m/s^2$

$V = u + at \Rightarrow \frac{54 \times 1000}{60 \times 60} = \left(\frac{36 \times 1000}{60 \times 60}\right) + 0.4t \Rightarrow t = 12.5\,sec.$

Q-7 A multiple unit electric train has 800 tonnes mass. The resistance to motion is 100N per tonne of the train mass. If the electric motors can provide 200 kN tractive force, how long does it take to accelerate the train to a speed of 90 Km/hr from rest.

Sol^n Given $m = 800$ tonne $= 800 \times 1000 = 800000$ kg

Resistance to motion $= 100$ N/tonne $= 100 \times 800 = 80000$ N $= 80$ kN

Tractive force $= 200$ kN

$V = 90$ Km/hr

$u = 0$ (as body is initially at rest)

$t = ?$

Net force available to move the train =

F = Tractive force − Resistance to motion

$= 200 - 80 = 120$ kN

Again $F = ma \Rightarrow a = \frac{F}{m} = \frac{120}{800} = 0.15\ m/s^2$

Again we know, $V = u + at$

$\Rightarrow \frac{90 \times 1000}{60 \times 60} = 0 + (0.15 \times t)$

$\Rightarrow t = 166.7$ sec.

Q-8 A man of mass 60 kg jumps downwards into a ~~train~~ swimming pool from a tower of height 20m. He was found to go down in water by 2m & then started rising. Find the average resistance of water. Neglect air resistance.

Sol^n Given mass $= m = 60$ kg

height of tower $= s = 20$ m

Considering motion of man from top of tower to water surface.

Here $u = 0$

distance $= s = 20$ m

we know that $V^2 = u^2 + 2gs$

$\Rightarrow V^2 = 0^2 + (2 \times 9.81 \times 20) = 392$

$\Rightarrow V = \sqrt{392} = 19.8$ m/s.

considering motion of man from water surface to the point ~~at~~ on water, he started rising.

Here $u = 19.8$ m/s, $V = 0$, $S = 2$ m

a = Retardation due to water resistance.

We know $v^2 - u^2 = 2as$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow 0 = 19.8^2 - (2a \times 2) \Rightarrow a = 98\ m/s^2$

Avg. resistance of water = $F = ma = 60 \times 98 = 5880N$.

Q-9 At a certain instant, a body of mass 10kg, falling freely under the force of gravity, was found to be falling at the rate of 20m/s. What force will stop the body i) in 2 sec.
ii) in 2m.

Sol^n Given $m = 10\ kg$ $F = ?$

$u = 20\ m/s$ i) $t = 2\ sec$

$v = 0$ ii) $s = 2m$

i) $v = u + at$

$\Rightarrow 0 = 20 + (a \times 2) \Rightarrow a = 10\ m/s^2$

As body is falling under force of gravity i.e. $a = 9.8\ m/s^2$

So the applied force must be able to produce an acc^n of $10 + 9.8 = 19.8\ m/s^2$.

$\therefore$ Force required to stop the body = $F = ma$

$= 10 \times 19.8 = 198N$

ii) $v^2 - u^2 = 2as$

$\Rightarrow 0 - 20^2 = 2a \times 2$

$\Rightarrow a = 100\ m/s^2$

The applied force must be able to produce an acc^n of $100 + 9.8 = 109.8\ m/s^2$.

So, force required to stop the body = $F = ma$

$= 10 \times 109.8 = 1098N$

Q-10 A body of mass 10kg is moving over a smooth surface, whose eqn of motion is $s = 5t + 2t^2$, where S is in meter & t is in sec. Find the magnitude of force responsible for the motion.

Sol^n

$m = 10kg$

$S = 5t + 2t^2$

$v = \frac{ds}{dt} = 5 + 2(2t) = 5 + 4t$

$a = \frac{d^2s}{dt^2} = \frac{d}{dt}(5 + 4t)$

$= 4$

So, $F = ma = 10 \times 4 = 40N$ Ans

Chapter-6 DYNAMICS

D-Alembert's Principle & Dynamic equilibrium:→

It states that "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction & the line of action may be found out by the method of graphic statics."

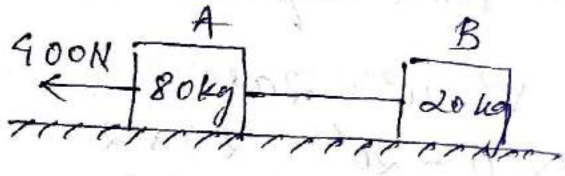
From Newton's 2nd law of motion, $F = ma$ (eqn of dynamics)

$\Rightarrow F - ma = 0$ (eqn of statics)

$F - ma = 0$ is also called as the <u>eqn of dynamic equilibrium.</u>

D-Alembert's principle is used for solving questions on dynamics which involve force & acceleration.

Q Two bodies A & B of mass 80kg & 20kg are connected by a thread & move along a rough horizontal plane under the action of force 400N applied to the first body of mass 80kg as shown in fig. $\mu = 0.3$. Determine the accn of two bodies & the tension in the thread, using D-Alembert's principle.

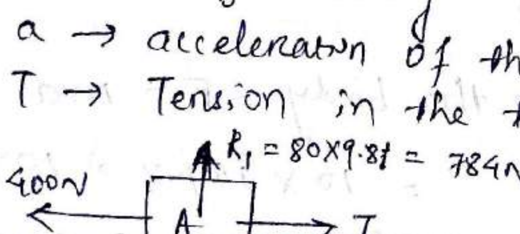


Soln Given data $m_A = 80\,kg$ $F = 400\,N$

$m_B = 20\,kg$ $\mu = 0.3$

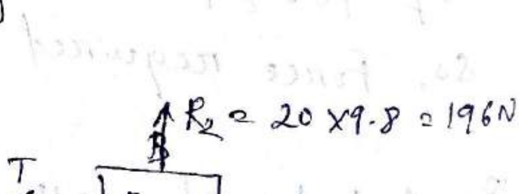
Let $a \rightarrow$ acceleration of the bodies

$T \rightarrow$ Tension in the thread



[FBD of body A]

[fig-1]



[FBD of body B]

(fig-2)

from fig-1 for equ^n of body A, horizontal force

$400 = \mu R_1 + T$

Let P_1 = Net or resultant horizontal force

so, $\cancel{P_1 = \mu R_1 + T - 400}$

$\cancel{= (0.3 \times 784) + T - 400}$

$P_1 = 400 - T - \mu R_1$

$= 400 - T - (0.3 \times 784) = 164.8 - T$ (acting towards left)

Again force causing acc^n to the body A = $m_A a = 80a$

Acc^n to D-Alembert's Principle,

$P_1 - M_A a = 0$

$\Rightarrow 164.8 - T - 80a = 0$

$\Rightarrow T = 164.8 - 80a$

Now, from fig-2, for Net horizontal force = P_2 (Let)

Let $P_2 = T - F_2 = T - \mu R_2 = T - (0.3 \times 196) = T - 58.8$

Again force causing acc^n to the body B = $m_B a = 20a$

Acc^n to D-Alembert's Principle,

$P_2 - M_B a = 0$

$\Rightarrow T - 58.8 - 20a = 0$

$\Rightarrow 164.8 - 80a - 58.8 - 20a = 0$

$\Rightarrow 100a = 106$

$\Rightarrow a = 1.06\ m/s^2$

Now $T = 164.8 - 80a$

$= 164.8 - (80 \times 1.06) = 80N$ Ans

Newton's Third law of Motion :→

It states that "To every action, there is always an equal & opposite reaction."

Ex- i) When a bullet is fired from a gun

(The bullet moves with a velocity is the action & in reaction, gun gives unpleasant shock to the man holding the gun).

ii> When swimmer tries to swim, he pushes the water backwards & the reaction of water pushes the swimmer forward.

iii> motion of boat

Law of conservation of momentum →

It states that, "The momentum is conserved in a system, in which the external force is zero."

Mathematically, Initial momentum = final momentum

$\Rightarrow MV = mv$

Ex- 1> Recoil of gun → when a bullet is fired from a gun, the opposite reaction of the bullet is called recoil of gun.

2> Motion of a boat → when a ~~body~~ boat boy pushes the water back with sticks, it in turn sets the boat in motion.

Recoil of gun →

Let M = mass of the gun

V = velocity of the gun with which it recoils

m = mass of the bullet

v = velocity of the bullet after explosion.

Momentum of the bullet after explosion = mv

Momentum of the gun = MV

Acc^n to law of conservation of momentum, $\boxed{MV = mv}$

Q-1 A m/c gun of mass 25 kg fires a bullet of mass 30 gm with a velocity of 250 m/s. find the velocity with which the m/c gun will recoil.

Soln Given $M = 25\,kg$ $v = 250\,m/s$.

$m = 30\,g = 0.03\,kg$ $V = ?$

Again $MV = mv$

$\Rightarrow 25 \times V = 0.03 \times 250 \Rightarrow V = \frac{7.5}{25} = 0.3\,m/s$.

Q-2 A bullet of mass 20g is fired horizontally with a velocity of 300 m/s, from a gun carried in a carriage. The combined mass of gun & carriage is 100g. The resistance to sliding of the carriage over the ice on

which it rests is 20N. Find
a) velocity, with which the gun will recoil.
b) distance, in which it comes to rest
c) time taken to do so.

sol^n Given $m = 20g = 0.02\,kg$ $M = 100kg$
$v = 300\,m/s$ $F = 20N$

a) we know that $MV = mv$
$\Rightarrow 100 \times V = 0.02 \times 300$
$\Rightarrow V = 0.06\,m/s$

b) Here $u = 0.06\,m/s$ & $v = 0$
Resisting force to sliding of carriage (F)
$F = Ma \Rightarrow 20 = 100 \times a \Rightarrow a = 0.2\,m/s^2$
Again $v^2 - u^2 = 2as$
$\Rightarrow 0 - 0.06^2 = 2 \times 0.2 \times S$
$\Rightarrow S = \frac{0.0036}{0.4} = 0.009m = 9\,mm$

c) we know that $v = u + at$
$\Rightarrow 0 = 0.06 + 0.2t$
$\Rightarrow t = \frac{0.06}{0.2} = 0.3$ sec.

Motion of a boat →

Boat man always pushes water back with the help of stick, which in return sets the boat in motion.

If the boat is at rest & the boat man runs on it, & dives off into the water, the boat will also move backward.

Let M = mass of boat
V = velocity of boat
m = mass of the boat man
v = velocity of " "

According to law of conservation of momentum, $\boxed{MV = mv}$

Q. Two men, standing on a floating boat, run in succession, along its length, with a speed of 4.2 m/s relative to the boat & dive off from the end. The weight of each man is 80 kg & that of the boat is 400kg. If the boat was initially at rest, find the final velocity of the boat. Neglect water friction.

Sol^n Given data $v = 4.2\,m/s$

$m = \cancel{20}\ 80$ kg (each)

$M_b = 400\,kg$

When first man dives off the boat, it will give some momentum to the boat as well as the 2nd man (who is still standing on the boat).

When the 2nd man also dives off the boat, it will also give some momentum to the boat. ~~So~~

So the total momentum gained by the boat is equal to the momentum given by the 1st man + momentum given by 2nd man to the boat.

Final momentum of the boat $= 400V$

Momentum given by the 1st man to the boat $= 80 \times 4.2 = 336$ kg-m/s.

" " 2nd man " $= 80 \times 4.2 = 336$ kg-m/s.

Final momentum = momentum by 1st man + momentum by 2nd man

$= 336 + 336 = 672$ kg-m/s.

Now, $400V = 672$

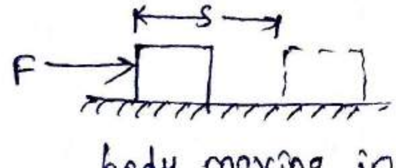
$\Rightarrow V = 1.68$ m/s.

Work, Power & Energy

Work (W) → Whenever force acts on a body & the body undergoes some displacement, then work is said to be done.

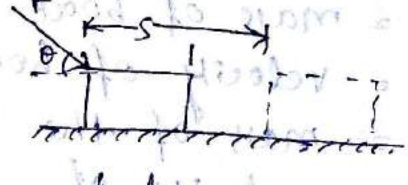
mathematically W = Force × distance

$\boxed{W = F \times S}$



body moving in dir^n of applied force

Here $W = FS$



body not moving in dir^n of applied force

Here $W = FS\cos\theta$

Unit

$W = F \times S = N \times m = 1$ Joule

$1\,KJ = 1000\,J = 1000\,N\text{-}m$

Q. A horse pulling a cart exerts a steady horizontal pull of 300N & walks at the rate of 4.5 kmph. How much work is done by the horse in 5 minutes.

Sol^n Given $F = 300N$, $V = 4.5$ Kmph $= 75$ m/min $t = 5$ min

Distance travelled ~~s~~ in 5 min =

$V = \frac{s}{t} \Rightarrow 75 = \frac{s}{5} \Rightarrow s = 75 \times 5 = 375m$

Workdone by the horse $= W = F \cdot S$

$= 300 \times 375 = 112500$ N-m

$= 112500J = 112.5$ KJ.

Q-2 A spring is stretched by 50mm by the application of force. Find the workdone, if the force required to stretch 1 mm of the spring is 10N.

Sol^n Given $S = 50$ mm   Stretching of spring = 1mm

$F = 10N$ $W = ?$

Force required to stretch the spring by 50 mm $= 10 \times 50 = 500N$

Avg. force $= \frac{500}{2} = 250$ N

workdone = Avg. force x distance $= 250 \times 50 = 12500$ N-mm

$= 12.5$ N-m $= 12.5$ J

Power → (P)

Rate of doing work is called power.

Mathematically $P = \frac{\text{Workdone}}{\text{Time}} = \frac{W}{t}$

Unit $P = \frac{W}{t} = \frac{N\text{-}m}{sec} =$ Watt $1KW = 10^3 W$

$1MW = 10^6 W$

Types of engine power →

1> Indicated power (IP)

It is the actual power generated inside the engine cylinder.

2> Brake power (BP)

Net output of the engine after overcoming frictional resistance is called brake power.

$BP = IP -$ Frictional resistance or losses

* Efficiency of the engine (η)

It is the ratio between BP to IP.

$\eta = \frac{BP}{IP}$

Q Find power of an engine, which can do work of 1200 Joule in 8 sec.

Sol^n Given $W = 1200J$ $P = \frac{W}{t} = \frac{1200}{8} = 150$ J/s.

$t = 8$ sec $= 150$ Watt

$P = ?$

Q-2 A railway engine of mass 20 tonnes is moving on a level track with a const. speed of 45 Kmph. Find the power of the engine, if the frictional resistance is 80 N/t. Take efficiency of the engine as 80%.

Soln Given $m = 20$ tonne

$v = 45$ kmph $= \frac{45 \times 1000}{60 \times 60} = 12.5$ m/s

Frictional resistance $= 80$ N/t $= 80 \times 20 = 1600$ N $= 1.6$ kN

$\eta = 80\% = 0.8$

Workdone by the railway engine in 1 sec = Resistance × distance

$= 1.6 \times 12.5 = 20$ KN-m/s $= 20$ KJ/s.

$\Rightarrow P = 20$ KW

Actual power of the engine $= BP = \frac{IP}{\eta} = \frac{20}{0.8} = 25$ KW.

Q-3 A train of weight 1000KN is pulled by an engine on a level track at a speed of 45 Kmph. The frictional resistance is 1% of the weight of the train. Find the power of the engine.

Soln Given weight = 1000 KN

$V = 45$ Kmph $= \frac{45 \times 1000}{60 \times 60} = 12.5$ m/s.

frictional resistance $= 1\%$ of $1000 = 0.01 \times 1000 = 10$ KN

Workdone in 1 sec = resistance × distance

$= 10 \times 12.5 = 125$ KN-m/s $= 125$ KJ/s

$\Rightarrow$ Power $= 125$ KW

Q-4 A locomotive draws a train of mass 400 tonnes, including its own mass, on a level ground with a uniform accn, until it acquires a velocity of 54 kmph in 5 minutes. If the frictional resistance is 40 N per tonne of mass & the air resistance varies with the square of velocity, find the power of the engine. Take air resistance as 500 N at 18 Kmph.

Soln Given mass $= m = 400$ tonne $= 400 \times 1000 = 400000$ kg

$v = 54$ Kmph $= \frac{54 \times 1000}{60 \times 60} = 15$ m/s

$t = 5$ min $= 5 \times 60 = 300$ sec

$F.R = 40$ N/tonne $= 40 \times 400 = 16000$ N $= 16$ KN

We know $V = u + at$

$\Rightarrow 15 = 0 + a \times 300 \Rightarrow a = \frac{15}{300} = 0.05$ m/s²

Force required for accn $= F = ma = 400000 \times 0.05 = 20000$ N $= 20$ KN

As air resistance varies with the square of velocity, so air resistance at 54 Kmph

$= 500 \left(\frac{54}{18}\right)^2 = 4500 N = 4.5 KN$

Total resistance = $16 + 20 + 4.5 = 40.5$ KN

workdone in 1 sec = Total resistance × Distance

$= 40.5 \times 15 = 607.5$ KN-m/s $= 607.5$ KJ/s

$P = 607.5$ KW

Energy :→

Capacity to do work is called energy. It is of different types like mechanical, electrical, chemical, heat, Light etc.

Unit N-m = Joule

mechanical energy →

It is the capacity to do mechanical work. It is of 2 types.

1> Potential energy (P.E) 2> Kinetic energy (K.E)

1> Potential energy →

It is The energy possessed by a body due to its position.

Ex- → A body raised to some height above the ground level. It can do work by falling on earth's surface

→ compressed air. It can do work by expansion.

→ compressed spring. It can do work in recovering its original shape.

Let m = mass of the body

h = height above datum level

workdone = weight × distance

$\Rightarrow \boxed{W = (mg)h}$

This workdone is stored in the body as P.E.

Q A man of mass 60kg dives vertically downward into a swimming pool from a tower of height 20m. He was found to go down in water by 2m & then started rising. Find the avg. resistance of the water. Neglect the air resistance.

Soln Given m = 60Kg

h = 20m

P.E of man before jumping = $mgh = 60 \times 9.8 \times 20 = 11760$ N-m

workdone by the avg resistance of water = Avg. resistance of water × depth of water

$= F \times 2 = 2F$ N-m

As total P.E of the man is used in workdone by the water.

so $11760 = 2f \Rightarrow f = 5880\,N$ Ans

Kinetic energy →

It is the energy possessed by a body due to its mass and velocity of motion.

Let m = mass of the body

u = Initial velocity "

F = Force applied on body to bring it to rest

a = retardation

s = distance travelled

Here $v = 0$

Workdone = $W = F \times S = ma \times s$ ——— ①

Again $v^2 - u^2 = 2as$

$\Rightarrow 0 - u^2 = -2as$ (retardation so a is -ve)

$\Rightarrow u^2 = 2as \Rightarrow as = \frac{u^2}{2}$

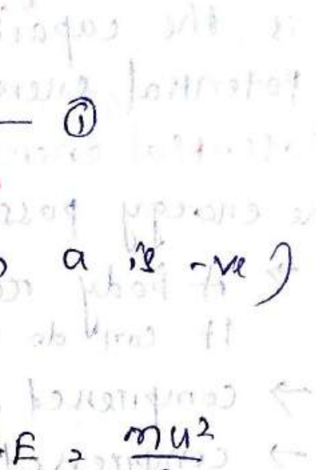
Now eqn ① becomes, $W = \frac{mu^2}{2} \Rightarrow K.E = \frac{mu^2}{2}$

In most cases u is taken as v

So, $\boxed{K.E = \frac{mv^2}{2}}$

Transformation of energy →

Consider a body just dropped on the ground from position A.



Let m = mass of the body

h = height " "

B & C are the positions of the body when it falls.

Position A

body is at rest. so $v = 0$

$K.E = \frac{1}{2}mv^2 = 0$

$P.E = mgh$

Total energy = $K.E + P.E = 0 + mgh = mgh$

Position B

Here body has moved distance y.

So, velocity of the body at B = $v = \sqrt{2gy}$

$v^2 - u^2 = 2as$

$\Rightarrow v^2 - 0 = 2gy$

$\Rightarrow v = \sqrt{2gy}$

$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m(\sqrt{2gy})^2 = \frac{1}{\cancel{2}}m\cancel{2}gy = mgy$

$P.E = mg(h-y)$

Total energy $= K.E + P.E$

$= mgy + mg(h-y)$

$= \cancel{mgy} + mgh - \cancel{mgy} = mgh$

Position C

$V = \sqrt{2gh}$

$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m(\sqrt{2gh})^2 = \frac{1}{\cancel{2}}m\cancel{2}gh = mgh$

$P.E = 0$

Total energy $= K.E + P.E$

$= mgh + 0 = mgh$

So in all position, total energy is const. i.e. mgh. This is called law of conservation of energy

Law of conservation of energy $\rightarrow$

It states that "Energy can neither be created nor be destroyed. It can only be transformed from one form to another."

Impulse (J) :$\rightarrow$

It is defined as the product of force & the time for which it is acting on a body.

mathematically $J = F \times t$

From Newton's 2nd law, $F = ma = m\left(\frac{V-u}{t}\right)$

$\Rightarrow Ft = m(V-u)$

$\Rightarrow \boxed{\text{Impulse} = \text{Change in momentum}}$

Impulse-Momentum equn

Unit of J = N x sec.

This equn. is used for solving dynamics problems involving force, velocity & time.

Q A pile hammer (driver) of mass 300kg falls on a pile. The hammer falls freely from a height of 6m & the hammer comes to rest in 1/80 sec. find avg. impulsive force of blow.

Soln Given $m = 300kg$

$u = 0$

We know $V^2 - u^2 = 2gh$

$\Rightarrow V^2 - 0 = 2gh$

$\Rightarrow V = \sqrt{2gh}$

So final velocity of hammer just before striking the pile $= V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 6} = 10.85$ m/s

Collision of Elastic bodies

The property of bodies, by virtue of which they rebound, after impact is called elasticity.
↓ rises up after striking the floor

→ The body which rebounds to a greater height is called more elastic than the body which rebounds to a lesser height.

→ If a body does not rebound at all after its impact, is called inelastic body.

Collision phenomenon →

When two elastic bodies collide

→ The bodies, immediately after collision, come momentarily to rest.

→ The two bodies tend to compress each other, so long as they are compressed to the max^m value.

→ The two bodies attempt to regain its original shape due to their elasticity. This process of regaining the original shape is called restitution.

The time taken by two bodies in compression, after the instant of collision is called time of compression & time for which restitution takes place is called time of restitution.

Time of collision = 2 (times of collision) + time of restitution

Law of conservation of momentum for collision →

It states that " The total momentum of two bodies remains constant after their collision or any other mutual action."

Mathematically, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

where m_1, u_1, v_1 → mass, Initial velocity & final velocity respectively of 1st body

m_2, u_2, v_2 → mass, initial velocity & final velocity respectively of 2nd body.

$\boxed{\text{Total Initial momentum = Total final momentum}}$

Newton's law of collision of elastic bodies →

It states that " When two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach."

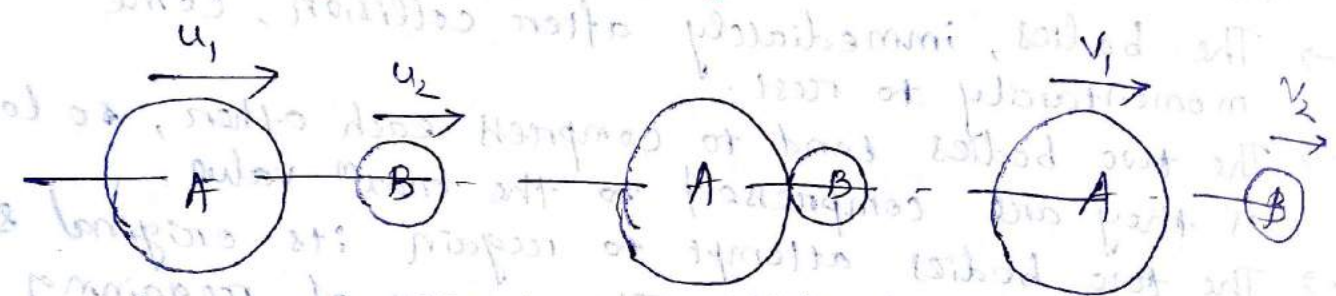
Mathematically, $v_2 - v_1 = e(u_1 - u_2)$

where $u_1, v_1 \rightarrow$ Initial & final velocity of 1^{st} body respectively.

$u_2, v_2 \rightarrow$ Initial & final velocity of 2^{nd} body respectively.

$e \rightarrow$ const. of proportionality

Coefficient of restitution →



before Impact During Impact After Impact

Consider two bodies A & B having direct ~~contact~~ impact.

Impact will occure if $u_1 > u_2$

Velocity of approach $= u_1 - u_2$

After Impact, separation of two bodies will takes place if $v_2 > v_1$

So, velocity of separation $= v_2 - v_1$

Acc^n to Newton's law of collision

Velocity of separation $= e \times$ velocity of approach

$\Rightarrow \quad v_2 - v_1 = e(u_1 - u_2)$

$\uparrow$ coefficient of restitution

$0 < e < 1$

If $e = 0 \Rightarrow$ two bodies are inelastic

$e = 1 \Rightarrow$ " " perfectly elastic

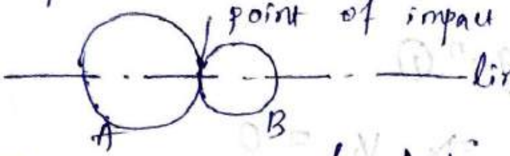
Types of collision →

when two bodies collide with one another, they are said to have impact. It is of 2 types

1> Direct Impact 2> Indirect or oblique impact.

1> Direct collision of two bodies →

If the two bodies, before impact are moving along the line of impact, the collision is called ~~as~~ direct impact.



Let m_1, m_2 → mass of body A & B respectively

u_1, u_2 → Initial velocity of " "

v_1, v_2 → final velocity of " "

from law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Q-1 A ball of mass 1 kg moving with a velocity of 2m/s impinges directly on a ball of mass 2kg at rest. The first ball after impinging, comes to rest. Find the velocity of the 2nd ball after the impact & coeff. of restitution

Sol^n Given $m_1 = 1 kg$ $m_2 = 2 kg$ $v_1 = 0$ $e = ?$

$u_1 = 2 m/s$ $u_2 = 0$ $v_2 = ?$

i) from law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow (1\times 2) + (2\times 0) = (1\times 0) + (2\times v_2)$$

$$\Rightarrow v_2 = 1 m/s.$$

ii) from law of collision of elastic bodies

$$(v_2 - v_1) = e\,(u_1 - u_2)$$

$$\Rightarrow 1 - 0 = e\,(2-0) \Rightarrow e = \frac{1}{2} = 0.5$$

Q-2 A ball overtakes another ball of twice its own mass & moving with 1/7 of its own velocity. If coeff. of restitution between the two balls is 0.75. Show that the first ball will come to rest after impact.

Sol^n Given $m_1 = M\,kg$ $u_1 = U$ $e = 0.75$

$m_2 = 2M\,kg$ $u_2 = \frac{U}{7}$

from law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow MU + \frac{2MU}{7} = Mv_1 + 2Mv_2$$

$\Rightarrow \frac{9MU}{7} = MV_1 + 2MV_2$

$\Rightarrow \frac{9U}{7} = V_1 + 2V_2$ —— ①

From law of conservation of elastic bodies.

$V_2 - V_1 = e(u_1 - u_2)$

$= 0.75\left(U - \frac{U}{7}\right) = \frac{9U}{14}$

$\Rightarrow V_2 = \frac{9U}{14} + V_1$

Putting value of V_2 in eqn ①

$\frac{9U}{7} = V_1 + 2\left(\frac{9U}{14} + V_1\right) \Rightarrow V_1 = 0$

So, the first ball will come to rest after impact.

Q-3 The masses of two balls are in the ratio of 2:1 & their velocities are in the ratio of 1:2, but in opposite dirn before impact. If $e = \frac{5}{6}$, prove that after impact each ball will move back with $\frac{5}{6}$th of its original velocity.

Sol" Given $m_1 = 2M$ $\quad u_1 = U$ $\quad e = \frac{5}{6}$

$m_2 = M$ $\quad u_2 = -2U$ (at opposite dirn)

From law of conservation of momentum

$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

$\Rightarrow (2M)U + M(-2U) = 2MV_1 + MV_2$

$\Rightarrow 0 = 2MV_1 + MV_2 \Rightarrow V_2 = -2V_1$ —— ①

From Law of collision of elastic bodies

$V_2 - V_1 = e(u_1 - u_2) = \frac{5}{6}(U - (-2U)) = \frac{5U}{2}$

$\Rightarrow (-2V_1) - V_1 = \frac{5U}{2} \Rightarrow -3V_1 = \frac{5U}{2} \Rightarrow V_1 = -\frac{5U}{6}$

-ve sign indicates that the dirn of V_1 is opposite to that of U. Thus the 1st ball will move back with $\frac{5}{6}$th of its original velocity.

Now eqn ① becomes $V_2 = -2V_1 = -2\left(-\frac{5U}{6}\right) = 2\left(\frac{5}{6}U\right)$

+ve sign indicates that the dirn of V_2 is same as that of V_1 or opposite to that of u_2.

So the 2nd ball will also move back with $\frac{5}{6}$th of its original velocity.