chp-01

Introduction

Reinforced Concrete

- Concrete is produced by mixing cement, coarse aggregate, fine aggregate (sand) & water mixed in a definite proportion.
- Fresh concrete is an easily workable plastic mixture & it flows easily so that it can be placed in a previously made formwork to cast beam, slab, column or any other desirable shape.
- When reinforcing steel is placed in the forms before fresh concrete is placed in the forms before fresh concrete is placed around it, the final solidified mass becomes the reinforced concrete.

Advantages of reinforced concrete

Following are the advantages of reinforced concrete -

- It has a very high compressive strength. The strength of concrete can be increased or decreased by using suitable proportion of ingredients.
- It is free from corrosion & weathering effect.
- It acts as a good fire proofing material.
- The structure made out of reinforced concrete are very rigid & have a low maintenance cost.
- It has a very long service life.

Disadvantages of reinforced concrete

Following are the disadvantages of reinforced concrete -

- It is weak in tension & hence cracks easily when subjected to tensile stress.
- It requires the form work to be kept for many days. The cost of formwork varies from 30-40 % of the total cost.
- It is not completely impervious.
- It shrinks and sets up shrinkage stresses.

Unit weight

For the purpose of design & estimate of loading, the unit weight of plain concrete with reinforced concrete with sand & gravel or crushed natural stone aggregate may be taken as -

Plain concrete - 24 kN/m³

~~rein~~ reinforced concrete - 25 kN/m³.

Loads on structure

- In general the loads on structure are classified as vertical or gravity loads, horizontal loads & longitudinal loads.
- The vertical loads are further classified as dead load, live load & impact load.
- The horizontal loads are classified as wind load & earthquake load.
- The longitudinal loads are considered in some special cases.

1. Dead Load

→ Dead loads are the loads due to self weight of the structure or structural member.

→ The dead loads are static loads & remain constant throughout the life of the structure.

→ e.g. – loads due to partition walls, flooring, roofs, false ceiling, fixtures etc.

2. Live Load

→ These are the loads which are not steady. Unlike the dead load they change their magnitude.

→ e.g – moving loads like persons, car etc and also movable loads like furniture.

3. Impact Load

→ These are the loads caused by vibration of live loads.

→ e.g – moving crane.

→ There is a difference between a person walking and a soldier marching. The person produces a live load while the soldier produces an impact load.

→ When live loads cause impact, it is usual in static analysis to increase the live load by some percentage depending on the type of impact.

4. Wind Load

These are the lateral loads depend on the velocity of wind. In different parts of the country, the velocity of wind can be different at different places.

5. Earthquake Load

→ These are the horizontal loads caused by earthquake.

→ The country is divided into 4 zones namely zone II, zone III, zone IV & zone V according to probable intensity of earthquake.

→ The earthquake forces ~~can be~~ on the structure shall be calculated in accordance with IS : 1893 – 2002.

6. Longitudinal Loads

→ These are caused by sudden stopping of moving loads.

→ A moving crane, moving truck etc. when abruptly stopped ~~cause~~ cause longitudinal loads.

Properties of reinforcing material

For any material to be used as reinforcement for concrete, it should possess the following properties:

1. It should possess high tensile strength.
2. It should be able to develop a good bond with concrete.
3. It should possess a high modulus of elasticity.
4. It should have same (or nearly same) temperature co-efficient of expansion and contraction as concrete to avoid the developement of thermal stresses.
5. It should be easily available.

Types of reinforcement

The different types of reinforcement used are :-

1. Mild steel
2. Medium tensile steel
3. Hot rolled deformed bars
4. High yield strength deformed (HYSD) bars
5. Hard drawn steel wire fabric

- cold twisted deformed (CTD) bars
- Thermo mechanically treated (TMT) bars.

Note

In all cases the modulus of elasticity of steel shall be

$$\boxed{E_s = 200\,kN/mm^2 = 2\times10^5\,N/mm^2}$$

Grade of Concrete

→ Generally 6 grades of concrete are used for RCC work i.e M_{15}, M_{20}, M_{25}, M_{30}, M_{35}, M_{40}

→ The number in the grade designation refers to the characteristic strength of concrete in a 15 cm cube in 28 days. & generally expressed as N/mm^2 and "M" stands for mix. & the characteristic compressive strength is denoted by "f_{ck}"

~~→ Here, M stands for mix and~~

Grade of concrete	characteristic compressive strength (f_{ck}) in N/mm^2
M_{15}	15
M_{20}	20
M_{25}	25
M_{30}	30
M_{35}	35
M_{40}	40

Modular Ratio (m)

- Modular ratio is the ratio of Young's modulus of elasticity of two materials in construction by composite material.
- In RCC work the materials are generally taken as Concrete & Steel.
- It is denoted by 'm'.
- $$m = \frac{\text{Young's modulus of elasticity of steel}}{\text{Young's modulus of elasticity of concrete}} = \frac{280}{3\sigma_{cbc}}$$

Grade	σ_{cbc}	m
M15	5 N/mm^2	18.66
M20	7 N/mm^2	13.33
M25	8.5 N/mm^2	10.98

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Working Stress Method

Permissible stresses :- In working stress method, the stresses in materials are not exceeded beyond their permissible values. The permissible stresses are found by using suitable factors of safety to the material's strength.

e.g :- For concrete in compression in bending, a factor of safety equal to 3 is considered on characteristic strength of concrete and a factor of safety equal to 1.8 is considered on the yield strength of mild steel reinforcement in tension due to bending.

Permissible Stresses in concrete

Grade of concrete	Permissible stress in compression, N/mm²		Permissible stress in bond (avg.) for plain bars in tension, N/mm²
	Bending (σ_{cbc})	Direct (σ_{cc})	(τ_{bd})
M_{20}	7.0	5.0	0.8
M_{25}	8.5	6.0	0.9
M_{30}	10.0	8.0	1.0

Note - The bond stress in column shall be increased by 25% for bars in compression.

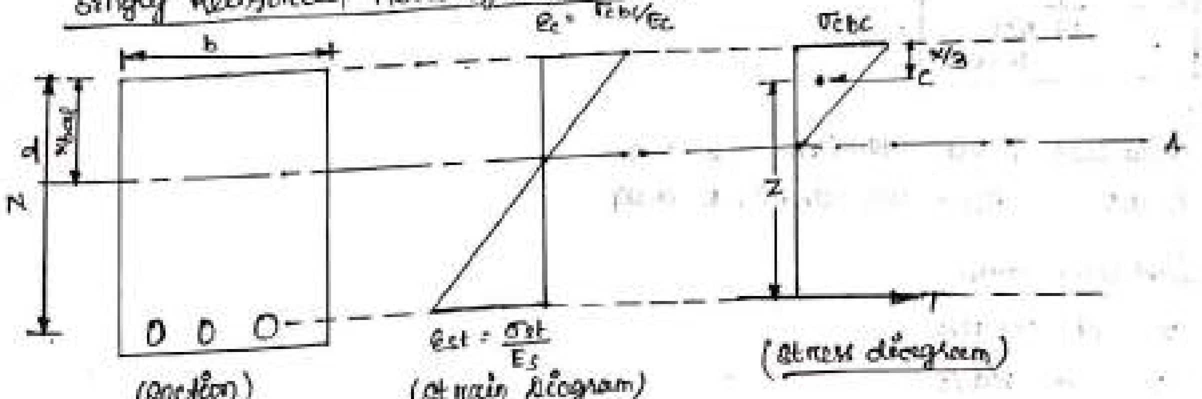
Modular Ratio (m)

The modular ratio 'm' of steel and concrete is defined as

$$m = \frac{\text{modulus of elasticity of steel}}{\text{modulus of elasticity of concrete}} = \frac{280}{3\sigma_{cbc}} \Rightarrow \frac{E_s}{E_c}$$

* Modulus of elasticity of concrete (E_c) = $5000\sqrt{f_{ck}}$

Singly Reinforced Rectangular Beam



(Section) (strain diagram) (stress diagram)

$A_{st,bal}$ = Reinforced area provided for balanced section.
σ_{cbc} = permissible stress in concrete in compression zone.
σ_{st} = permissible stress in steel in tension zone.
E_c = Modulus of elasticity of concrete
E_s = Modulus of elasticity of steel.
e_c = Strain in concrete
e_{st} = Strain in steel.

b = width of beam

d = effective depth of beam

D = overall depth of beam

m = modular ratio = E_s / E_c

x_{bal} = Depth of NA i.e the distance from extreme position of fibre.

z = Lever arm i.e the distance between centroid of compressive force to the centroid of tensile force.

* To find NA

On strain diagram, ΔMNO & ΔOPQ are similar Δ, so

$$\frac{x_{bal}}{d - x_{bal}} = \frac{\sigma_{cbc}/E_c}{\sigma_{st}/E_s}$$

$$\Rightarrow \frac{x_{bal}}{d - x_{bal}} = \frac{\sigma_{cbc} \times E_s}{\sigma_{st} \times E_c}$$

$$\Rightarrow \frac{x_{bal}}{d - x_{bal}} = m \times \frac{\sigma_{cbc}}{\sigma_{st}}$$

$$\Rightarrow x_{bal}\,\sigma_{st} = m\sigma_{cbc}(d - x_{bal})$$

$$\Rightarrow x_{bal}\,\sigma_{st} = m\sigma_{cbc}\,d - m\sigma_{cbc} \times x_{bal}$$

$$\Rightarrow x_{bal}(\sigma_{st} + m\sigma_{cbc}) = m\sigma_{cbc}\,d$$

$$\Rightarrow x_{bal} = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right) d$$

$$\Rightarrow \boxed{x_{bal} = Kd} \quad (\because K = \text{NA constant})$$

$$K = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}$$

$$\Rightarrow K = \frac{m\sigma_{cbc}/m\sigma_{cbc}}{\sigma_{st}/m\sigma_{cbc} + m\sigma_{cbc}/m\sigma_{cbc}}$$

$$\Rightarrow \boxed{K = \frac{1}{1 + \frac{\sigma_{st}}{m\sigma_{cbc}}}}$$

Note

for mild steel ; $\sigma_{st} = 140\ N/mm^2$, $K = 0.4$.

for Fe 415 , $\sigma_{st} = 230\ N/mm^2$, $K = 0.29$

* To find lever arm

$$z = d - x_{bal}/3$$

$$\Rightarrow z = d - Kd/3$$

$$\Rightarrow z = d(1 - K/3)$$

$$\Rightarrow \boxed{z = dj}$$

$j = 1 - K/3$ = lever arm constant

* **To find total forces**

compressive force (C) = comp. stress × area

$= \sigma_{cbc} \times \frac{1}{3} \times b \times x_{bal}$

$\Rightarrow \boxed{C = 0.5\alpha \cdot \frac{b\, x_{bal}\, \sigma_{cbc}}{2}}$

Total tension (T) = stress × Area

$\Rightarrow \boxed{T = \sigma_{st} \times A_{st,bal}}$

* **To find Moment of resistance of the section**

As it is a balanced section, hence comp. & tens. force will be equal.

** MR = comp. force × lever arm

$\Rightarrow MR = \frac{1}{2} \times b \times \sigma_{cbc} \times x_{bal} \times jd$

$\Rightarrow MR = \frac{1}{2} \times b \times \sigma_{cbc} \times kd \times jd$

$\Rightarrow MR = \frac{1}{2} \times \sigma_{cbc} \times kj \times bd^2$

$\Rightarrow \boxed{MR = Q_{bal} \times bd^2 = M_{bal}}$ (comp.)

** MR = tens. force × lever arm

$\Rightarrow \boxed{MR = \sigma_{st} \times A_{st,bal} \times jd}$ (tens)

* **To find Steel area**

For a balanced section

$M_{bal} = A_{st,bal} \times \sigma_{st} \times jd$

$\Rightarrow \boxed{A_{st,bal} = \frac{M_{bal}}{\sigma_{st}\, jd}}$

$P_{t,lim} = \frac{A_{st,bal}}{bd} \times 100$

$\Rightarrow P_{t,lim} = \frac{M_{bal}}{\sigma_{st}\, j\, bd^2} \times 100$

$= \frac{\frac{1}{2} \times \sigma_{cbc} \times kj\, \cancel{bd^2}}{\sigma_{st}\, \cancel{j\, bd^2}} \times 100$

$= \frac{\frac{1}{2}\, \sigma_{cbc}\, k \times 100}{\sigma_{st}}$

$\Rightarrow \boxed{P_{t,lim} = \frac{50\, \sigma_{cbc}\, k}{\sigma_{st}}}$

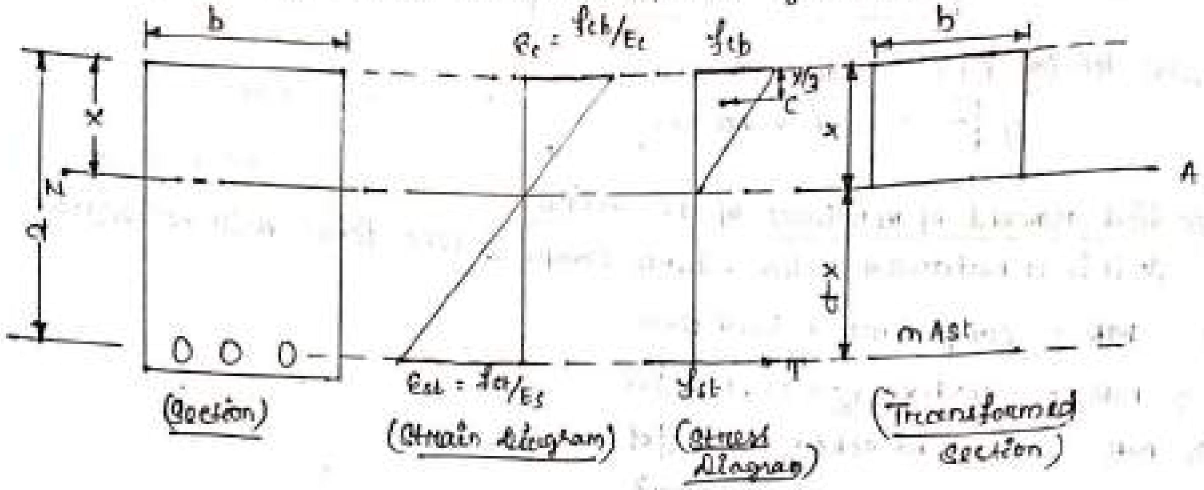
* **To design balanced section**

$M = M_{bal} = Q_{bal} \times bd^2$

$\Rightarrow \boxed{d = \sqrt{\frac{M_{bal}}{Q_{bal} \times b}}}$

Transformed Area Method

→ A transformed area is a area in which the steel area is replaced by an equivalent concrete area. The transformed ~~area~~ section may be of steel when concrete is replaced by steel or it may be of concrete when the steel area is replaced by concrete.



→ In fig. the actual concrete in tension zone is absent because we have assumed that concrete carry tensile force. thus all tensile force will be carried by steel.

→ Let f_{st} & f'_{cb} be the stresses in steel and concrete respectively at the level of centroid of steel.

Strain in concrete = strain in steel

$\Rightarrow \dfrac{f'_{cb}}{E_c} = \dfrac{f_{st}}{E_s}$

$\Rightarrow f_{st} = f'_{cb} \times \dfrac{E_s}{E_c}$

$\Rightarrow \boxed{f_{st} = m f'_{cb}}$

∴ Now $\boxed{\text{force in steel} = m f'_{cb} \times A_{st,bal}}$ —— ①

If the steel is replaced by an equivalent concrete area, the equivalent concrete will carry the same force.

∴ Now the $\boxed{\text{force in equivalent concrete} = \text{transformed area} \times f'_{cb}}$ —— ②

Equating ① & ②, we get

transformed area $\times f'_{cb} = m f'_{cb} \times A_{st,bal}$

$\Rightarrow \boxed{\text{transformed area} = m \cdot A_{st,bal}}$

* To find NA

As the theory of simple bending can be applied, the neutral axis is the centroidal axis of the transformed section. To determine the centroidal axis, the moments of composite area may be taken about any selected axis. eg. top of the section. Then the formula $\bar{x} = \dfrac{\Sigma Ax}{\Sigma A}$ or $\bar{y} = \dfrac{\Sigma Ay}{\Sigma A}$ can be applied. In present case, it is easier to take moments of transformed area about NA itself. Hence $\bar{x} = 0$

There,
$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = 0$$
$$\Rightarrow \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = 0$$
$$\Rightarrow \frac{b \cdot x \cdot x/2 + m A_{st}\{-(d-x)\}}{A_1 + A_2} = 0$$
$$\Rightarrow \frac{b x^2}{2} - m A_{st}(d-x) = 0$$
$$\Rightarrow \frac{b x^2}{2} = m A_{st}(d-x)$$

* Stress in concrete

from strain diagram, $\frac{f_{cb}/E_c}{f_{st}/E_s} = \frac{x}{d-x}$

$$\Rightarrow f_{cb} = \frac{f_{st}}{E_s/E_c} \times \frac{x}{d-x}$$
$$\Rightarrow \boxed{f_{cb} = \frac{f_{st}}{m} \times \frac{x}{d-x}}$$

* Stress in steel

$$\boxed{f_{st} = \frac{M}{A_{st}(d - x/3)}}$$
(where, $d - x/3 = Z$)

* MOMENT OF RESISTANCE

** MR of compression side / concrete

$$\boxed{MR = \sigma_{cbc} \cdot \frac{b x}{2} \cdot Z}$$

** MR of tension side / steel

$$\boxed{MR = \sigma_{st} \cdot A_{st} \cdot Z}$$

Note :- whichever is smaller will be the moment of resistance of the section.

* Numericals

Type - 1 (To find the depth of NA & specify type of beam)

Step-1 :- * If the section and actual stresses are given of the materials. Find out depth of NA using eqn.

$\boxed{x = Kd}$ where, $\boxed{K = \frac{1}{1 + \frac{f_{st}}{m f_{cb}}}}$

* If the section & steel area are provided. Find out depth of NA by taking moments of transformed areas about NA.

$$\boxed{b \cdot x \cdot x/2 = m \cdot A_{st}(d-x)}$$

Step-2 :- Find out the depth of NA for balanced section, also known as depth of critical NA using eqn.

$\boxed{x = Kd}$, where, $\boxed{K = \frac{1}{1 + \frac{\sigma_{st}}{m\, \sigma_{cbc}}}}$

Step-3

1. $x_{actual} < x_{critical}$; under-reinforced section
2. $x_{actual} = x_{critical}$; Balanced section
3. $x_{actual} > x_{critical}$; Over-reinforced section.

Q. A RCC beam 250 mm wide × 520 mm effective depth is reinforced with 3 nos of 16 mm diameter bars. Findout the ~~type~~ depth of NA & type of beam. The materials are M_{20} grade of concrete & HYSD reinforcement of grade Fe 415.

Solution :- Data given,

$b = 250$ mm

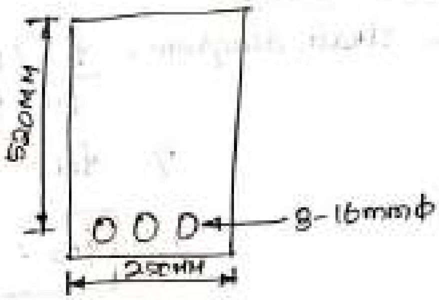
$d = 520$ mm

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

$A_{st} = 3 \times \pi/4 \times (16)^2$

$= 603 mm^2$



Let x be the depth of Neutral axis.

Taking moments of transformed area about neutral axis,

$b \cdot x \cdot x/2 = m A_{st}(d-x)$

$\Rightarrow 250 \times \frac{x^2}{2} = 13.33 \times 603(502-x)$ $\left[\because m \text{ for } M_{20} \text{ grade of concrete} = 13.33\right]$

$\Rightarrow 125x^2 = 4035070.98 - 8037.99x$

$\Rightarrow 125x^2 + 8037.99x - 4035070.98 = 0$

$\Rightarrow x^2 + 64.3x - 32280.56 = 0$

$\Rightarrow x = 150$ mm

Depth of critical NA $= K.d$

$= 0.29 \times 520$

$= 150$ mm

$$\left(K = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}} = \frac{1}{1+\frac{230}{13.33\times 7}} = 0.29\right)$$

$\therefore$ $x_{actual} = x_{critical}$

$\therefore$ Here, the beam is balanced. (H)

Type-2 (To findout ~~depth of NA~~ MR of a given section)

Step-1 :- If the section & actual stresses in the materials are given, find out depth of NA using given eqn.

$\boxed{x = Kd}$ where, $\boxed{K = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cb}}}}$

If the section & steel area are provided, findout NA by using following eqn.

$\boxed{b.x.x/2 = m A_{st}(d-x)}$

Step-2 :- findout depth of NA for balanced section, depth of critical NA by using eqn

$\boxed{x = Kd}$ where, $\boxed{K = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}}}$

Step 3 :- 1. $x_{actual} < x_{critical}$; under - reinforced

2. $x_{actual} = x_{critical}$; Balanced

3. $x_{actual} > x_{critical}$; Over - reinforced

Step 4 :- Then findout MR.

1. If UR secⁿ ($x_{actual} < x_{critical}$)

$$\boxed{MR = A_{st} \cdot \sigma_{st} \cdot (d - x/3)}$$

2. If OR secⁿ ($x_{actual} > x_{critical}$)

$$\boxed{MR = \frac{1}{2} \sigma_{cbc} \cdot b \cdot x \; (d - x/3)}$$

Q: A RCC beam 300 mm wide × 550 mm effective depth is reinforced with 4 nos of 16 mm diameter bars in tension. The materials are M 20 grade of concrete & HYSD bars of grade Fe 415. The permissible stresses in ~~bending~~ concrete in bending compression and steel in tension are 5.6 N/mm² & 210 N/mm² respectively. Findout the MR of the section.

Solution :- Given,

$b = 300$ mm

$d = 550$ mm

$A_{st} = 4 \times \pi/4 \times (16)^2$

$= 804 \text{ mm}^2$

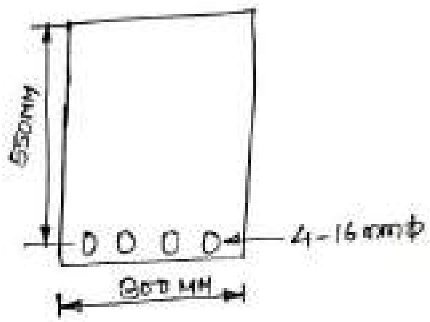
$\sigma_{st} = 210 \text{ N/mm}^2$

$\sigma_{cbc} = 5.6 \text{ N/mm}^2$

$m = \dfrac{280}{3\sigma_{cbc}}$

$= \dfrac{280}{3 \times 5.6}$

$= 16.66$



x actual

$b \cdot x \cdot x/2 = m A_{st} (d - x)$

$\Rightarrow 300 \times x^2/2 = 16.66 \times 804 (550 - x)$

$\Rightarrow 150 x^2 = 7367052 - 13394.64 x$

$\Rightarrow 150 x^2 + 13394.64 x - 7367052$

$\Rightarrow x_{actual} = 181.4$ mm

Critical depth of NA ($x_{critical}$)

$x_{critical} = Kd$

$K = \dfrac{1}{1 + \dfrac{\sigma_{st}}{m \sigma_{cbc}}} = \dfrac{1}{1 + \dfrac{210}{16.66 \times 5.6}} = 0.30$

$= 0.30 \times 550$

$= 165$ mm

Here, $x_{actual} > x_{critical}$; O-R section

$MR = \frac{1}{2} \times \sigma_{cbc} \times b \cdot x \; (d - x/3)$

$= \frac{1}{2} \times 5.6 \times 300 \times 181.4 \left(550 - \frac{181.4}{3}\right)$

$= 74592131.2 \text{ N-mm}^2$

$= 74.59$ KNM (Ans)

Methods of Design

IS:456, permits 3 methods of design. they are:-

1. Limit state method
2. Working stress method
3. Ultimate load method

1. Limit state Method — chp-02 & 03

→ The acceptable limit for the safety & serviceability requirement before failure occurs is known as limit state.

→ In this method of design, the structure is designed to withstand safely all loads liable to act on it throughout its life.

→ The structure also has to be checked for the serviceability requirements such as limitations on deflection and cracking.

→ It may be noted that the concept of limit state analysis is not applicable to a brittle material. Concrete is a brittle material however reinforced concrete posses some ductility due to presence of reinforcing steel, so limit state method can therefore applied to RCC structure.

i. Limit state of collapse

This limit state refers to the strength of the structure. A structure or it's parts should be strong enough to resist the applied design load. this is called limit state of collapse.

ii. Limit state of serviceability

This limit state is introduced to prevent deflection & cracking.

I. Deflection :- Excessive deflection that can reduce the efficiency of the structure must be avoided.

II Cracking :- → Concrete structures have cracks. However if the width of cracks are larger, the appearance of the structure

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will be affected. Also water & gas from atmosphere can cause rusting of reinforcement.

→ for normal concrete structure the surface crack width of 0.3mm is acceptable.

Objectives of design & detailing of a structure

The objectives of designing & detailing of a structure are :-

1. Durability
2. Serviceability.

Modulus of elasticity of concrete

$$\boxed{E_c = 5000\sqrt{f_{ck}}}$$

Flexural strength

$$\boxed{f_{cr} = 0.7\sqrt{f_{ck}}}$$

Q. Find the modulus of elasticity & flexural strength of M_{20} grade of concrete :-

Solⁿ :- Given, $f_{ck} = 20\ N/mm^2$

$$E_c = 5000\sqrt{f_{ck}} = 5000\times\sqrt{20} = 22,360.68\ N/mm^2$$

$$f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\times\sqrt{20} = 3.1304\ N/mm^2$$

Service Load (Axial load)

Service loads are the actual loads that the structure will be subjected which are not factored.

~~Design Load~~

Factored Load

The load subjected ~~to~~ by multiplying a characteristic load by an appropriate partial safety factor is known as factored load.

Partial Safety factor

Steel = 1.15

Concrete = ~~1.15~~ 1.5

Q. A service load of 200 kN is applied on a concrete structure. Find the factored load ?

$$\text{factored load} = \text{Service Load} \times 1.5$$
$$= 200\times 1.5$$
$$= 300\ kN$$

Assumptions in Limit State design

Following are the assumptions made in Limit State.

1. Plane sections normal to the axis remain after bending. This assumption means that strain at any point on the cross-section is directly proportional to its distance from neutral axis.

2. The maximum strain in the concrete at the outermost compression fibre is taken as 0.0035 in bending.

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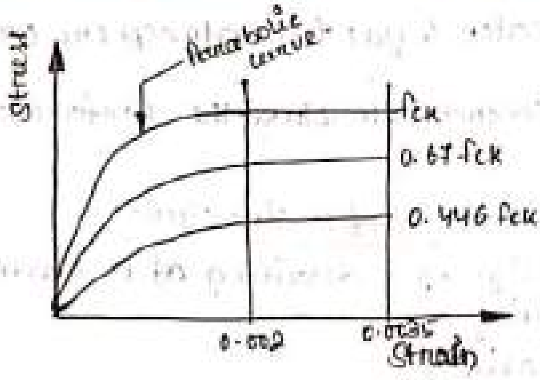


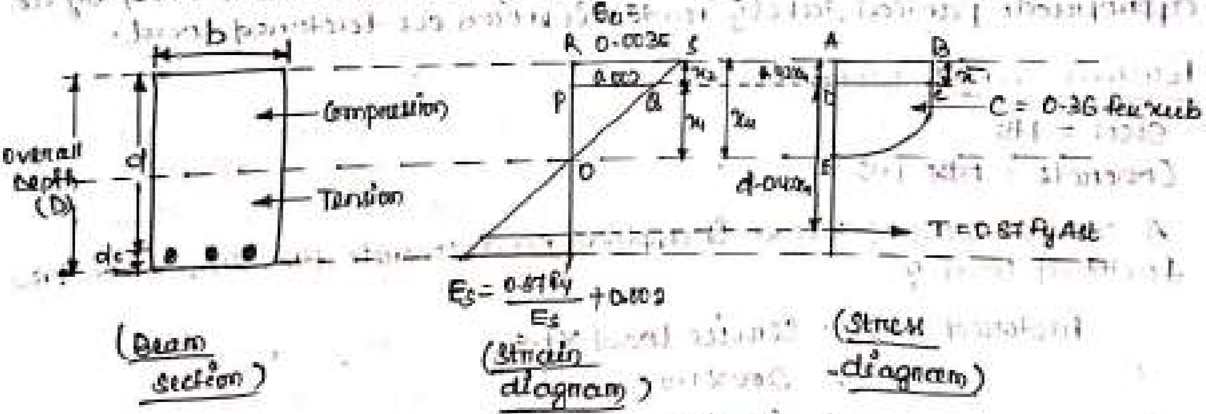
3. The stress strain diagram of concrete is parabolic from strain value of zero to 0.002 & corresponding stress value of zero to fck. The stress now remains constant & strain increase to 0.0035 as shown in fig. Since the concrete is a brittle material, the compressive strength of concrete shall be taken as 0.67 fck. Then applying the partial safety factor for material, $\gamma m = 1.5$, the design flexural strength of concrete shall be $\frac{0.67 fck}{1.5} = 0.446 fck$

4. The tensile strain of concrete is ignored.

5. The maximum strain in the tensile reinforcement in the section at failure shall not be less than $\frac{fy}{1.15 E_s} + 0.002$

Bar	fy (N/mm²)
1. Mild steel	250
2. Fe 415	415
3. Fe 500	500

<u>Stress strain curve of concrete</u>



d = effective depth

dc = clear cover

xu = neutral axis constant

In strain curve ~~triangle~~ ΔOPQ & ΔORS are similar triangles.

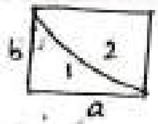
So,
$$\frac{0.002}{0.0035} = \frac{x_1}{x_u}$$
$$\Rightarrow \frac{20}{35} = \frac{x_1}{x_u}$$
$$\Rightarrow \frac{4}{7} = \frac{x_1}{x_u}$$
$$\Rightarrow \boxed{x_1 = 4/7\, x_u}$$

$$x_2 = x_u - x_1$$
$$\Rightarrow x_2 = x_u - 4/7\, x_u$$
$$\Rightarrow x_2 = \frac{7x_u - 4x_u}{7}$$
$$\Rightarrow \boxed{x_2 = 3/7\, x_u}$$

In stress curve the area of ABCD rectangle is $0.446\, f_{ck} \times x_2$
$$= 0.446\, f_{ck} \times 3/7\, x_u$$
$$= 0.1911\, f_{ck}\, x_u$$

Area of DCE parabola $= 2/3 \times 0.446\, f_{ck} \times 4/7\, x_u$
$$= 0.169\, f_{ck}\, x_u$$
$$\simeq 0.17\, f_{ck}\, x_u$$

* Note



1. $A = ab/3$
   $\bar{x} = b/4$
2. $A = 2/3\, ab$; $\bar{x} = b/4$

Now, total area = Area of ABCD rectangle + Area of DCE parabola
$$= 0.19\, f_{ck}\, x_u + 0.17\, f_{ck}\, x_u$$
$$= 0.36\, f_{ck}\, x_u$$

Location of compressive force from the extreme edge

(Here the total compressive force = total area = $0.36\, f_{ck}\, x_u \times b$ (unit length i.e. $d=1$))

$$\bar{x} = \frac{\Sigma A x}{\Sigma A}$$
$$\Rightarrow \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$
$$\Rightarrow \bar{x} = \frac{0.19\, f_{ck}\, x_u \times (x_2/2) + 0.17\, f_{ck}\, x_u\,(x_2 + 3/8\, x_1)}{0.19\, f_{ck}\, x_u + 0.17\, f_{ck}\, x_u}$$
$$\Rightarrow \boxed{\bar{x} = 0.416\, x_u}$$

b. Tensile stress

Tensile stress $= 0.87 f_y$

Tensile force = stress × Area of steel
$$= 0.87\, f_y\, A_{st}$$

Now equating the compressive force to tensile force, we get

Compressive force = tensile force
$$\Rightarrow 0.36\, f_{ck}\, x_u\, b = 0.87\, f_y\, A_{st}$$
$$\Rightarrow \boxed{x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b}}$$

* f_y = characteristic strength of steel
f_{ck} = characteristic strength of concrete

Limiting value of strain of steel

1. for mild steel

we know,

$$\text{strain } \epsilon_{us} = \frac{f_y}{1.15 E_s} + 0.002$$

for mild steel $f_y = 250 \text{ N/mm}^2$

So,

$$\epsilon_{us} = \frac{250}{1.15 \times 2\times10^5} + 0.002$$

$$\Rightarrow \boxed{\epsilon_{us} = 0.00308}$$

2. for Fe-415 ; $f_y = 415 \text{ N/mm}^2$

$$\epsilon_{us} = \frac{415}{1.15 \times 2\times10^5} + 0.002$$

$$\Rightarrow \boxed{\epsilon_{us} = 0.00380}$$

3. for Fe-500, $f_y = 500 \text{ N/mm}^2$

$$\epsilon_{us} = \frac{500}{1.15 \times 2\times10^5} + 0.002$$

$$\Rightarrow \boxed{\epsilon_{us} = 0.00417}$$

*

Type of steel	Strain value
for $f_y = 250 \text{ N/mm}^2$	0.00308
for $f_y = 415 \text{ N/mm}^2$	0.00380
for $f_y = 500 \text{ N/mm}^2$	0.00417

*

Type of Steel	x_u max
$f_y = 250 \text{ N/mm}^2$	0.531d
$f_y = 415 \text{ N/mm}^2$	0.479d
$f_y = 500 \text{ N/mm}^2$	0.456d

Limiting value of x_u max

1. for mild steel

ΔORS & ΔMNO are similar triangles

So,

$$\frac{0.0035}{x_u \max} = \frac{0.00308}{d - x_u \max}$$

$$\Rightarrow x_u \max = \frac{0.0035}{0.00308}(d - x_u \max)$$

$$\Rightarrow x_u \max = 1.13(d - x_u \max)$$

$$\Rightarrow x_u \max + 1.13 x_u \max = 1.13d$$

$$\Rightarrow \boxed{x_u \max = 0.531d}$$

2. for Fe 415

$$\frac{0.0035}{x_u \max} = \frac{0.00380}{d - x_u \max}$$

$$\Rightarrow x_u \max = \frac{0.0035}{0.00380}(d - x_u \max)$$

$$\Rightarrow x_u \max = 0.92d - 0.92 x_u \max$$

$$\Rightarrow \boxed{x_u \max = 0.479d}$$

3. for Fe-500

$$\frac{0.0035}{x_u \max} = \frac{0.00417}{d - x_u \max}$$

$$\Rightarrow x_u \max = \frac{0.0035}{0.00417}(d - x_u \max)$$

$$\Rightarrow x_u \max = 0.839(d - x_u \max)$$

$$\Rightarrow \boxed{x_u \max = 0.456d}$$

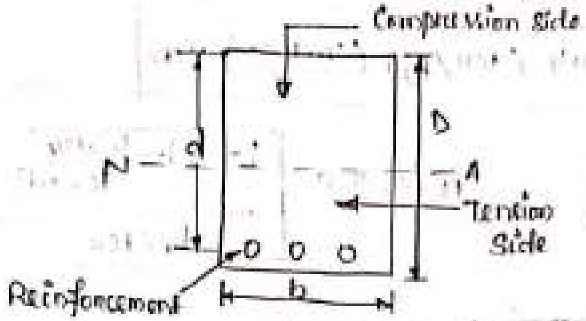
* $x_u$max is the max^m depth of NA at failure cond^n at which concrete reaches to a max^m strain of 0.0035. At this cond^n steel will be in plastic zone with continuous yielding or deformation.

* M_{ulim} is the moment resistance of balanced sec^n.

Classification of Beams :- The beams are classified as :-

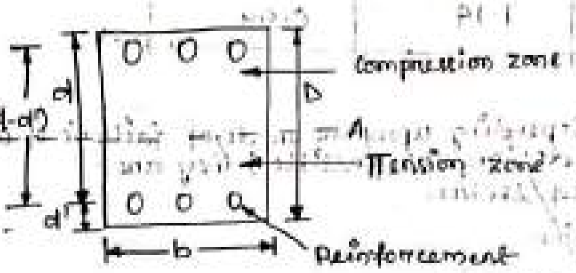
1. Singly reinforced & Doubly reinforced beams
2. Rectangular & flanged beams.

1. **Singly reinforced beam** :- We know that the reinforcement is provided in tension zone while in compression zone concrete resist the applied force. If the concrete is strong enough to resist the applied bending in compression, then such beams are called singly reinforced beam.



d: Effective depth
D: overall depth

2. **Doubly reinforced beam** :- If the concrete in compression zone is unable to withstand all the compression applied to it, it is reinforced in compression zone also. When a beam is additionally reinforced in compression zone then it is called doubly reinforced beam.



3. Rectangular or **Flanged beam** :- The cross-section of a beam may have flanged i.e 'T' shape.

Effective cover :- It is defined as the minimum distance between the surface of concrete to the outside of the reinforcement. It is also called as the clear cover.

Effective Depth :- It is defined as the distance from extreme compression fibre to the centroid of the tensile reinforcement. It is denoted by 'd'.

Overall Depth

$$\boxed{\text{Overall depth (D)} = \text{Effective depth} + \text{bar radius} + \text{Effective cover}}$$

Ex :- Let the effective depth is 500 mm and 22 mm diameter of steel is used. Find the overall depth.?

Soln :- Assume, effective depth = 20 mm

$d = 500\,mm$

$r = d/2 = 22/2 = 11\,mm$

$\therefore$ overall depth $= D = 500 + 11 + 20 = 531\,mm$ . . (A)

Limiting value of percentage of steel

We know,

$$x_{umax} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\Rightarrow \frac{x_{umax}}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

$$\Rightarrow \frac{A_{st}}{bd} = \frac{0.36 f_{ck} x_{umax}}{0.87 f_y d}$$

$$\Rightarrow \boxed{\frac{A_{st}}{bd} \times 100 = 0.414 (f_{ck}/f_y)\left(\frac{x_{umax}}{d}\right) \times 100}$$

$$\boxed{P_t \text{ limit} = \frac{A_{st}}{bd} \times 100} \qquad P_t = 50 \left[\frac{1-\sqrt{1-\frac{4.6 M_u}{f_{ck} bd^2}}}{f_y/f_{ck}}\right]$$

Pt limit

fck	fy = 250	fy = 415	fy = 500
M-15	1.32	0.72	0.57
M-20	1.76	0.96	0.76
M-25	2.20	1.19	0.94

Type of ~~section~~ Design: Depending upon the mode of failure of a beam, 3 types of design are possible; they are:-

- ~~Under reinforced section~~
- ~~Balanced section~~
- ~~Over reinforced section~~

Moment of Resistance

It is the maximum permissible moment resisted by the beam when it is subjected to an external force.

Types of section

Depending upon the mode of failure of a beam, 3 types of sections are possible, they are:-

1. Under reinforced section
2. Balanced section
3. Over reinforced section

1. Under reinforced section

i. It is the section in which $x_u < x_{umax}$

ii. $M_u < M_{umax}$

iii. $P_t < P_t$ limit

iv. Here in the limit of collapse strain in steel is not more than $f_y/1.5E_s + 0.002$ & stress in steel is taken as $0.87 f_y$

v. $\boxed{M_u = 0.87 f_y A_{st} (d - 0.416 x_u)}$

2. Balanced section

i. $x_u = x_{u\,max}$

ii. $p_t = p_{t\,limit}$

iii. $M_u = M_{u\,max}$

iv. The maximum stress in concrete is $0.446 f_{ck}$

v. The maximum strain in concrete is 0.0035.

vi. $\boxed{M_u = 0.36 f_{ck} x_u b \,(d - 0.416 x_u)}$

3. Over - reinforced section

i. $x_u > x_{u\,max}$

ii. $p_t > p_{t\,limit}$

iii. $M_u > M_{u\,max}$

iv. But this type of beam is avoided for the design purpose because the beam may crack without giving prior warning & is dangerous

Mulimit for various Grades of steel

$$M_{u\,limit} = 0.36 f_{ck} x_{u\,max} b (d - 0.416 x_{u\,max})$$

$$= 0.36 f_{ck} x_{u\,max} bd \left(1 - 0.416 \frac{x_{u\,max}}{d}\right)$$

$$= 0.36 f_{ck} \frac{x_{u\,max}}{d} \times bd \times d \left(1 - 0.416 \frac{x_{u\,max}}{d}\right)$$

So, $\boxed{M_{u\,limit} = 0.36 \left(\frac{x_{u\,max}}{d}\right)\left(1 - 0.416 \frac{x_{u\,max}}{d}\right) f_{ck} bd^2}$

for mild steel

$f_y = 250\ N/mm^2$

$$M_{u\,limit} = 0.36 \left(\frac{0.531d}{d}\right)\left(1 - 0.416 \times \frac{0.531d}{d}\right) f_{ck} bd^2$$

$\Rightarrow \boxed{M_{u\,limit} = 0.149 f_{ck} bd^2}$

for Fe-415

$f_y = 415\ N/mm^2$

$$M_{u\,limit} = 0.36 \left(\frac{0.479d}{d}\right)\left(1 - 0.416 \times \frac{0.479d}{d}\right) f_{ck} bd^2$$

$\Rightarrow \boxed{M_{u\,limit} = 0.138 f_{ck} bd^2}$

for Fe-500

$f_y = 500\ N/mm^2$

$$M_{u\,limit} = 0.36 \left(\frac{0.456d}{d}\right)\left(1 - 0.416 \times \frac{0.456d}{d}\right) f_{ck} bd^2$$

$\Rightarrow \boxed{M_{u\,limit} = 0.133 f_{ck} bd^2}$

$$Q_{lim} = \frac{M_{u\,lim}}{bd^2} = \frac{0.36\left(\frac{x_{u\,max}}{d}\right)\left(1 - 0.416 \frac{x_{u\,max}}{d}\right) f_{ck} bd^2}{bd^2}$$

$\Rightarrow \boxed{Q_{lim} = 0.36 (x_{u\,max}/d)(1 - 0.416 \times x_{u\,max}/d) f_{ck}}$

Limiting moment of resistance factor Q_{lim} N/mm² for singly reinforced Rectangular section.

fck (N/mm²)	fy (N/mm²)			
	250	415	500	550
15	2.22	2.07	2.00	1.94
20	2.96	2.76	2.66	2.58
25	3.70	3.45	3.33	3.23
30	4.44	4.14	3.99	3.87

Types of problems in singly reinforced beam

Type-1 To find out the depth of neutral axis & specify the type of beam

Steps

I. find A_{st}

II. find x_u from $x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b}$ = Depth of neutral axis

III. find $x_{u\,max}$ using limiting value of $\dfrac{x_{u\,max}}{d}$

IV. If $x_u < x_{u\,max}$; under-reinforced
$x_u = x_{u\,max}$; Balanced
$x_u > x_{u\,max}$; over-reinforced

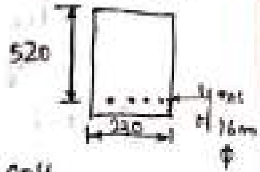
or

find Percentage of reinforcement (P_t) & shall be compared with $P_{t\,lim}$

If $P_t < P_{t\,lim}$; under-reinforced
$P_t = P_{t\,lim}$; Balanced
$P_t > P_{t\,lim}$; over-reinforced

Problem

Q. A rectangular beam 230 mm wide & 520 mm effective depth is reinforced with 4 nos. of 16 mm dia bars. Findout the depth of neutral axis and specify the type of beam. The materials are M20 grade of concrete & HYSD reinforcement of grade Fe 415. Also findout the depth of neutral axis if the reinforcement is increased to 4 nos. of 20 mm dia bars.



Soln:- Case-1 :

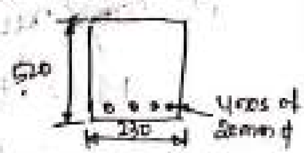
$$A_{st} = 4 \times \pi/4 \times (16)^2 = 804 \text{ mm}^2$$

$$\text{Depth of neutral axis} = x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 804}{0.36 \times 20 \times 230} = 175.3 \text{ mm}$$

$$\text{Limiting value of neutral axis} = x_{umax} = 0.479 d = 0.479 \times 520 = 249 \text{ mm}$$

∴ Here, $x_u < x_{umax}$, hence the section is under-reinforced.

Case-2



$$A_{st} = 4 \times \pi/4 \times (20)^2 = 1256.6 \text{ mm}^2$$

$$\text{Depth of neutral axis} = x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 1256.6}{0.36 \times 20 \times 230} = 274 \text{ mm}$$

$$\text{Limit value of neutral axis} = x_{umax} = 0.479 d = 0.479 \times 520 = 249 \text{ mm}$$

∴ Here, $x_u > x_{umax}$; hence the section is over reinforced.

Type-2 (To findout moment of resistance for a given section)

Steps

i. Findout the depth of NA & the type of beam

ii. For under reinforced & balanced section, obtain MR by using eqn.

$$\boxed{M_u = 0.36 \frac{x_u}{d}\left(1 - 0.416 \frac{x_u}{d}\right) f_{ck} b d^2}$$

or

$$\boxed{M_u = 0.87 f_y A_{st} (d - 0.416 x_u)}$$

or

$$\boxed{M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d}\right)}$$

iii. For over-reinforced section, obtain MR by using eqn.

$$M_{ulim} = Q_{lim} b d^2$$

$$\boxed{M_{ulim} = 0.36 \frac{x_{umax}}{d}\left(1 - 0.416 \frac{x_{umax}}{d}\right) f_{ck} b d^2}$$

Q. A singly reinforced rectangular beam of width 230 mm & 460 mm depth is reinforced with 3 nos. 20 mm dia bars. Findout the factored moment of resistance of the section. The materials are M_{20} grade of concrete & HYSD reinforcement of grade Fe 415. Also findout the factored moment of resistance if it is reinforced with 5 nos. of 20 mm dia bars.

Solution :-

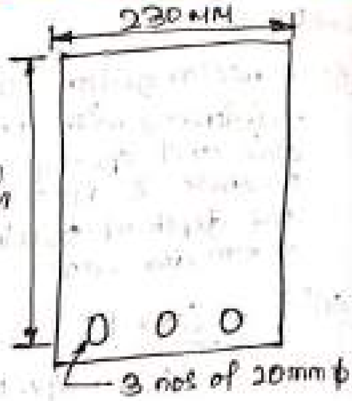
Data Given,

$b = 230$ mm

$d = 460$ mm

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$



Case-1

$A_{st} = 3 \times \pi/4 \times (20)^2 = 942\ mm^2$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b} = \frac{0.87 \times 415 \times 942}{0.36 \times 20 \times 230} = 205.4 \text{ mm}$$

$x_{u\,max} = 0.479\,d = 0.479 \times 460 = 220.3$ mm

$\because x_u < x_{u\,max}$, hence it is an under-reinforced section.

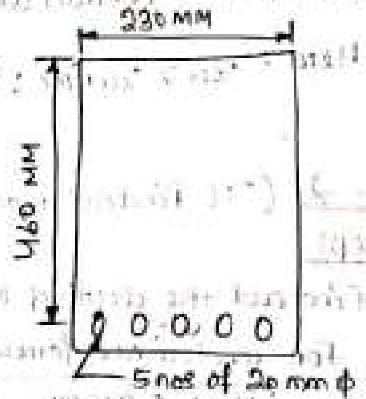
$M_u = 0.87 f_y A_{st} (d - 0.416 x_u)$

$= 0.87 \times 415 \times 942 \,(460 - 0.416 \times 205.4)$

$= 127389087.8$ N-mm

$= 127.38$ KN-M

Case-2



$A_{st} = 5 \times \pi/4 \times (20)^2 = 1571\ mm^2$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b} = \frac{0.87 \times 415 \times 1571}{0.36 \times 20 \times 230}$$

$= 342.5$ mm

$x_{u\,max} = 0.479\,d = 0.479 \times 460 = 220.3$ mm

$\therefore x_u > x_{u\,max}$, hence it is an over-reinforced section.

$$M_u = 0.36 \frac{x_{u\,max}}{d}\left(1 - 0.416 \frac{x_{u\,max}}{d}\right) f_{ck}\, b d^2$$

$$= 0.36 \times \frac{0.479\,d}{d}\left(1 - 0.416 \frac{0.479\,d}{d}\right) \times 20 \times 230 \times (460)^2$$

$= 134400493.5$ N-mm

$= 134.4$ KNM

Type-3 (To design a singly reinforced rectangular section for a given width & applied factored moment).

1. The width is usually decided by the functional requirements. To decide the depth, first determine the balanced depth.

$$\boxed{d_{bal} = \sqrt{\frac{M_u}{Q_{lim} \times b}}}$$

where,

$$\boxed{Q_{lim} = 0.36\, x_u (1 - 0.416\, x_u) f_{ck}}$$

Note

If the balanced area is found with this depth, difficulty will arise while providing the reinforcements. Selecting exact number of reinforcement with available diameter is hardly possible. We cannot provide lesser area. If we provide larger area, the section becomes over-reinforced. Therefore under-reinforced section should be followed by using larger depth than balanced one.

2. The following procedure may be adopted as one of the alternatives for depths upto 1000 mm to arrive a practical solution.

i. Assume 5% larger effective depth for $d \leq 500$ mm (case-1) & 10% larger depth for $1000\text{ mm} \geq d \geq 500$ mm (case-2).

ii. Assume 1 layer of 20 mm diameter bars for (case-1) & two layers of 20 mm diameter bars for (case-2)

iii. If the clear cover to main reinforcement is 30 mm effective cover $= 30 + 10 = 40$ mm for case-1 (assume 1 layer of 20 mm ϕ)

$= 40 + 20 = 60$ mm for case-2 (assume 2 layer of 20 mm ϕ)

iv. The overall depth can be obtained as

$D_{overall} = d + 40$ mm for case-1

$= d + 60$ mm for case-2

v. The value thus obtained shall be rounded upto nearest 25 mm.

vi. Now, $d = D - 40$ mm case-1

$d = D - 60$ mm case-2

vii. Determine $\frac{M_u}{bd^2}$, P_t & A_{st} using eqn of P_t as per the case of under-reinforcement section. Also determine $A_{st,lim}$

viii Select the bar size & number such that $A_{st} < A_{st,lim}$.

Q. Design a singly reinforced rectangular beam for an applied factored moment of 120 KNM. Assume the width of the section as 230 mm. The materials are M 20 grade of concrete & HYSD reinforcement of grade Fe 415.

Solution :- Data given,

$M_u = 120$ KNM

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

$b = 230$ mm

$Q_{lim} = 2.76$

$P_{t,bal} = 0.96$

} Check the table & IS 456

Balanced depth, $d = \sqrt{\frac{M_u}{Q_{lim} \times b}} = \sqrt{\frac{120 \times 10^6}{2.76 \times 230}} = 434.8 \text{ mm}$

Increase the depth by 5% and add 40 mm effective cover.

$$D = 1.05 \times 434.8 + 40 = 496.5 \text{ mm}$$

Rounding up 'D' to nearest multiple of 25 mm = 500 mm = D

D = 500 mm. (Assuming one layer of 20 mm dia bars)

~~d = 500 - 30(cover) - 10 (ϕ/2)~~

$d = D - c.c - \phi/2$ [$\because$ c.c = clear cover, ϕ = diameter of bar]

$= 500 - 30 - 20/2$

$= 460 \text{ mm}$

$$P_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 120 \times 10^6}{20 \times 230 \times (460)^2}}}{415/20} \right]$$

$= 0.824$

$\therefore A_{st}$ required $= \frac{P_t \times b \times d}{100} = \frac{0.824 \times 230 \times 460}{100} = 871.792 \text{ mm}^2 \simeq 872 \text{ mm}^2$

$A_{t,lim} = \frac{P_{t,bal} \times b \times d}{100} = \frac{0.96 \times 230 \times 460}{100} = 1015.68 \text{ mm}^2 \simeq 1016 \text{ mm}^2$

A_{st}, provided = ~~Provide~~ Assuming 3 nos of 20 mm dia bars

$= 3 \times \pi/4 \times (20)^2$

$= 942 \text{ mm}^2$

$\therefore$ Thus, $A_{st,Required} < A_{st,provided} < A_{st,lim}$

Finally we have $b \times D = 230 \text{ mm} \times 500 \text{ mm}$ (Ans)

Type-4 (To find the steel area for a given factored moment)

Steps:

1. For a given ultimate moment (also known as factored moment) & assumed width of section, findout 'd' from the eq^n

$$\boxed{d = \sqrt{\frac{M_u}{Q_{lim} \times b}}}$$

This is a balanced section & balanced steel area may be foundout using the following eq^n

$$\boxed{M_u = 0.87 f_y A_{st} (d - 0.416 x_u)}$$

Alternatively $P_{t,lim}$ may be obtain from table.

2. For a given factored moment, width & depth of section

$$\boxed{M_{u,lim} = Q_{lim}\, b d^2}$$

If,

i. $M_u < M_{u,lim}$: design as U-R section

ii. $M_u = M_{u,lim}$; design as balanced section

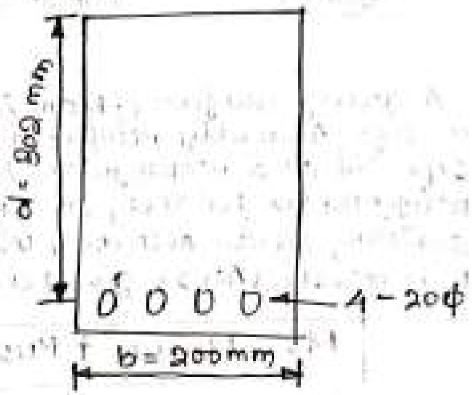
iii. $M_u > M_{u,lim}$: redesign the sec^n either increasing the dimensions of section or design as doubly-reinforced beam.

For U-R section, the steel area can be obtained by using equation

$M_u = 0.87 f_y A_{st} (d - 0.416 x_u)$
$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{b d f_{ck}}\right)$
$P_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}}{f_y / f_{ck}}\right]$

Q. A rectangular singly reinforced beam is subjected to a bending moment of 36 kNM at working loads. The width of the beam is 200 mm. Find the depth & steel area for balanced design. The materials are M20 grade of concrete and mild steel reinforcement.

<u>Solution</u> :- Data Given,



$b = 200 mm$

$M_u = 1.5 \times 36$

$= 54 kNM$

$= 54 \times 10^6$ N-mm

$f_{ck} = 20 N/mm^2$

$f_y = 250 N/mm^2$

$Q_{lim} = 2.96$

$$d = \sqrt{\frac{M_u}{Q_{lim} \times b}}$$

$$= \sqrt{\frac{54 \times 10^6}{2.96 \times 200}}$$

$= 302$ mm

$M_u = 0.87 f_y A_{st} (d - 0.416 x_u)$

$\Rightarrow 54 \times 10^6 = 0.87 \times 250 \times A_{st} (302 - 0.416 \times 0.531)$ $\left(\because \frac{x_u}{d} = \frac{x_{u,max}}{d} = 0.531 d\right)$

$\Rightarrow 54 \times 10^6 = 51175 \, A_{st}$

$\Rightarrow A_{st} = 1055 \, mm^2$

Assuming 20mm dia bars are used.

$$\text{No. of bars} = \frac{A_{st}}{\frac{\pi}{4} \times (\phi)^2}$$

$$= \frac{1055}{\frac{\pi}{4} \times (20)^2}$$

$= 3.35$ nos

$\simeq 4$ nos. (A)

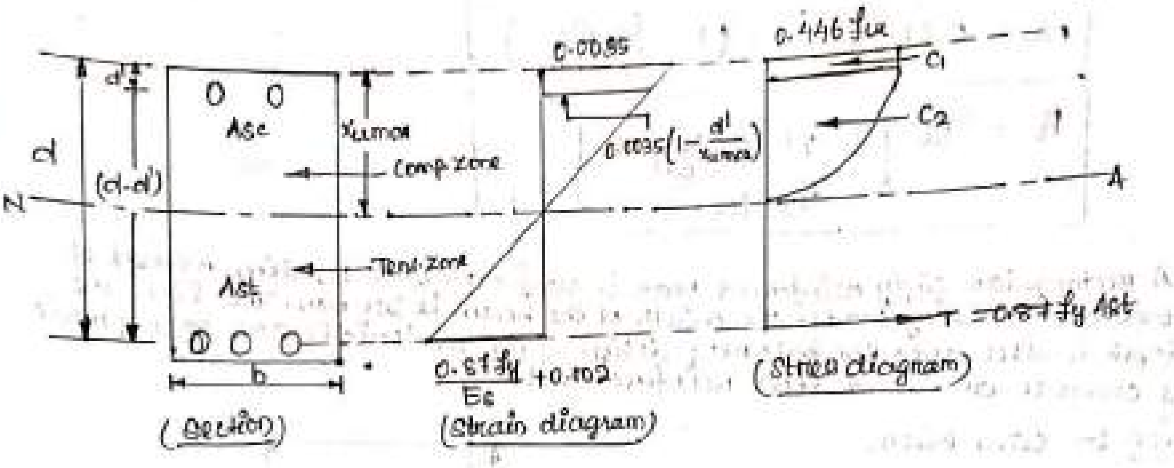
Doubly Reinforced Beams

If the applied moment is greater than the M.R of a singly reinforced section, there can be 3 alternatives.

(1) If possible, increase the dimensions of the section, preferably depth.

(2) Higher grade concrete can be used to increase the M.R of the section.

(3) Steel reinforcement may be added in compression zone to increase the M.R of the section. This is known as doubly reinforced section.



→ A doubly reinforced beam section, strain & stress diagram are shown in fig. A doubly reinforced beam subjected to a moment Mu can be expressed as a rectangular section with tension reinforcement (Ast,lim) reinforced for balanced condition giving moment of resistance (Mu,lim) + an auxiliary section reinforced with compression reinforcement (Asc) & tensile reinforcement (Ast2) giving a moment of resistance Mu2 such that.

$$\boxed{M_u = M_{u,lim} + M_{u2}}$$

→ For the moment Mu lim the tension steel Ast,lim is found out as explained for singly reinforced beams. For the additional moment Mu2 the additional tension steel and compression steel are provided such that they give a couple of moment Mu2.

→ Let the compression reinforcement be provided at a depth "d'" from the extreme compression fibre. Then lever arm for additional moment will be "d-d'".

→ considering tension steel

$$\boxed{M_{u2} = A_{st2} \times 0.87 f_y (d-d')}$$

→ Considering compression steel

$$\boxed{M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d-d')}$$

where,
A_{st2} = area of add^n tens. reinforcement
A_{sc} = area of comp. reinforcement
f_{sc} = stress in " "
f_{cc} = comp. stress in concrete at level of comp. steel.

→ Now,
additional tension = additional compression

$\Rightarrow 0.87 f_y A_{st2} = A_{sc} (f_{sc} - f_{cc})$

$$\Rightarrow \boxed{A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y}}$$

→ Total tensile reinforcement $\boxed{A_{st} = A_{st,lim} + A_{st2}}$

→ We know.

Total compression = Total tension

$\Rightarrow C_1 + C_2 = T$

$\Rightarrow \boxed{0.36 f_{ck} x_u b + A_{sc}(f_{sc} - f_{cc}) = 0.87 f_y A_{st}}$ using the formula findout x_u.

→ $f_{cc} = 0.446 f_{ck}$

$f_{sc} = d'/d$.

f_y (N/mm²)	d'/d			
	0.05	0.10	0.15	0.20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370
550	458	441	419	380

Types of Problems

Type-1 (To findout the MR of a given section)

Steps

1. Findout x_u from eqn.

$$\boxed{(0.36 f_{ck} x_u b) + A_{sc}(f_{sc} - f_{cc}) = 0.87 f_y A_{st}}$$

2. Find x_{umax} & type of beam.
3. Findout MR from

$$\boxed{MR = 0.36 f_{ck} x_u b (d - 0.416 x_u) + A_{sc}(f_{sc} - f_{cc})(d - d')}$$

or $\boxed{MR = 0.87 f_y A_{st} Z}$

Note :- If it's an over-reinforced section, then use x_{umax} instead of x_u.

Q. Find the factored moment of resistance of a beam section 230 mm x 460 mm effective depth reinforced with 2-16mm dia bars as compression reinforcement at an effective cover of 40 mm and 4-20mm diameter bars as tension reinforcement. The materials are M20 grade of concrete & mild steel.

Solution :- Data Given.

$b = 230$ mm

$d = 460$ mm

$d' = 40$ mm

$f_{ck} = 20$ N/mm²

$f_y = 250$ N/mm²

$A_{sc} = 2 \times \pi/4 \times (16)^2 = 402$ mm²

$A_{st} = 4 \times \pi/4 \times (20)^2 = 1256$ mm²

$f_{sc} = d'/d = 40/460 = 217$ N/mm²

$f_{cc} = 0.446 f_{ck} = 8.92$ N/mm² (as the value is very less so neglecting) we know,

$$0.36 f_{ck} x_u b + A_{sc} f_{sc} = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times x_u \times 230 + 402 \times 217 = 0.87 \times 250 \times 1256$$

$$\Rightarrow x_u = \frac{0.87 \times 250 \times 1256 - 402 \times 217}{0.36 \times 20 \times 230}$$

$$\Rightarrow x_u = 112.29 \text{ mm}$$

$$x_{umax} = 0.531 d$$

$$\Rightarrow x_{umax} = 0.531 \times 460$$

$$\Rightarrow x_{umax} = 244.26 \text{ mm}$$

$\because x_u < x_{umax}$; hence the section is under-reinforced.

$$MR = 0.87 f_y A_{st} \times z$$

$$= 0.87 f_y A_{st} (d - 0.416 x_u)$$

$$= 0.87 \times 250 \times 1256 (460 - 0.416 \times 112.29)$$

$$= 112901841 \text{ N-mm}$$

$$= 112.90 \text{ KNM} \quad \text{(Ans)}$$

Type - 2 (To find out reinforcement for flexure for a given section and factored moment)

Steps

1. Find out $M_{u,lim}$ and $A_{st,lim}$ for a given section by using the eqn.

$$M_{u,lim} = Q_{lim} bd^2$$

$$\Rightarrow \boxed{M_{u,lim} = 0.36 f_{ck} b\, x_{umax} (d - 0.416 x_{umax})}$$

and $\boxed{A_{st,lim} = \dfrac{M_{u,lim}}{0.87 f_y (d - 0.416 x_{umax})}}$

2. Obtain moment $\boxed{M_{u2} = M_u - M_{u,lim}}$

3. Find compression steel from equation

$$\boxed{M_{u2} = A_{sc}(f_{sc} - f_{cc})(d - d')}$$ Neglecting f_{cc}

$$\boxed{A_{sc} = \frac{M_{u2}}{f_{sc}(d - d')}}$$

4. Corresponding tension steel A_{st2} may be found out from

$$\boxed{A_{st2} = \frac{A_{sc} f_{sc}}{0.87 f_y}}$$

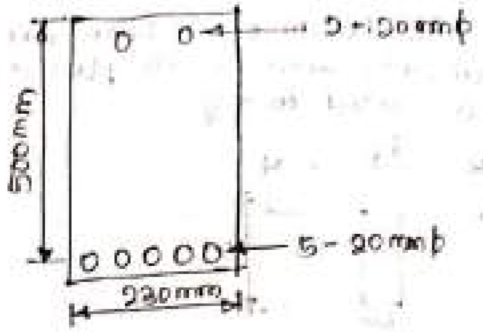
5. $\boxed{A_{st} = A_{st,lim} + A_{st2}}$

6. Provide reinforcement.

7. Find x_u, x_{umax} type of beams and MR for designed section.

Q: A rectangular beam of size 230mm × 500 mm effective depth is subjected to a factored moment of 200 KNM. Find the reinforcement for flexure. The materials are M20 grade of concrete and HYSD reinforcement of grade Fe 415.

Solution :- Data given,



$b = 230$ mm

$d = 500$ mm

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

$M_u = 200$ KNM

$= 200 \times 10^6$ N-mm

$M_{u,lim} = Q_{lim}\, b d^2$

$= 2.76 \times 230 \times (500)^2$

$= 158700000$ N-mm

$= 158.7$ KNM

$M_{u2} = 200 - 158.7 = 41.3$ KNM

Let the compression reinforcement be provided at an effective cover of 50mm.

$d'/d = 50/500 = 0.1$

$\therefore f_{sc} = 353\ N/mm^2$

$$A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y (d - 0.416\, x_{umax})} = \frac{158.7 \times 10^6}{0.87 \times 415 (500 - 0.416 \times 239.5)} = 1098\ mm^2$$

$x_{umax} = 0.479\, d$

$= 239.5$ mm

$$A_{sc} = \frac{M_{u2}}{f_{sc}(d - d')} = \frac{41.3 \times 10^6}{353(500-50)} = 260\ mm^2$$

$$A_{st2} = \frac{A_{sc} f_{sc}}{0.87 f_y} = \frac{260 \times 353}{0.87 \times 415} = 254\ mm^2$$

$A_{st} = A_{st,lim} + A_{st2} = 1098 + 254 = 1352\ mm^2$

Providing 20 mm dia bars

No. of bars @ comp. zone $= \frac{260}{\frac{\pi}{4} \times 20^2} = 0.80 \simeq 2$ nos

No. of bars @ tens. zone $= \frac{1352}{\frac{\pi}{4} \times 20^2} = 4.30 \simeq 5$ nos

A_{st}, provided $= 5 \times \frac{\pi}{4} \times 20^2 = 1570\ mm^2$

A_{sc}, provided $= 2 \times \pi/4 \times 20^2 = 628\ mm^2$

$$x_u = \frac{0.87 f_y A_{st} - A_{sc}(f_{sc})}{0.36 \times f_{ck} \times b} = \frac{0.87 \times 415 \times 1570 - 628 \times 353}{0.36 \times 20 \times 230} = 208\ mm$$

$\therefore x_u < x_{umax}$; hence U-R sec^n.

$M_R = 0.87 f_y A_{st} (d - 0.416 x_u)$

$= 0.87 \times 415 \times 1570 (500 - 0.416 \times 208)$

$= 234375983$ N-mm

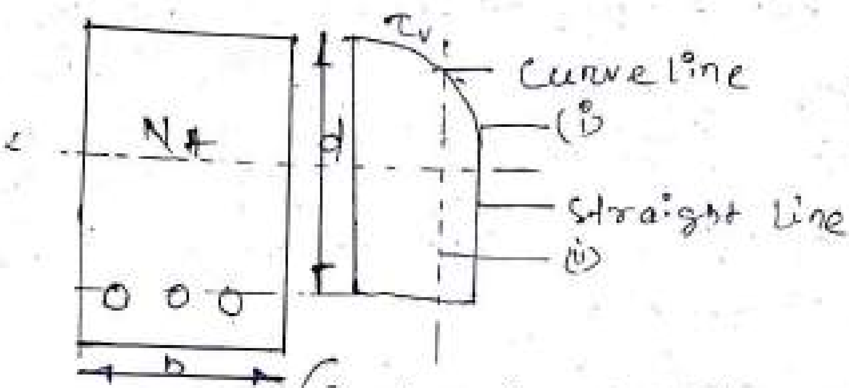
$= 234.37$ KNM (H)

Chapter - 4

Shear, Bond and Devlopment

Shear Stress (τ_v)

The distribution of shear in reinforced concrete rectangular T & L - beams of uniform and varying depths depends on the distribution of the normal stress.

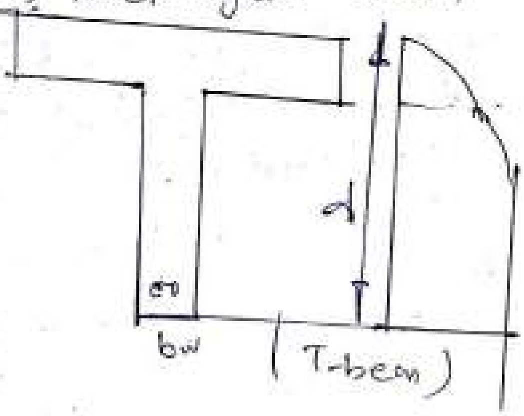


(Rectangular beam)

(i Actual distribution

(ii) Average distribution

(a) Rectangular beam



(T-beam)

Design shear strength of reinforced Concrete:-

The shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maxm value $\tau_{c\,max}$ depending on the grad of concrete.

Minimum Shear Reinforcement

Minimum shear reinforcement has to be provided when τ_v is less than τ_c given in 40.3 of IS. 456.

The minimum shear reinforcement in the form of stirrups shall be provided

$$\frac{A_{sv}}{b\,s_v} \geq \frac{0.4}{0.87 f_y}$$

where A_{sv} = total cross-sectional area of stirrup

s_v = stirrup spacing along the length of member.

b = breadth of the beam

f_y = Characteristic strength of the stirrup reinforcement in N/mm2 which shall not be taken greater than 415 N/mm2

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Design of shear Reinforcement (Cl. 40.4 of IS 456)

When τ_v is more τ_c given in shear reinforcement shall be provided in any of

(a) vertical stirrup

(b) Bent-up bars along with stirrups..

(c) Inclined stirrups.

$$V_{us} = V_u - \tau_c bd$$

(a) vertical stirrups

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

(b) for inclined stirrups

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}(\sin\alpha + \cos\alpha)$$

(c) for single bar or single group of parallel bars

$$V_{us} = 0.87 f_y A_{sv} d \sin\alpha$$

where A_{sv} = total cross-sectional area of stirrup legs

s_v = spacing of stirrup

τ_v = nominal shear stress

τ_c = design shear strength,

b = width

f_y = characteristic strength of the stirrup

α = angle, d = effective depth

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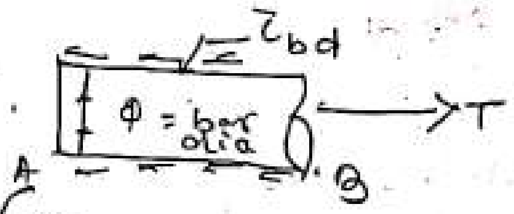
Bond

The bond between steel and concrete is very important and essential so that they can act together without any slip in a loaded structure. with the perfect bond betn them, the plane section of a beam remains plane even after bending.
the length of a member required to develop the full bond is called <u>anchorage length</u>. The bond is measured by bond stress.



(Development length of bar)

tensile force $T = (\pi\phi^2 \sigma_s/4)$
where ϕ is the nominal diameter of the bar
σ_s = tensile stress
τ_{bd} = resistance force
L_d = length

resistance force $= \pi\phi \, L_d \, (\tau_{bd})$
$= \pi\phi^2 \sigma_s / 4$

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}}$$

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$fd = 0.87 fy$

Problem - 2

A reinforced concrete bean is supported on two walls 250mm thick, spaced at a clear distance of 6m. The bean carries a super-imposed load of 9.8 KN/m. design using M20 of concrete and Fe500 bar,

for Fe415 steel $f_s = 0.58 \times 415 = 240 N/mm^2$

Also $l/d = 20 \times 1 \Rightarrow d = l/20 = 6000/20 = 300$

Overall depth

$= 400 + 25 + 8 + 0.5 \times 20 = 443mm$

Assume $b = 250mm$ $\approx 450mm$

Load calculation

$DL = 0.25 \times 0.45 \times 1 \times 25 = 2.81 KN/m$

$LL = 9.8 KN/m$

total Load $= 2.81 + 9.8 = 12.61 KN/m$

$W_u = 1.5 \times W = 18.91 KN/m$

effective span $= (6 + 0.25/2 + 0.25/2)$

$= 6.25m$

Calculation of BM

$BM = \frac{W_u l^2}{8} = \frac{18.91 \times 6.25^2}{8} = 92.376 KN.m$

Max SF $= \frac{W_u l}{2} = 59.12 KN$

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$M_u = 0.138 f_{ck} b d^2$

$$d = \sqrt{\frac{92.37 \times 10^6}{0.138 \times 20 \times 250}} = 365.89$$

depth = 407 mm

development Length

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd} \times 1.6} = \frac{20 \times 0.87 \times 415}{4 \times 1.2 \times 1.6}$$

$$= 940.23 \text{ mm}$$

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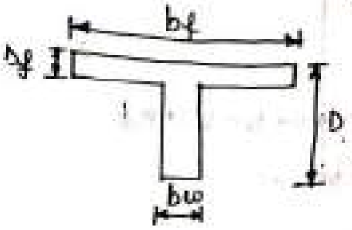
Flanged Beams

Definition

It's simply a rectangular beam cast monolithically with the slab shaped as 'T'.

A "tee beam or ell beam" can be considered as a rectangular beam with dimensions $b_w \times D$ plus a flange of size $(b_f - b_w) \times D_f$. This is indicated in fig.



Advantages

The various advantages of T-beam are as follows :-

1. Since the beam is cast monolithically with the slab, the flange also takes up the compressive stresses which means, it will be more effective in resisting the sagging moment acting on the beam.

2. Better head room, this is direct outcome of the first point since the depth of the beam can be considerably reduced.

3. For longer spans, T-beams are usually preferred rather than rectangular beam as the deflection is reduced to a good extent.

Disadvantages

The various disadvantages of T-beam are as follows :-

1. There is considerable increase in the shear stress at the junction of the flange and the web of the beam due to the change in cross-section. So casting should be done very carefully to ensure both are bonded well.

2. Since the beam slab is monolithic (rigid), it becomes very weak in resisting lateral shear forces. cracks develop quickly. Hence usually in earthquake prone zones using t-beams for high rise building is reinforced with mechanical stiffness in the junction.

3. There will be small savings in steel too (not a significant amount though).

Position of Neutral Axis

For a flanged beam, the neutral axis either (a) lies in flange or (b) lies in web. For a given section, to decide whether the neutral axis lies in flange or web, the flange force and the total tension may be compared as explained below.

As a first approximation, let us assume that the neutral axis lies at the bottom of flange.

Now, total compression. total tension

$F_{tc} = 0.36 f_{ck} b_f D_f$; $F_{ts} = 0.87 f_y A_{st}$

Then,

1. If $F_{tc} > F_{ts}$; NA lies in flange
2. If $F_{tc} = F_{ts}$; NA lies at the bottom of the flange.
3. If $F_{tc} < F_{ts}$; NA lies in the web

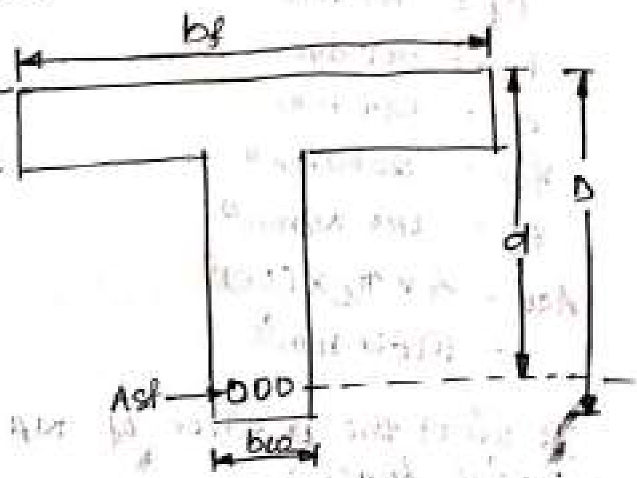
Then.

1. $x_u < x_{umax}$; the section is under-reinforced
2. $x_u = x_{umax}$; the section is balanced
3. $x_u > x_{umax}$; the section is over-reinforced

Derivation

Neutral axis lies in flange ($x_u < D_f$)

b_f = width of flange
b_w = width of web
D_f = Depth of flange
d = effective depth
D = overall depth
A_{st} = Area of steel
d_c = clear cover



When the neutral axis lies in the flange, the size of the compression zone becomes $b_f \times x_u$. As concrete does not resist any tension, the width of tension zone has no effect on the M.R. of the section. Therefore, this beam can be throughout of as a rectangular beam of dimensions $b_f \times d$. The formulae derived for rectangular beams shall be applied. They are summarised below.

Let,
$M_{u,lim\,T}$ = moment resisting capacity of a flanged beam.

* For a singly reinforced flanged beam

i. Equating total compression & total tension

$$\boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}}$$

ii. For under-reinforced section

$$\boxed{M_u = 0.87 f_y A_{st} (d - 0.416 x_u)}$$

or

$$\boxed{M_u = 0.36 f_{ck} b_f x_u (d - 0.416 x_u)}$$

iii. For balanced or over reinforced section

$$\boxed{M_{u,lim\,T} = 0.36 f_{ck} b_f x_{umax} (d - 0.416 x_{umax})}$$

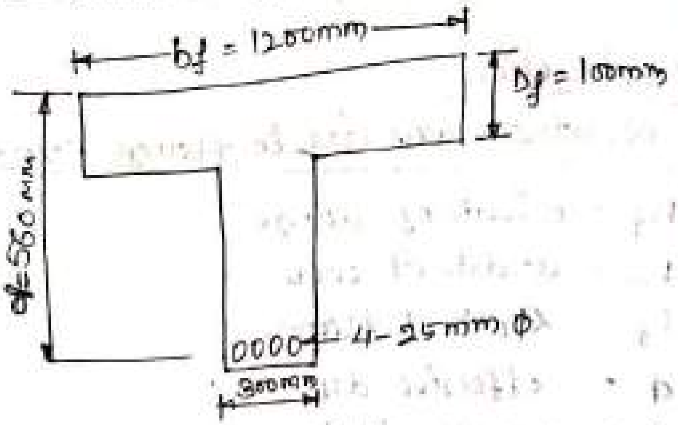
or

$$\boxed{M_{u,lim\,T} = 0.87 f_y A_{st,lim} (d - 0.416 x_{umax})}$$

Q. ~~A tee beam of of~~

Q. A tee beam of effective flange width 1200 mm, thickness of slab 100mm, width of rib 300 mm and effective depth of 560mm is reinforced with 4 nos. of 25 mm diameter bars. Calculate the factored moment of resistance. The materials are M20 grade of concrete and HYSD reinforcement of grade Fe415.

Solution :- Data Given,



$b_f = 1200$ mm

$D_f = 100$ mm

$b_w = 300$ mm

$d = 560$ mm

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

$A_{st} = 4 \times \pi/4 \times (25)^2$

$= 1964 mm^2$

To find the position of NA, compare flange compression and tensile force.

$F_{tc} = 0.36 f_{ck} b_f D_f$

$= 0.36 \times 20 \times 1200 \times 100$

$= 864000$ N

$= 864$ KN

$F_{ts} = 0.87 f_y A_{st}$

$= 0.87 \times 415 \times 1964$

$= 709102.2$ N

$= 709.1$ KN

$\therefore F_{tc} > F_{ts}$; NA lies in flange.

Equating the forces

Total compression = Total tension

$\Rightarrow 0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

$\Rightarrow x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$

$\Rightarrow x_u = \dfrac{0.87 \times 415 \times 1964}{0.36 \times 20 \times 1200}$

$\Rightarrow x_u = 82.07$ mm

$x_{umax} = 0.479 d$

$= 0.479 \times 560$

$= 268.24$ mm

$\therefore x_u < x_{umax}$; hence the section is under-reinforced

$M_u = 0.87 f_y A_{st} (d - 0.416 x_u)$

$= 0.87 \times 415 \times 1964 (560 - 0.416 \times 82.07)$

$= 372887688.7$ N-mm

$= 372.88$ KNM

Slab

→ Slabs are plate elements having the depth 'D' much smaller than it's span & width. They usually carry an uniformly distributed load and form the floor or roof of the building.

→ Like beams, slabs may also be simply supported, cantilever or continuous depending upon the support conditions. They are classified according to the system of supports used as under:

1. One way spanning slab
2. Two way spanning slab
3. Flat slabs supported directly on columns without beams.
4. Grid slabs
5. Circular and other slabs
6. Ribbed and waffle slabs.

Difference between oneway slab and two way slab

Oneway slab

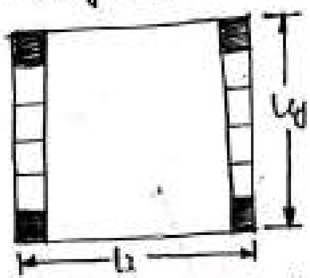
i) If longer span divided by shorter span & exceeds 2, that type of slab is called oneway slab.

$$L_y / L_x > 2$$

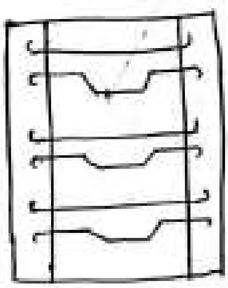
L_y = longer span

L_x = shorter span

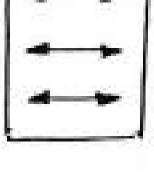
ii) Oneway slab is supported by beams only in two sides.



iii) In oneway slab, the main bar is provided in one direction only.



iv) Carries load along one direction.



Two way slab

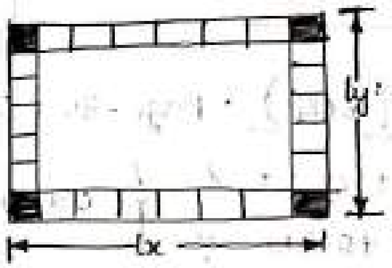
i) If longer span is divided by shorter span & doesnot exceeds 2 that type of slab is called two way slab.

$$L_y / L_x < 2$$

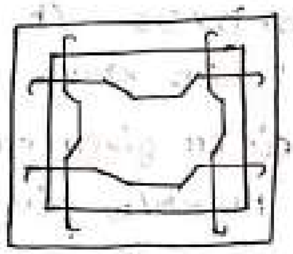
L_y = longer span

L_x = shorter span

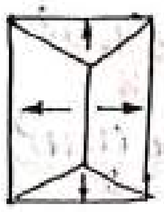
ii) Two way slab is supported by beams in four sides.



iii) In two way slab the main bar is provided in both directions.



iv) It carries load along both direction.



Design of oneway slab

Given data

i. Size of slab
ii. Floor finish
iii. Live load
iv. f_{ck}, f_y.

Step-1 :- (check the type of slab)

1. $l_y/l_x > 2$ (oneway slab)
2. $l_y/l_x < 2$ (two way slab)

Step-2 :- (Depth of slab)

$$\boxed{d = \frac{\text{Shorter span}}{\text{Basic value} \times MF}}$$

* Basic value = Page 37 (2.3/2.1)

for
S.S.B = 20
C.B = 7.
Continuous = 26

* MF = page 38 ; from the value of
$f_s = 0.58 f_y \frac{Ast\ Req}{Ast\ pro.}$

Normally Ranges from 1.2-1.6

$$\boxed{D = d + \phi/2 + \text{clear cover}}$$

* clear cover = 15 mm

* ϕ
for slab = 8mm - 15mm
for beam = 16 - 24 mm

Step-3 :- (Eff. length of slab)

Page 35 (22.2)

1. $l_{eff} = l + d$
2. $l_{eff} = l + 0.3m$

} whichever is lesser

Step-4 :- (BM / Mu)

Take b = 1m
Dead load = $(b \times D \times 25)$ kN/m
Live load = either given or take 4 kN/m
Floor finish = 1.5 kN/m
Total load (W) = D.L + L.L + F.F
W_u = Total load × 1.5

BM (M_u) = $\frac{W_u (l_{eff})^2}{8}$ (S.S.B)

$\frac{W_u (l_{eff})^2}{2}$ (C.B)

Step-5 (check for 'd')

for balanced section

$\boxed{M_u = M_{u\,lim}}$ → find "d_{cal}"

$$\boxed{d_{cal} = \sqrt{\frac{M_u}{Q_{lim} b}}}$$

If
$d_{cal} < d$ (safe)
$d_{cal} > d$ (Redesign)

∴ Normally d ranges from 100-150 m

Step-6 (Area of steel) * Main Bar

$$\boxed{P_t = 50\left[\frac{1-\sqrt{1-\frac{4.6 M_u}{f_{ck} b d^2}}}{f_y/f_{ck}}\right]}$$

$$\boxed{A_{st} = \frac{P_t\, b d}{100}}$$

check for min^m Ast

HYSD (415, 500) = $A_{st\,min}$ = 0.12 % bD
Mild steel ϕ = $A_{st\,min}$ = 0.15 % bD

→ check Ast in relation Ast min & ~~taken~~ ~~calculated taken~~ Ast > Ast min, (safe)

→ Assume bar diameter & find spacing from $\boxed{\frac{\pi/4 \times \phi^2 \times b}{A_{st} \text{ or } A_{st\,min}} = S_v}$

check for spacing

(i) 3d (ii) 300mm (iii) S_v
Take whichone is less

* **Distribution Bar**

HYSD (415, 500) = $A_{st\,min}$ = 0.12% bD
Mild steel = $A_{st\,min}$ = 0.15 % bD

→ Assume bar diameter and find spacing from $\boxed{\frac{\pi/4 \times \phi^2 \times b}{A_{st\,min}} = S_v}$

check for spacing

(i) 5d (ii) 450 mm (iii) S_v
Take which one is less.

Step-7 (check for shear)

$V_u = \frac{W_u L}{2}$ (SSB) or W_u (C.B)

$\tau_v = \frac{V_u}{bd}$

τ_v in relation τ_c

τ_c = Table 19

$P_t = \frac{100\, A_{st\, pro (main)}}{bd}$

$\tau_v < \tau_c$ (ok)

Step-8 (check for deflection)

$$\boxed{\left(\frac{l_{eff}}{d}\right)_{Pro} \leq \left(\frac{l_{eff}}{d}\right)_{Per}}$$

$\left(\frac{l_{eff}}{d}\right)_{Per}$ = Basic value × K find k from Page - 40 ; table - 14

$$\boxed{f_s = 0.58 f_y \frac{A_{st\,Req\,(main)}}{A_{st\,pro\,(main)}}}$$

Q. Design a simply supported slab of size ~~...~~ 3m×6.2m for a living room of a residential building. Take floor finish as 1.5 kN/m. Use M20 grade of concrete & Fe415 steel.

Solution

Given data,

size = 3m × 6.2m

floor finish = 1.5 kN/m

live load = 4 kN/m

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

$\frac{l_y}{l_x} > 2$

$\Rightarrow \frac{6.2}{3} = 2.06 > 2$ (oneway slab)

$d = \frac{\text{Shorter span}}{\text{Basic value} \times MF}$

$= \frac{3 \times 10^3}{20 \times 1.5}$

$= 100 mm$

$D = d + \phi/2 + c.c$

$= 100 + \frac{10}{2} + 15$

$= 120 mm$

Leff

(i) $L + d = 3 \times 10^3 + 100 = 3100 mm = 3.1 m$

(ii) $L + 0.3m = 3 + 0.3 = 3.3m$

$\therefore$ Leff = 3.1 m

Total Load

(i) $D.L = D \times 25 = .120 \times 25 = 3 kN/m$

$\therefore$ Total Load (W) = D.L + L.L + F.F = 3+4+1.5 = 8.5 kN/m

$W_u = 1.5 \times W = 1.5 \times 8.5 = 12.75 kN/m$

$(M_u)\ BM = \frac{W_u (L_{eff})^2}{8} = \frac{12.75 \times (3.1)^2}{8} = 15.31 kNM$

$d_{cal} = \sqrt{\frac{M_u}{Q_{lim} \times b}} = \sqrt{\frac{15.31 \times 10^6}{0.138 \times 20 \times 1000}} = 74.47 mm$ ~~...~~

$\therefore$ $d_{cal} < d$ (safe)

$\therefore$ d = 100 mm

Main Bar

$$P_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}}{f_y / f_{ck}} \right] = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 15.31}{20 \times 1000 \times (100)^2}}}{415/20} \right] = 0.47$$

$A_{st} = \frac{0.47 \times 1000 \times 100}{100} = 470 mm^2 \simeq 500 mm^2$

$$\boxed{\begin{array}{l} A_{st\,min} = 0.12\% \,bD = \frac{0.12}{100} \times 1000 \times 120 \\ \quad = 144 mm \\ \therefore A_{st} > A_{st\,min}\ (\text{safe}). \end{array}}$$

~~$A_{st\,min} = 0.12\%\, bD = \frac{0.12}{100} \times 1000 \times 120$~~

Assume 10mm ϕ bars, no. of bars $= \frac{A_{st}}{\frac{\pi}{4} \times (\phi)^2} = \frac{500}{\frac{\pi}{4} \times (10)^2} = 6.36 \simeq 7$ nos

$A_{st\,prov} = \frac{\pi}{4} \times \phi^2 \times$ nos of bars $= \frac{\pi}{4} \times (10)^2 \times 7 = 549.77 mm^2 \simeq 550 mm^2$

Spacing $= \frac{\pi/4 \times \phi^2 \times b}{A_{st\,pro}} = \frac{\pi/4 \times (10)^2 \times 1000}{550} = 142.79 mm \simeq 140 mm$

Check for spacing

1. $3d = 3\times100 = 300$ mm (II) 300 mm (III) 140 mm

$\therefore$ Spacing = 140 mm

$\therefore$ 10 mm diameter bars @ 140 mm c/c is provided.

Distribution Bars

$$A_{st} = 0.12\%\ bD = \frac{0.12}{100}\times1000\times120 = 144\ mm^2 \simeq 150\ mm^2$$

Use 8mm ϕ bars.

$$\text{no of bars} = \frac{150}{\pi/4\times(8)^2} = 2.98 \simeq 3\text{nos}$$

$$\text{spacing} = \frac{\pi/4\times(8)^2\times1000}{150} = 335\ mm$$

$$A_{st\,pro} = 3\times\pi/4\times(8)^2 = 150\ mm^2$$

$\therefore$ check spacing

(I) $5d = 5\times100 = 500$ mm (II) 450 mm (III) 335 mm

$\therefore$ spacing = 335 mm

$\therefore$ provide 8 mm ϕ bars @ 335 mm c/c.

check for shear

$$V_u = \frac{W_u\, l_{eff}}{2} = \frac{12.75\times3.1}{2} = 19.76\ kN$$

$$\tau_v = \frac{V_u}{bd} = \frac{19.76\times10^3}{1000\times100} = 0.1976$$

$$\tau_c = 0.28$$

$\therefore \tau_v < \tau_c$ (OK)

check for deflection

$$\left(\frac{l_{eff}}{d}\right)_{act} < \left(\frac{l_{eff}}{d}\right)_{per}$$

$$\left(\frac{l_{eff}}{d}\right)_{per} = 20\times1.7 = 34$$

$$\left(\frac{l_{eff}}{d}\right)_{act} = \frac{3.1\times10^3}{100} = 31$$

$$\therefore \left(\frac{l_{eff}}{d}\right)_{act} < \left(\frac{l_{eff}}{d}\right)_{per} \quad (OK).$$

Stair case

Q. Design a dog-legged stair case by LSM for a residential building of ceiling height of 3.5 m & height of each flight is to be kept 1.2 m. Choose suitable rise & trade. Use M20 grade of concrete & Fe 415 steel.

Solution :- Data given,

Height of each flight = 1.2 m

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

Assuming L.L = 3000 N/mm²

Assuming Riser = 150 mm

No. of Riser required = $\frac{1200}{150}$ = 8 nos

No. of tread in each flight = no. of rise − 1
= 8 − 1
= 7

Let the size of each tread = 270 mm

so, 7×270 = 1890 mm ≃ 1900 mm = 1.9 m (Length of all trades)

Let the thickness of waist be 220 mm

Let the width of landing be 1.60 m = 1600 mm

Let the wall thickness = 350 mm

Design constant

For M20 & Fe 415

$R_u = 0.36 f_{ck} \left(\frac{x_{umax}}{d}\right)\left(1 - 0.416 \frac{x_{umax}}{d}\right)$

$= 0.36 \times 20 \times 0.479 \times (1 - 0.416 \times 0.479)$

$= 2.76$

Design of flight AB

Dead load of waist slab (w) = 25t = 25×220 = 5500 N/mm²

weight along horizontal = $\frac{w \cdot \sqrt{T^2 + R^2}}{T} = \frac{5500\sqrt{(270)^2 + (150)^2}}{270}$

$= 6291.77$ N/mm²

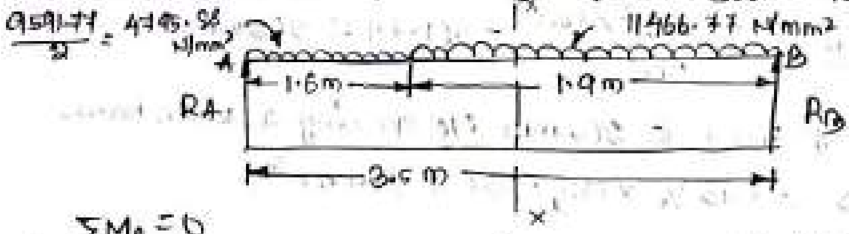
weight of steps = 12.5×R = 12.5×150 = 1875 N/mm²

Live load = 3000 N/mm²

Finishing load (12.5 mm thick) = 24×12.5 = 300 N/mm²

Load on steps of AB flight = 6291.77 + 1875 + 3000 + 300 = 11466.77 N/mm²

Load on landing = 6291.77 + 3000 + 300 = 9591.77 N/mm²



$\Sigma M_A = 0$

$\Rightarrow -R_B(3.5) + 11466.77 \times \frac{1.9^2}{2} + 4795.88 \times 1.6 \times \left(1.9 + \frac{1.6}{2}\right) = 0$

$\Rightarrow R_B = 41415.72 / 3.5$

$\Rightarrow R_B = 11833$ N

Total upward load = Total downward load

$\Rightarrow R_A + R_B = 4795.85 \times 1.6 + 11466.77 \times 1.9$

$\Rightarrow R_A = 29460.371 - 11833$

$\Rightarrow R_A = 17627.371 \text{ N}$

SF at xx'

$-R_A + 11466.77(x) = 0$

$\Rightarrow -17627.371 + 11466.77(x) = 0$

$\Rightarrow x = \frac{17627.371}{11466.77} = 1.53 \text{ m}$

BM at xx'

$R_A(x) - 11466.77 \times \frac{(1.53)^2}{2} = 0$

$\Rightarrow 17627.371(1.53) - 11466.77 \times \frac{(1.53)^2}{2}$

$\Rightarrow 13548.44 \text{ N-m}$

$M_u = 1.5 \times BM$

$= 1.5 \times 13548.44$

$= 20322.66 \text{ N-m}$

$d = \sqrt{\frac{M_u}{R_u b}} = \sqrt{\frac{20322.66 \times 10^3}{2.76 \times 1000}} = 85.80 \text{ mm} \simeq 86 \text{ mm}$

Provide an overall depth (D) = 220 mm

Use 10 mm ϕ rod with clean cover of 20 mm

$\therefore d = 220 - \frac{10}{2} - 20 = 195 \text{ mm} > d$ (okay)

$A_{st} = 0.5 \frac{f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right](bd)$

$= 0.5 \times \frac{20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 20322.66 \times 10^3}{20 \times 1000 \times (195)^2}}\right] \times 1000 \times 195$

$= 298.26 \text{ mm}^2$

$\simeq 300 \text{ mm}^2$

No. of bars $= \frac{A_{st}}{\pi/4 \times \phi^2} = \frac{300}{\pi/4 \times (10)^2} = 3.81 \simeq 4$ nos

Spacing $= \frac{\pi/4 \times (\phi)^2 \times b}{A_{st}} = \frac{\pi/4 \times (10)^2 \times 1000}{300} = 261.79 \simeq 262 \text{ mm}$

$\therefore$ Use 4 nos of 10 mm ϕ bars @ 262 mm c/c spacing as main bars.

A_{st} (dist) $= 0.12\%\ bD = 0.12\% \times 1000 \times 220 = 264 \text{ mm}^2$

Use 8 mm ϕ bars.

No. of bars $= \frac{264}{\pi/4 \times (8)^2} = 5.25 \simeq 6$ nos

spacing $= \frac{\pi/4 \times (8)^2 \times 1000}{264} = 190 \text{ mm}$

$\therefore$ Use 8 mm ϕ bars 6 nos @ 190 mm c/c spacing as dist. bars.

$\therefore$ Similarly design DE flight just like flight AB.

Column

- A compression member whose effective length exceeds three times to it's least lateral dimension is termed as column.
- If the effective length is less than three times it's least lateral dimension, it is termed as pedestal.
- A column is a very important component of a structure. Column support beams which in term support walls & slabs.
- Failure of a column results in the collapse of a structure.
- The column is divided into 2 types — (a) Short column
  (b) Long column

(a) Short column

- When the slenderness ratio is less than or equal to 12, it is known as short column.
- Short column are those which fails primarily due to failure of material.
- Short columns have buckling but of a limited value.

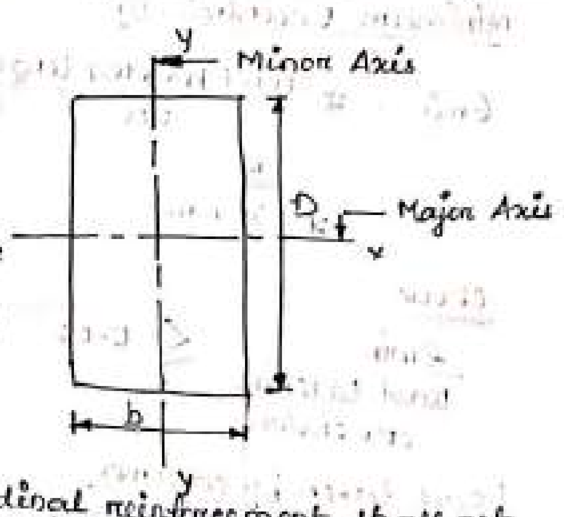
$$\frac{l_{ex}}{D} \leq 12$$

(b) Long column

- When the slenderness ratio is greater than 12, it is known as long column.
- Long columns are those which fails due to material failure and also due to buckling.
- Long columns have ~~predominate~~ predominant buckling value

$$\frac{l_{ey}}{D} > 12$$

l_{ex} = Eff. length along major axis.
l_{ey} = Eff. length along minor axis
D = Depth in respect of major axis.
b = width of the member.



Reinforcement Requirements

* Longitudinal Reinforcement

- The cross-sectional area of longitudinal reinforcement shall not be less than 0.8 % of the gross cross-sectional area of the column.
- The minimum number of longitudinal bars provided in a column shall be 4 in rectangular column & 6 in circular column.
- The diameter of bars shall not be less than 12 mm.
- RCC columns having helical reinforcement shall have at least 6 bars of longitudinal reinforcement within the helical reinforcement.
- Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm.

Transeverse Reinforcement

→ The longitudinal reinforcement should be laterally tied by transeverse links to provide a restrained against outward buckling of each of the longitudinal bars.

→ The diameter of longitudinal bars shall not be less than 12 mm & the diameter of transeverse reinforcement shall not be less than 1/4th of the diameter of longitudinal bars or 6mm.

→ The ends of the transeverse links shall be properly anchored.

Spacing or pitch

The pitch or spacing of transeverse reinforcement shall not be more than the least of the following :-

i. The least lateral dimension of the compression member.

ii. 16 times the diameter of the longitudinal reinforcement bars to be tied.

iii. 300 mm.

iv. 48 times the diameter of the transeverse reinforcement bars.

Diameter

The diameter of the polygonal links or ties shall not be less than 1/4th of the diameter of the longitudinal bars and in no case less than 6 mm.

Cover

The longitudinal reinforcing bar in a column shall have concrete cover, not less than 40 mm, nor less than diameter of bar whichever is greater.

Minimum Eccentricity

$$e_{min} = * \frac{\text{Unsupported Length}}{500} + \frac{\text{Least Lateral dimension}}{30}$$

or

$$* \; 20 \text{ mm}$$

whichever is greater.

Check

$$\frac{e_{min}}{\text{least Lateral dimension}} \leq 0.05 \quad (\text{OK})$$

Load taken by column

$$\boxed{P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}}$$

where,

P_u = factored Load

A_c = Area of concrete

A_{sc} = Area of longitudinal bars

$$\boxed{A_c = A_g - A_{sc}}$$

A_g = Gross-cross-sectional area of column

Assumptions made for design

The following assumptions are made for the limit state of collapse in compression.

i. Plane section normal to the axis remain plane after bending.

ii. The relationship between stress-strain distribution in concrete shall be assumed to be parabolic. The maximum compressive stress at the extreme compression fibre is $0.446 f_{ck}$.

iii. The tensile strength of the concrete is ignored.

iv. The stresses in the reinforcement are deprived from the representation stress-strain curve for the type of steel used. ~~typical~~

v. The maximum compressive strain in concrete in axial compression is taken as 0.002.

vi. The maximum compressive strain at highly compressed extreme fibre in concrete subjected to axial compression & bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at least compressed extreme fibre.

Q. A short RCC column 450 x 450 mm is provided with 4 nos. bars of 18 mm diameter. If the unsupported length of the column is 3m. Find the ultimate load for the column. Use M_{25} grade of concrete & Fe 415 steel. Check for eccentricity & design for transverse reinforcement of the bar.

Solution Data given,

$A_g = 450 \times 450 = 202500\ mm^2$

$A_{sc} = 4 \times \pi/4 \times (18)^2 = 1017.87\ mm^2 \simeq 1018\ mm^2$

$A_c = A_g - A_{sc} = 202500 - 1018 = 201482\ mm^2$

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

unsupported length $= 3m = 3 \times 10^3\ mm$

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$= 0.4 \times 15 \times 201482 + 0.67 \times 415 \times 1018$

$= 1491946.9\ N$

$\simeq 1491944\ kN$

Eccentricity

(i) $\frac{\text{unsupported length}}{500} + \frac{\text{lateral dimension}}{30}$

$= \frac{3000}{500} + \frac{450}{30}$

$= 21\ mm$

(ii) 20 mm

$\therefore e_{min} = 21\ mm$

Check

$\frac{e_{min}}{\text{least lateral dimension}} = \frac{21}{450} = 0.041 < 0.05$ (OK)

Transeverse Reinforcement

(i) $1/4 \times 18 = 4.5\ mm$

(ii) 6 mm

$\therefore$ So diameter of transeverse reinforcement is 6 mm.

Spacing

(i) 450 mm (ii) $16 \times 18 = 288$ mm (iii) 300 mm (iv) $48 \times 6 = 288$ mm

$\therefore$ Hence provide 6mm diameter bar @ 288 mm c/c spacing.

Q. A short RCC column is to carry a factored load of 1900 kN. If the column is to be square. Assume $e_{min} < 0.05D$. Design the column. The columns are M_{20} grade of concrete & mild steel.

Solution :- Data given.

$P_u = 1900$ kN

$f_{ck} = 20$ N/mm^2

$f_y = 250$ N/mm^2

Assuming,

$e_{min} = 20$

$e_{min} = 0.05D$

$\Rightarrow D = \frac{20}{0.05}$

$\Rightarrow D = 400$ mm

i.e 400 mm × 400 mm size column.

As there is no restriction on the size of column. We can assume 0.8% steel is used.

$$\boxed{A_{sc} = 0.008 A_g}$$

$A_c = A_g - A_{sc}$

$\Rightarrow A_c = A_g - 0.008 A_g$

$$\Rightarrow \boxed{A_c = 0.992 A_g}$$

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$\Rightarrow 1900 \times 10^3 = 0.4 \times 20 \times 0.992 A_g + 0.67 \times 250 \times 0.008 A_g$

$\Rightarrow 1900 \times 10^3 = 7.936 A_g + 1.34 A_g$

$\Rightarrow A_g = \frac{1900 \times 10^3}{9.276}$

$\Rightarrow A_g = 204829.668$ mm^2

$\Rightarrow A_g \simeq 204930$ mm^2

Hence the size of the column $= \sqrt{204930} = 452.69$ mm $\simeq 453$ mm

$\therefore$ So adopt 453 mm × 453 mm square column

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$\Rightarrow 1900 \times 10^3 = 0.4 \times 20 \times [(453 \times 453) - A_{sc}] + 0.67 \times 250 \times A_{sc}$

$\Rightarrow 1900 \times 10^3 = 1641672 - 8 A_{sc} + 167.5 A_{sc}$

$\Rightarrow 159.5 A_{sc} = 258328$

$\Rightarrow A_{sc} = 1619.61$ mm^2 $\simeq 1620$ mm^2

Assuming 16mm dia bars are used.

So, no. of bars $= \frac{1620}{\frac{\pi}{4} \times (16)^2} = 8$ nos

Lateral tie

(I) $\frac{1}{4} \times 16 = 4mm$

(II) 6mm

∴ 6mm diameter ~~bars~~ lateral ties are used.

Spacing

(I) 453mm (II) $16 \times 16 = 256$ mm (III) 300 mm (IV) $48 \times 6 = 288$ mm

∴ Hence 6mm diameter tie bars used @ 256mm c/c spacing.

Q. A reinforced concrete column of effective length 2.75m carries an axial load of 1600 kN. Design the column using M_{20} grade of concrete & Fe415 steel.

Solution Data given,

Effective length = 2.75m

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

$P_u = 1600 kN \times 1.5$

$= 2400 kN$

Assume steel area is 2% of gross cross-sectional area.

$A_{sc} = 2\% \, A_g$

$\Rightarrow \boxed{A_{sc} = 0.02 A_g}$

$A_c = A_g - A_{sc}$

$\Rightarrow A_c = A_g - 0.02 A_g$

$\Rightarrow \boxed{A_c = 0.98 A_g}$

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$\Rightarrow 2400 \times 10^3 = 0.4 \times 20 \times 0.98 A_g + 0.67 \times 415 \times 0.02 A_g$

$\Rightarrow 2400 \times 10^3 = 13.401 A_g$

$\Rightarrow A_g = 179091 \, mm^2$

Assume square column.

Hence size of column $= \sqrt{179091}$

$= 423$ mm

$\simeq 425$ mm

i.e the size of column = 425mm × 425mm

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$\Rightarrow 2400 \times 10^3 = 0.4 \times 20 \times (425 \times 425 - A_{sc}) + 0.67 \times 415 \times A_{sc}$

$\Rightarrow 2400 \times 10^3 = 1445000 - 8 A_{sc} + 278.05 A_{sc}$

$\Rightarrow A_{sc} = \dfrac{2400 \times 10^3 - 1445000}{278.05 - 8}$

$\Rightarrow A_{sc} = 3536 \, mm^2$

Use 25 mm dia bars are used.

∴ No. of bars $= \dfrac{3536}{\frac{\pi}{4} \times (25)^2} = 7.20$ nos $\simeq 8$ nos

e_{min}

I. $\frac{2.75\times10^3}{500} + \frac{425}{30} = 19.66$ mm

II. 20 mm

$\therefore$ $e_{min} = 20$ mm

Check

$\frac{20}{425} = 0.04 < 0.05$ (OK)

Transverse Reinforcement

I. $\frac{1}{4}\times 25 = 6.25$ mm

II. 6 mm

$\therefore$ Hence use 8 mm dia bars.

Spacing

(I) 425 mm (II) $16\times25 = 400$ mm (III) 300 mm (IV) $48\times8 = 384$ mm

$\therefore$ Hence use 8 mm dia bars @ 300 mm c/c spacing.

Circular column

→ The strength of a column with helical reinforcement satisfying the requirements given shall be taken as 1.05 times the strength of similar member with lateral ties.

$$\boxed{P_u = 1.05\,(0.4 f_{ck} A_c + 0.67 f_y A_{sc})}$$

Dia of core (D_k) = Dia of column $- 2\times$ c.c $+ 2\times$ dia of helical reinforcement

Area of core (A_k) = $\frac{\pi}{4}\times(D_k)^2$

Dia of core corresponding to the centre of helical bar (d_h) = $= D_k -$ Dia of helical bar

Check for validity of formula used

Consider one pitch length of the column

Length of helix per pitch length $= \sqrt{\pi(d_h)^2 + p^2}$

where, p = Pitch ($\because$ Assume $p = 45$ mm)

Volume of helix per pitch length = $a_{sh}\times$ length of helix per pitch length

Volume of core per pitch length = Area of core $\times$ Pitch

$$\frac{\text{Volume of helical reinforcement}}{\text{Volume of core}} \geq 0.36\left(\frac{A_g}{A_k}-1\right)\left(\frac{f_{ck}}{f_y}\right) \quad \text{(OK)}$$

where,
A_g = Gross c-s area of the secn
A_k = Area of core

Pitch of helical Reinforcement

(i) $P \not> 75$ mm

(ii) $P \not> 1/6$ (dia of column)

(iii) $P \not< 25$ mm

(iv) $P \not< 3$ (dia of helical reinforcement)

∴ So assume Pitch (P) = 45 mm

Spacing

(i) The least lateral dimension of column
(ii) 16 x dia of longitudinal reinforcement
(iii) 300 mm
(iv) 48 x dia of helical reinforcement

} whichever is less

Diameter of helical reinforcement

(i) $D \not< 1/4$ x dia of longitudinal reinforcement

(ii) $D \not< 5$ mm

∴ Assume 6mm - 8mm dia for helical reinforcement.

Q. Determine the safe axial load for a column of 400mm dia, reinforced with 6 bars of 25mm dia as longitudinal steel. It is provided with 8mm dia helical reinforcement at a pitch of 45 mm. Use M_{20} grade of concrete & fe 415 steel.

Solution :- Data given,

dia of column = 400 mm

$A_{sc} = 6 \times \pi/4 \times (25)^2$

$= 2945 \text{ mm}^2$

Dia of helical reinforcement = 8 mm

Pitch (P) = 45 mm

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ N/mm}^2$

$A_g = \frac{\pi}{4} \times (400)^2$

$= 125663.7 \text{ mm}^2$

$= 125664 \text{ mm}^2$

$A_c = A_g - A_{sc}$

$= 125664 - 2945$

$= 122719 \text{ mm}^2$

$P_u = 1.05\,(0.4 f_{ck} A_c + 0.67 f_y A_{sc})$

$= 1.05\,(0.4 \times 20 \times 122719 + 0.67 \times 415 \times 2945)$

$= 1890640$ N

$= 1890.64$ KN

Axial Load $= P_u / 1.5$

$= 1890.64 / 1.5$

$= 1260.42$ KN

Check

Dia of core (D_K) = ϕ of column − 2×c.c + 2× ϕ of helical reinforcement

$= 400 - 2\times 40 + 2\times 8$

$= 336$ mm

Area of core (A_K) = $\frac{\pi}{4}\times (D_K)^2$

$= \frac{\pi}{4}\times (336)^2$

$= 88668$ mm^2

ϕ of column corresponding to the centre of helical bar (d_h) =

$= D_K - \phi$ of helical reinforcement

$= 336 - 8$

$= 328$ mm

Length of helix per pitch length = $\sqrt{\pi(d_h)^2 + P^2}$

$= \sqrt{\pi(328)^2 + (45)^2}$

$= 583$ mm

volume of helix per pitch length = 50× Length of helix per pitch length

$= 50\times 583$

$= 29150$ mm^3

volume of core per pitch length = $A_K \times$ Pitch

$= 88668 \times 45$

$= 3990060$ mm^3

Check

$$\frac{\text{vol}^m \text{ of helix}}{\text{vol}^m \text{ of core}} \geq 0.36\left(\frac{A_g}{A_K} - 1\right)\left(\frac{f_{ck}}{f_y}\right)$$

$$\Rightarrow \frac{29150}{3990060} \geq 0.36\left(\frac{125664}{88668} - 1\right)\left(\frac{20}{415}\right)$$

$\Rightarrow 0.0073 > 0.0072$ (ok)

Spacing

(i) 400 mm (ii) $16\times 25 = 400$ mm (iii) 300 mm (iv) $48\times 8 = 384$ mm

∴ use 8mm dia helical reinforcement @ 300 mm c/c spacing. (H)

2.

Q. Design a circular column to carry an axial load of 1500 kN. The column has an effective length of 2.50m. Use M_{20} grade of concrete & Fe 415 steel.

Solution

Data given.

Axial load (P) = 1500 kN

Effective length (L) = 2.5 M

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

Assume steel area is 2% of gross cross-sectional area of the column.

$A_{sc} = 2\%$ of A_g

$\Rightarrow A_{sc} = \frac{2}{100} \times A_g$

$\Rightarrow \boxed{A_{sc} = 0.02 A_g}$

$A_c = A_g - A_{sc}$

$\Rightarrow A_c = A_g - 0.02 A_g$

$\Rightarrow \boxed{A_c = 0.98 A_g}$

factored load $= 1.5 \times 1500$

$\Rightarrow P_u = 2250$ kN

$P_u = 1.05 (0.4 f_{ck} A_c + 0.67 f_y A_{sc})$

$\Rightarrow 2250 \times 10^3 = 1.05 (0.4 \times 20 \times 0.98 A_g + 0.67 \times 415 \times 0.02 A_g)$

$\Rightarrow 14.07 A_g = 2250 \times 10^3$

$\Rightarrow A_g = 159914.71\ mm^2$

$\Rightarrow A_g \simeq 159915\ mm^2$

Assume circular column is used.

So, the size of the column is

$$d = \sqrt{\frac{159915 \times 4}{\pi}}$$

$$= 452\ mm$$

$\therefore$ Size of the column $= \frac{\pi}{4} \times (452)^2$

$A_c = A_g - A_{sc}$

$= \frac{\pi}{4} \times (452)^2 - A_{sc}$

$= 160460 - A_{sc}$

$P_u = 1.05 (0.4 f_{ck} A_c + 0.67 f_y A_{sc})$

$\Rightarrow 2250 \times 10^3 = 1.05 [0.4 \times 20 \times (160460 - A_{sc}) + 0.67 \times 415 \times A_{sc}]$

$\Rightarrow 2250 \times 10^3 = 1347864 - 8.4 A_{sc} + 291.95 A_{sc}$

$\Rightarrow 283.55 A_{sc} = 902136$

$\Rightarrow A_{sc} = 3181\ mm^2$

Assume 25mm ϕ bar is used

So, no. of bars $= \frac{3181}{\frac{\pi}{4} \times (25)^2} = 6.48$ nos $\simeq 7$ nos

Dimension of helical reinforcement

1. $D \nless \frac{1}{4}$ (ϕ of longitudinal reinforcement)

$\Rightarrow 452 \nless \frac{1}{4}(25)$

$\Rightarrow 452 \nless 6.25$ mm

2. $D \nless 5$ mm

$452 \nless 5$ mm

$\therefore$ So assume 8mm dia of helical reinforcement.

Check

D_k = ϕ of column − 2×clear cover + 2×ϕ of helical reinforcement

$= 452 - 2\times 40 + 2\times 8$

$= 388$ mm

$A_k = \frac{\pi}{4}(D_k)^2$

$= \frac{\pi}{4}(388)^2$

$= 118237 \text{ mm}^2$

$d_h = D_k - \phi$ of helical reinforcement

$= 388 - 8$

$= 380$ mm

Length of helix per pitch length $= \sqrt{\pi d_h^2 + P^2}$ ($\because$ Assume P = 45mm)

$= \sqrt{\pi \times (380)^2 + (45)^2}$

$= 675$ mm

Volume of helix per pitch length = 50 × length of helix per pitch length

$= 50 \times 675$

$= 33750 \text{ mm}^3$

Volume of core per pitch length = $A_k \times$ pitch

$= 118237 \times 45$

$= 5320665 \text{ mm}^3$

Check

$$\frac{\text{volume of helix}}{\text{volume of core}} \geq 0.36\left(\frac{A_g}{A_k} - 1\right)\left(\frac{f_{ck}}{f_y}\right)$$

$$\Rightarrow \frac{33750}{5320665} \geq 0.36\left(\frac{159915}{118237} - 1\right)\left(\frac{20}{415}\right)$$

$\Rightarrow 0.00634 > 0.00611$ (Ok)

Spacing

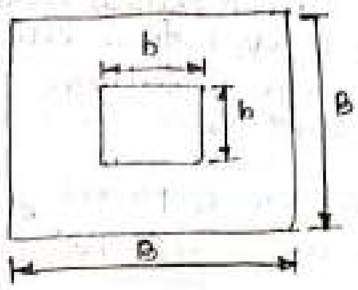
(i) 452 mm (ii) 16×25 = 400mm (iii) 300mm (iv) 48×8 = 384 mm

$\therefore$ So use 8mm dia helical reinforcement @ 300mm c/c spacing. (d)

Design of a square footing

Given data

P_u, Soil bearing capacity (SBC)
size of column, f_{ck}, f_y.



Step-1 :-
Add 10% load as self weight of footing.

Step-2 (calculate area of footing)

$$\boxed{A = \frac{P + 10\%\,P}{SBC}}$$

where, P = axial load.

sides of footing = $\boxed{B = \sqrt{A}}$

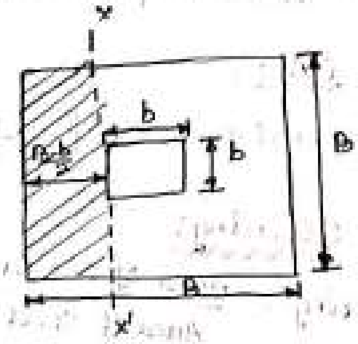
Step-3

Reaction of soil / Pressure (q_u) (for factored load) $\boxed{= \frac{P_u}{A} = \frac{1.5 \times P}{A}}$

Step-4 (Calculation of minimum depth)

This include (I) BM (II) Oneway shear (III) Twoway shear

(I) BM calculation



$$M_u = q_u \left(\frac{B-b}{2}\right) \times \frac{1}{2}\left(\frac{B-b}{2}\right)$$

$$\boxed{M_u = q_u \frac{(B-b)^2}{8}}$$

or $\boxed{M_u = \frac{q_u L^2}{8}}$

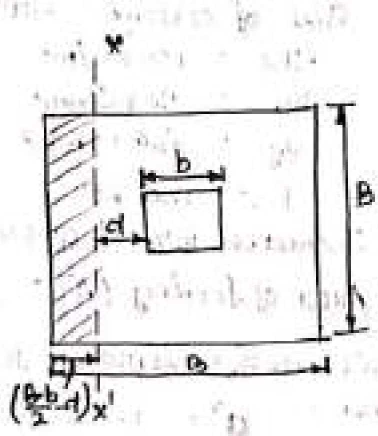
Moment at limiting moment as per IS 456 : 2000 ; Page - 96

$$\boxed{M_{ulim} = 0.36\, f_{ck} \left(\frac{x_{umax}}{d}\right)\left(1 - 0.416\frac{x_{umax}}{d}\right) b d^2}$$

or
$$M_{ulim} = 0.149\, f_{ck}\, bd^2 \quad (Fe\ 250)$$
$$= 0.138\, f_{ck}\, bd^2 \quad (Fe\ 415)$$
$$= 0.133\, f_{ck}\, bd^2 \quad (Fe\ 500)$$

(II) Oneway shear

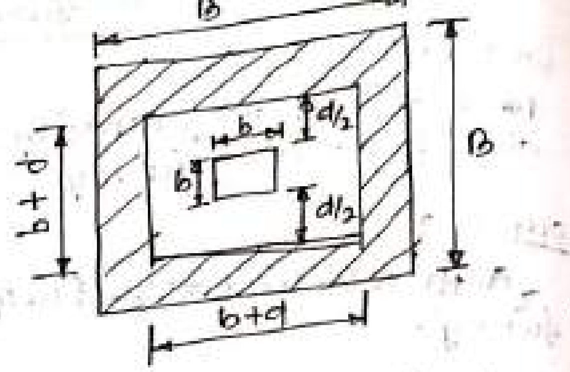
For the critical section is cut at a distance 'd' from the column face, then design shear as per IS code.



$$\boxed{V_u = q_u B \left[\left(\frac{B-b}{2}\right) - d\right]}$$

V_u = Soil pressure.

III. Double shear



→ Two way shear also called punching shear. when critical section is at a distance ($d/2$) from column face.

→ This type of failure occur when the depth of section is less.

Perimeter of punching shear = $4(b+d)$
Area of concrete resisting punching force (A) = $4(b+d) \times d$
Force of punching Q_u is given by $Q_u = q_u \times$ shear area. $Q_u = q_u \times [B^2 - (b+d)^2]$

∴ Punching shear stress, τ_p is given by

$$\boxed{\tau_p = \frac{Q_u}{A} = 0.25\sqrt{f_{ck}}}$$

∴ The value is permissible value of τ_p.

Step-5 (Ast)

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{bd f_{ck}}\right]$$

Step-6 (Spacing)

$$\text{Spacing} = \frac{\text{Area of one bar}}{\text{Area of steel}} \times 1000$$

Q. Design an isolated footing for a column of 400 mm × 400 mm subjected to an axial load of 1800 kN. The safe bearing capacity of soil is 150 kN/m². Use M_{20} grade of concrete & Fe415 steel.

Solution :- Data given,

Size of column = 400mm × 400mm

SBC = 150 kN/m²

f_{ck} = 20 N/mm²

f_y = 415 N/mm²

P = 1800 kN

Increase 10% of given load,

$$\text{Area of footing }(A) = \frac{P + 10\% \text{ of } P}{SBC} = \frac{1800 + 0.1 \times 1800}{150} = 13.20\, m^2$$

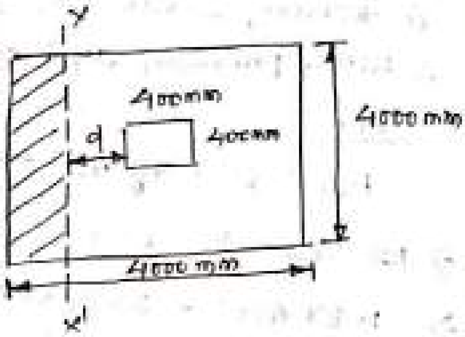
we design square footing,

then, $a^2 = 13.20\, m^2$

$\Rightarrow a = \sqrt{13.20} = 3.63m \simeq 4\,m$

∴ Hence provide 4m × 4m size of footing.

$q_u = \frac{P_u}{a^2} = \frac{1.5 \times 1800}{4 \times 4} = 168.75\ kN/m^2 = 0.168\ N/mm^2$



$V_u = q_u B \left(\frac{B-b}{2} - d \right)$

$= 0.168 \times 4000 \left(\frac{4000-400}{2} - d \right)$

$= 672 (1800 - d)$

Assuming 0.2% of steel, from M_{20} grade of concrete

$\tau_c = 0.32\ N/mm^2$ [Page-73 (table 19)]

$V_u = \tau_c B d = 0.32 \times 4000 \times d = 1280d$

$\Rightarrow V_u = 672(1800 - d)$

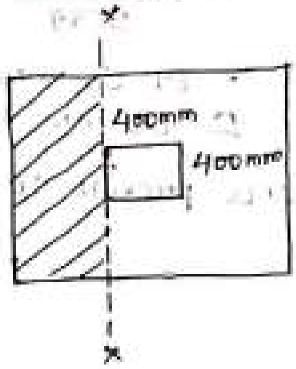
$\Rightarrow 1280d = 672(1800-d)$

$\Rightarrow 1280d = 1209600 - 672d$

$\Rightarrow d = \frac{1209600}{1952}$

$\Rightarrow d = 619.67\ mm$

$\Rightarrow d \simeq 620\ mm$



Check for bending

$M_{u\,lim} = 0.138 f_{ck} B d^2$ (for 415)

$= 0.138 \times 20 \times 4000 \times (620)^2$

$= 4243.77 \times 10^6\ N\text{-}mm$

$M_u = q_u B \frac{(B-b)^2}{8}$

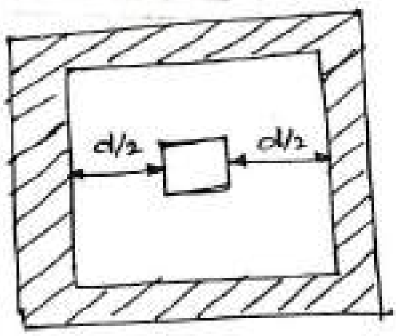
$= 0.168 \times 4000 \times \frac{(4000-400)^2}{8}$

$= 1088.64 \times 10^6\ N\text{-}mm$

$\therefore M_u < M_{u\,lim}$; hence provided depth is sufficient.

Check for two way shear

critical section at d/2 distance from the face of column in two way shear



Perimeter of critical section

$= 4(b+d)$

$= 4(400+620)$

$= 4080\ mm$

Area of critical section = Perimeter × depth

$= 4080 \times 620$

$= 2529600\ mm^2$

Shear stress in two way shear $= \frac{\text{upward pressure in shaded area}}{\text{Area of critical section}}$

$= \frac{0.168(4000 \times 4000 - 1020 \times 1020)}{4080 \times 620}$

(But according to IS code) $= 0.99\ N/mm^2$

Maximum permitted shear stress $= 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118\ N/mm^2$

Shear stress in two-way shear < Max^m permitted shear stress (OK)

∴ Hence provided depth of footing is OK.

Ast

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{B d f_{ck}}\right]$$

$$\Rightarrow 1088.64 \times 10^6 = 0.87 \times 415 \times A_{st} \times 620 \left[1 - \frac{A_{st} \times 415}{4000 \times 620 \times 20}\right]$$

$$\Rightarrow 1.87 A_{st}^2 - 223851 A_{st} + 1088.64 \times 10^6 = 0$$

$$\Rightarrow A_{st} = 5078.7 \text{ mm}^2$$

$$\Rightarrow A_{st} \simeq 5079 \text{ mm}^2$$

Assume 20 mm dia bar is used.

$$\text{No. of bars} = \frac{5079}{\frac{\pi}{4} \times (20)^2} = 16 \text{ nos}$$

$$\text{Spacing} = \frac{\text{Area of one bar}}{A_{st}} \times B$$

$$= \frac{\pi/4 \times (20)^2}{5079} \times 4000$$

$$= 61.85 \text{ mm}$$

$$\simeq 62 \text{ mm}$$

∴ Hence provide 16 nos of 20 mm dia bars @ 62 mm c/c spacing