LECTURE NOTE ON
CONTROL SYSTEM ENGINEERING
6TH SEM ELECTRICAL ENGINEERING

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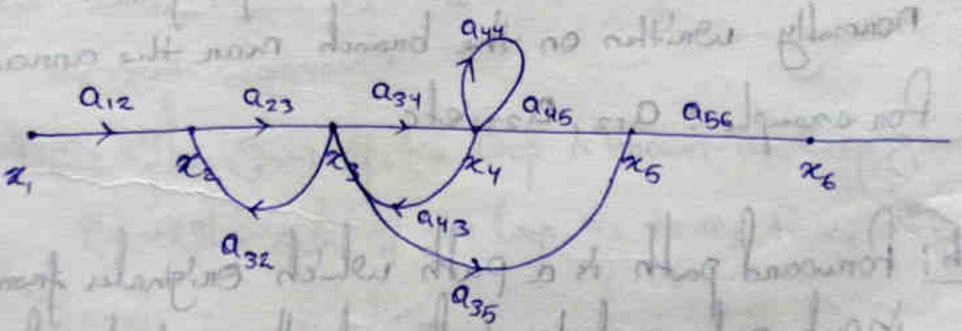
Signal flowgraph :

*) The process of block diagram reduction technique is time Consuming because at every stage modified block diagram is to be redrawn.

*) A simple method was developed by S.J. Mason which is known as ~~single~~ signal flow graph. This method is very simple and does not required any reduction technique.

*) Signal flow graph is a diagram ~~which represents~~ a applicable to linear system

*) A signal flowgraph is a diagram which represent a set of simultaneous equations.



*) Signal flow graph Consist of nodes and these nodes are connected by a directed line called branches. Every branch of signal flow graph having an arrow, which represent the flow of signal.

The following term are associated with the signal flow graph.

1. <u>Input node and source node</u>: An input node which has only outgoing branches. For example x_1 is the input node.

2. <u>Output node or sink node</u>: An output node that has only one or more incoming branches e.g. x_6 is the output node.

3. <u>Mixed nodes</u>: A node having incoming and outgoing branches is known as mixed node. For example x_2, x_3, x_4 and x_5 are mixed node.

4. <u>Transmittance</u>: Transmittance also known as transfer function, which is normally written on the branch near the arrow. For example: a_{12}, a_{23} etc.

5. <u>Forward path</u>: Forward path is a path which originates from the input node and terminates at the output node and along which no node is traversed more than once.

   Example: i) x_1 to x_2 to x_3 to x_4 to x_5 to x_6

   ii) x_1 to x_2 to x_3 to x_5 to x_6

6. <u>Loop</u>: Loop is a path that originates and terminates on the same node and along which no other node is traversed more than once.

   E.g. x_2 to x_3 to x_2

   x_3 to x_4 to x_3

7. <u>Self loop</u>: It is a path which originates and terminates on the same node

   E.g. x_4 to x_4

8. <u>Path gain</u>: The product of the branch gain along the path is called path gain.

   E.g. The gain of the path x_1 to x_2 to x_3 to x_4 to x_5 to x_6 is $a_{12} \cdot a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{56}$

9. <u>Loop gain</u>: The gain of the loop is known as loop gain

   E.g. The gain of the loop x_2 to x_3 to x_2 is $a_{23} \cdot a_{32}$

10. <u>Non-touching loop</u>: Non touching loop having no common node, branch and path. ~~for e-~~

    E.g. The loops x_2 to x_3 and x_4 to x_4 are non-touching loop.

Scanned with CamScanner

Comparison of block diagram and signal flow graph method:

Block diagram	SFG
*) Applicable to linear time invariant system only.	*) Applicable to linear time invariant system.
*) each element is represented by block	*) each variable is represented by node.
*) Summing point and takeoff point are separate	*) summing and takeoff point are absent.
*) self loop do not exist.	*) self loop can be exist.
*) it is time consuming method	*) required less time by using mason gain formula.
*) block diagram required in each and every step.	*) at each step it is not necessary to draw SFG.
*) transfer function of the element is shown inside the corresponding block.	*) transfer function is shown along the branches connecting the node.
*) feed-back path is present.	*) feed back loop are used

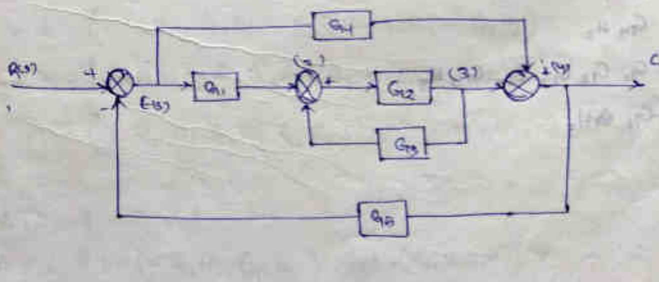
Construction of signal flow graph from block diagram:

Rule ① All variables summing point and takeoff point are represented by nodes.

Rule ② If a summing point is placed before a take off point in the direction of signal flow, in such case represent the summing point and takeoff point by a single node.

Rule ③ If a summing point is placed after a takeoff point in the direction of signal flow, in such case represent the summing point and takeoff point by seperate node connected by a branch having transmittance unity.

Consider the block diagram the corresponding



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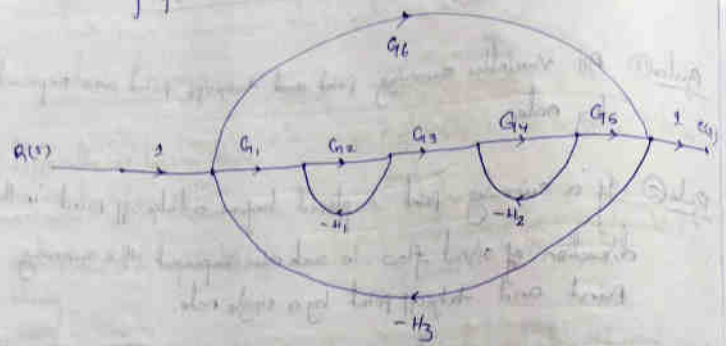
Q) Find transfer function?



Forward path:

$P_1 = G_1, G_2\, G_3\, G_4\, G_5$

$P_2 = G_6$

Loop:

$L_1 = -G_2 H_1$

$L_2 = -G_4 H_2$

$L_3 = -G_1 G_2 G_3 G_4 G_5 H_3$

$L_4 = -G_6 H_3$

2 Non touching loop:

~~$L_1 = -G_2 H_1$~~

~~$L_2 \neq H_1$~~

$L_1 L_4 = -G_2 H_1 - G_6 H_3 = G_2 H_1 G_6 H_3$

$L_2 L_4 = G_4 H_2 G_6 H_3$

$L_1 L_2 = G_2 H_1 G_4 H_2$

3 Nontouching loop:

$L_1 L_2 L_4 = -G_2 H_1\, G_4 H_2\, G_6 H_3$

~~$\Delta = 1 - ((-G_2 H_1) + (-G_4 H_2))$~~

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_4 + L_2 L_4 + L_1 L_2) - (L_1 L_2 L_4)$

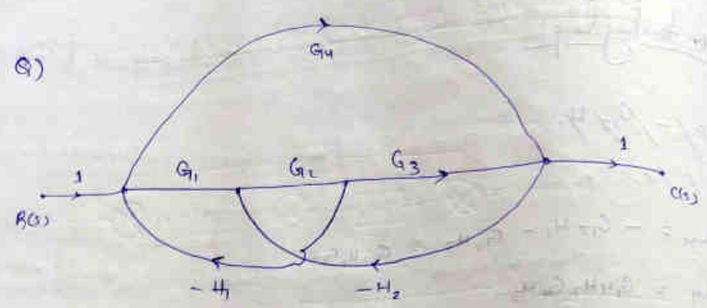
$$\Delta = 1 - (-G_2 H_1 - G_4 H_2 - G_1 G_2 G_3 G_5 H_3 - G_6 H_3)$$
$$+ (G_2 H_1 G_6 H_3 + G_4 H_2 G_6 H_3 + G_2 H_1 G_4 H_2)$$
$$- (-G_2 H_1 G_4 H_2 G_6 H_3) =$$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - (-G_2 H_1 - G_4 H_2) + (G_2 H_1 G_4 H_2) =$

$= 1 + G_2 H_1 -$

Q)



Forward path:

$P_1 = G_1 G_2 G_3$

$P_2 = G_4$

Loop:

$L_1 = -G_1 G_2 H_1$

$L_2 = -G_2 G_3 H_2$

$\Delta = 1 - [L_1 + L_2]$

$= 1 - [(-G_1 G_2 H_1) + (-G_2 G_3 H_2)]$

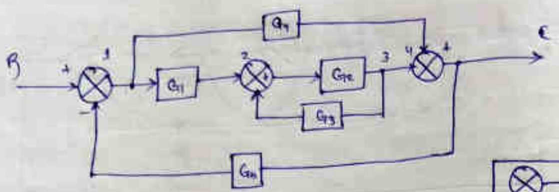
$= 1 + G_1 G_2 H_1 + G_2 G_3 H_2$

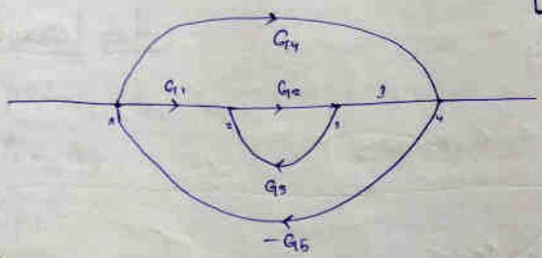
$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$$T = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2}$$

Construction of signal flow graph from block diagram:





Forward path:

$P_1 = G_1 G_2$

$P_2 = G_4$

Loop:

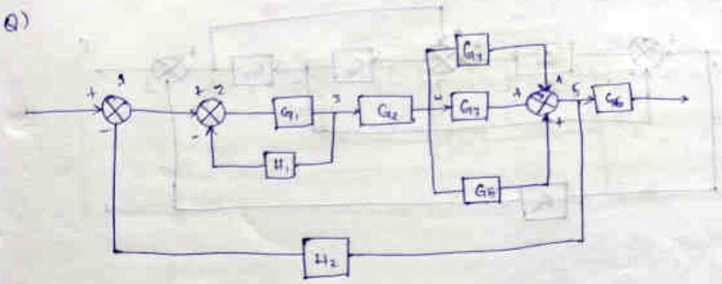
$L_1 = G_2 G_3$

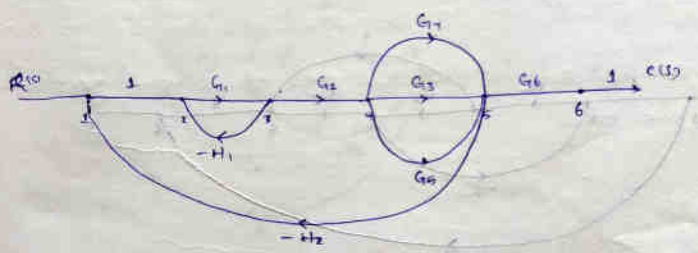
$L_2 = -G_1 G_2 G_5$

$L_3 = -G_4 G_5$

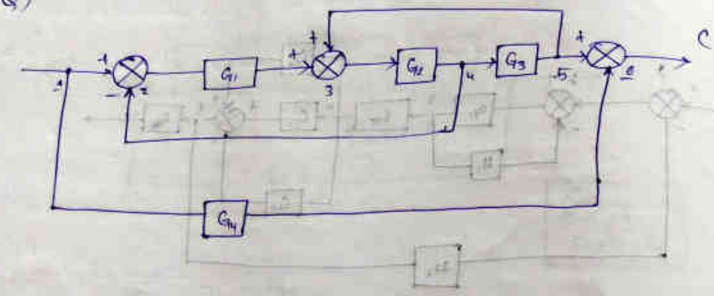
Non touching loop:

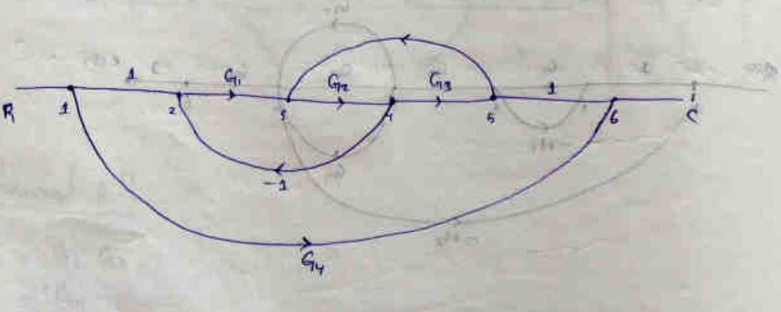
Q)



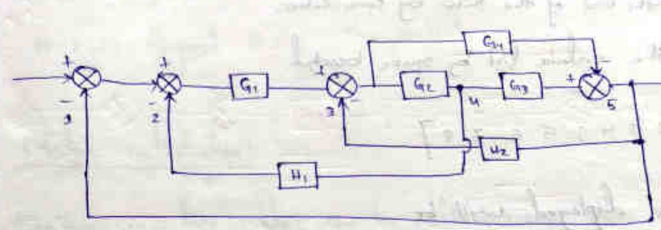


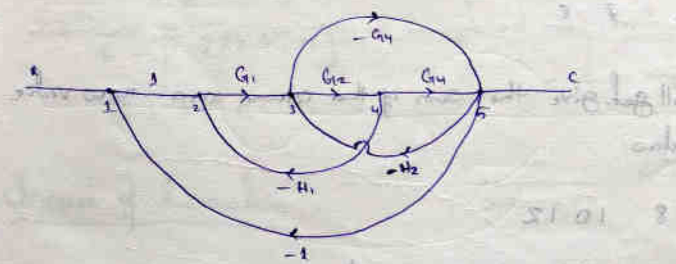
Q)





Q)





MATLAB

*) seperate the element of the row by space or Comma
*) seperate the end of the row by semi-colon.
*) Surround the entire list by square bracket

A= [1 2 3 4 ; 5 6 7 8]

The matrix displayed will be

A= 1 2 3 4
5 6 7 8

Sum (A) will give the sum of the column as a row vector as given below

6 8 10 12

A' give the transpose of the Matrix

1 5
2 6
3 7
4 8

A(1,1) Respond 1 as the output
A(2,3) Respond 7 as the output
A(2,2) Respond 6 as the output

Matrix multiplication:

) Two compatible matrix can be multiply using the operator ''

B= [1 2 3 4]
C = [1 ; 2 ; 3 ; 4]
D = B * C

Inverse of the matrix:

>> A = [1 0 0 ; 0 1 0 ; 0 0 1]

>> det (A)
1

>> inv(A)
1 0 0
0 1 0
0 0 1

The eigenvalue can be calculated using eig function.

```
>> eig(A)

Ans> 1
     1
     1
```

Transfer function:

*) Transfer function can be defined in MATLAB using the numerator and denominator matrix.

$$T.F = \frac{5}{s^2+2s+10}$$

The sequence of command will be like this

```
>> A = [5]
>> B = [1 2 10]
>> tf (A,B)
```

*) The co-efficient in the numerator and denominator of the matrix should be in the increasing order of the power towards the left of the matrix.

```
>> num = [2, 1, 10]
>> den = [1]
```

transfer function

$$\cancel{T.F = \frac{1}{2s^2+1s+10}}$$

$$T.F = \frac{2s^2+s+10}{1}$$

Root:

*) The function root 'c' compute the polynomial root whose element are vector 'C'

```
>> num = [2 1 10]
>> roots(num)
   -0.25 + 2.2220i
   -0.25 - 2.2220i
```

2-marks

Chapter ①

1) Define T.F
2) Define characteristic equation
3) Define masons gain formula
4) Define signal flow graph
5) Define loop
6) Define Node and branch
7) Define the forward path.
8) Why we use signal flow graph method.
9) Write drawback of block diagram reduction method
10) Write Two properties of signal flow graph

5 mark or 7 marks: →

Chapter ②

1) Explain property of signal flow graph
2) Comparison of block diagram and signal flow graph method
3) Explain Construction of signal flow graph from block diagram with example
4) Explain difference between open loop Control system and Closed loop Control System.

Date: 10/01/19

Time response analysis:

Standard test signal:

*) The standard test signal used in Control system are
→ step signal
→ Ramp signal
→ parabolic signal
→ impulse signal
→ sinusoidal signal

*) The input to the Control system is called excitation and the o/p of the Control system is called response.

*) The standard input or test signal or excitation applied to the Control system for finding the time response are step, Ramp and parabolic signal

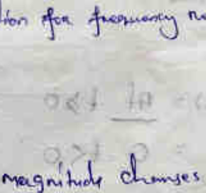
*) The sinusoidal signal is used as excitation for frequency response analysis.

Step signal:

*) The step signal is signal whose magnitude changes from zero to another level (say A) in a negligible time (No time)

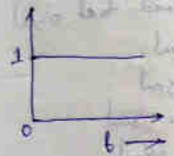
*) It is represented as

$$r(t) = A\,u(t) = A \quad t \geq 0$$
$$= 0 \quad t < 0$$



Unit step signal:

*) if magnitude of A is unity then step signal is known as unit step signal.

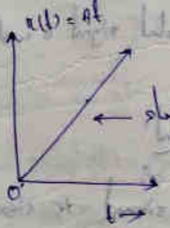


$L\{u(t)\} = L\{1\} = 1/s$

Ramp signal:

*) It is also called Constant Velocity signal.

*) Ramp signal is a signal which starts from 0 and increases linearly with time.



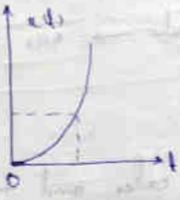
$r(t) = At \quad t \geq 0$
$\quad = 0 \quad t < 0$

$L\{r(t)\} = L(t) = \frac{1}{s^2}$

Parabolic signal:

*) It is also called acceleration signal

*) It is graphically represented by.



$r(t) = \frac{1}{2} A t^2 \quad t > 0$
$\quad = 0 \quad , t < 0$

*) Parabolic signal is integral of ramp signal and ramp signal is integral of step signal.

$L\{\frac{1}{2}At^2\} = \frac{1}{s^3}$

• Impulse signal:

*) An impulse signal is defined as a rectangular pulse having very short duration and infinite amplitude

*) The area under impulse is one

$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

$\delta(t) = 0 \quad t \neq 0$
$\quad = \infty \quad , t = 0$

$L\{\delta(t)\} = 1$

$$\boxed{\begin{array}{l} \text{step} \rightarrow R(s) \rightarrow \frac{1}{s} \\ \text{Ramp} \rightarrow R(s) \rightarrow \frac{1}{s^2} \\ \text{Parabolic} \rightarrow R(s) \rightarrow \frac{1}{s^3} \\ \text{Impulse} \rightarrow R(s) \rightarrow 1 \end{array}}$$

Q) What is order and type of a system?

Order:

*) The highest power of (s) or highest degree of (s) in the characteristic equation is called order of the system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$\boxed{1 + G(s)H(s) = 0}$ characteristic equation

$G(s) = \frac{s+1}{s+3}$ $H(s) = 1$

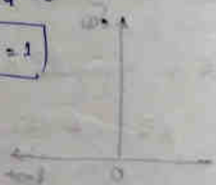
$1 + G(s)H(s) = 0$

$\Rightarrow 1 + \frac{s+1}{s+3} = 0$

$\Rightarrow \frac{s+3+s+1}{s+3} = 0$

$\Rightarrow 2s+4 = 0$

$\boxed{\text{Order} = 1}$



Q) $G(s) = \frac{s+1}{s(s+2)}$

$1 + G(s)H(s) = 0$

$\Rightarrow 1 + \frac{s+1}{s(s+2)} = 0$

$\Rightarrow 1 + \frac{s+1}{s(s+2)} = 0$

$\Rightarrow \frac{s(s+2)+(s+1)}{s(s+2)} = 0$

$\Rightarrow s^2+2s+s+1 = 0$

Order = 2

Q) $G(s) = \frac{s+1}{s^2(s+3)(s+5)}$ $\boxed{\text{4th order}}$

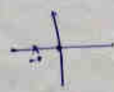
Type of the system:

*) The number of pole at the origin in the openloop transfer function is equal to type of the system

$G(s) = \frac{s+1}{s+2}$

Pole $s+2 = 0$

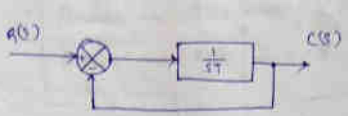
$s = -2$



$\boxed{\text{type} = 0}$

01 - 15/03/19

Derive the expression for the time response of a first order system.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

$G(s) = \frac{1}{sT}$

$H(s) = 1$ (unit feedback)

$$\frac{C(s)}{R(s)} = \frac{1/sT}{1 + \frac{1}{sT}\cdot 1}$$

$$= \frac{1/sT}{1 + 1/sT} = \frac{1/sT}{\frac{sT+1}{sT}} = \frac{1}{sT+1}$$

$$\boxed{C(s) = \cancel{\frac{1/sT}{1+1/sT}} \left(\frac{1}{sT+1}\right) R(s)}$$

Q) Derive expression for time response of a first order system to unity step input

$R(s) = \frac{1}{s}$

$$C(s) = \left(\frac{1}{sT+1}\right)\cdot\frac{1}{s}$$

$$C(s) = \frac{1}{s(sT+1)}$$

$$C(s) = \frac{1}{sT(s+1/T)}$$

$$C(s) = \frac{1/T}{s(s+1/T)}$$

using partial fraction method

$$C(s) = \frac{1/T}{s(s+1/T)} = \frac{A}{s} + \frac{B}{s+1/T}$$

$$A = s \times \frac{1/T}{s(s+1/T)}\bigg|_{s=0} = \frac{1/T}{1/T} = 1$$

$$B = (s+1/T)\cdot\frac{1/T}{s(s+1/T)}\bigg|_{s=-1/T} = \frac{1/T}{-1/T} = -1$$

$$C(s) = \frac{1/T}{s(s+1/T)} = \frac{A}{s} + \frac{B}{s+1/T}$$

$$C(s) = \frac{1}{s} - \frac{1}{s+1/T}$$

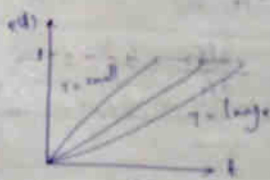
Taking Inverse laplace,

$$C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

$$L^{-1}\{C(s)\} = L^{-1}\{\tfrac{1}{s}\} - L^{-1}\left\{\frac{1}{s+\frac{1}{T}}\right\}$$

$\Rightarrow c(t) = 1 - e^{-\frac{1}{T}t}$

$\Rightarrow \boxed{c(t) = 1 - e^{-t/T}}$



T = Time Constant of system

$c(t) = 1 - e^{-0/T} = 1 - 1 = 0$

$c(t) = 1 - e^{-\infty/T} = 1 - 0 = 1$

The smallest time constant, indicate faster the

Steady state error

$$e_{ss}(t) = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty}\left[r(t) - c(t)\right]$$

$$= \lim_{t\to\infty}\left[1 - (1 - e^{-t/T})\right]$$

$$= \left[1 - (1 - e^{-\infty})\right]$$

$$= \left[1 - (1 - 0)\right]$$

$$= 1 - 1 = 0$$

b) In steady state the output track the input with a steady state error (0), so it is an excellent system for step input

Q) Time response of the 1st order system with unit ramp function.

$$C(s) = \frac{1}{sT+1} R(s)$$

Input of the unit ramp $R(s) = \frac{1}{s^2}$

$$C(s) = \left(\frac{1}{sT+1}\right)\frac{1}{s^2}$$

$$C(s) = \frac{1}{s^2(sT+1)}$$

$$C(s) = \frac{1}{s^2 T(s+1/T)} = \frac{1/T}{s^2(s+1/T)}$$

using partial fraction method

$$C(s) = \frac{1/T}{s^2(s+1/T)} = \frac{As+B}{s^2} + \frac{C}{(s+1/T)}$$

$$\boxed{\frac{1}{T} = (As+B)(s+1/T) + Cs^2}$$

$$\boxed{\frac{1}{T} = As(s+1/T) + B(s+1/T) + Cs^2} \quad \text{—(i)}$$ Comparing Numerator.

Putting $s=0$ in eqn(1)

$$\frac{1}{T} = B\frac{1}{T}$$

$$\Rightarrow \boxed{B=1}$$

$$s+1/T = 0$$

$$s = -1/T$$

Putting $s=-1/T$ in eqn (i)

$$\frac{1}{T} = -\frac{A}{T}\left(-\frac{1}{T}+\frac{1}{T}\right) + B\left(-\frac{1}{T}+\frac{1}{T}\right) + \left(\frac{C}{T^2}\right)$$

$$\frac{1}{T} = \frac{C}{T^2}$$

$$\Rightarrow T^2 = CT$$

$$\Rightarrow \boxed{T=C}$$

Comparing the Co-efficient of s^2 on both side.

$$0 = A+C$$

$$\Rightarrow \boxed{A = -C = -T}$$

$$C(s) = \frac{-Ts+1}{s^2} + \frac{T}{(s+1/T)}$$

$$= \frac{-T}{s} + \frac{1}{s^2} + \frac{T}{s+1/T}$$

taking inverse laplace transfore

$$L^{-1}\{C(s)\} = L^{-1}\left\{-\frac{T}{s}\right\} + L^{-1}\left\{\frac{1}{s^2}\right\} + L^{-1}\left\{\frac{T}{s+1/T}\right\}$$

$$L^{-1}\{C(s)\} = -TL^{-1}\left\{\frac{1}{s}\right\} + L\left\{\frac{1}{s^2}\right\} + TL^{-1}\left\{\frac{1}{s+1/T}\right\}$$

$$c(t) = -T \cdot 1 + t + T \cdot e^{-t/T}$$

$$\boxed{c(t) = -T + t + Te^{-t/T}}$$

error signal $e(t) = [r(t) - c(t)]$

$$= t - \left[-T + t + Te^{-t/T}\right]$$

$$= \cancel{t} + T - \cancel{t} - Te^{-t/T}$$

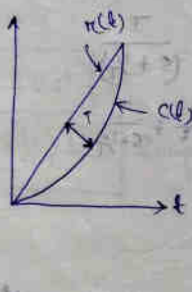
$$e(t) = T\left(1 - e^{-t/T}\right)$$

$$e_{ss} = \lim_{t\to\infty} e(t)$$

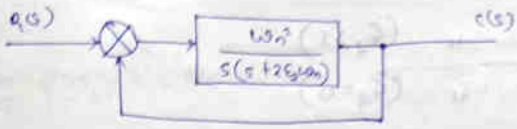
$$= \lim_{t\to\infty} \left(T - Te^{-t/T}\right)$$

$$= \left(T - Te^{-\infty/t}\right)$$

$$\boxed{e_{ss} = T}$$



Second order system:



$$G(s) = \frac{\omega_n^2}{s(s+2\xi_2\omega_n)} \qquad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{\frac{\omega_n^2}{s(s+2\xi_2\omega_n)}}{1 + \frac{\omega_n^2}{s(s+2\xi_2\omega_n)} \cdot 1}$$

$$= \frac{\frac{\omega_n^2}{s(\cancel{s+2\xi_2\omega_n})}}{\frac{s^2+2s\xi_2\omega_n+\omega_n^2}{s(\cancel{s+2\xi\omega_n})}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi_2\omega_n s + \omega_n^2}}$$

ξ_2 = damping ratio or factor

ω_n = undamped natural frequency.

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*) under damped system ($\xi_2 < 1$)

*) Critical " " ($\xi_2 = 1$)

*) over " " ($\xi_2 > 1$)

*) undamped " " ($\xi_2 = 0$)

Q) Find the ~~re~~ time responce of a second order system subtected to a unit step input function

The o/p for the system is given by

$$C(s) = \left(\frac{\omega_n^2}{s^2 + 2\xi_2 \omega_n s + \omega_n^2} \right) R(s)$$

Here input of a step function $= 1/s$

$$C(s) = \left(\frac{\omega_n^2}{s^2 + 2\xi_2 \omega_n s + \omega_n^2} \right) \frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi_2 \omega_n s + \omega_n^2)}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi_2 \omega_n s + \omega_n^2)} = \frac{A}{s} + \frac{Bs + C}{(s^2 + 2\xi_2 \omega_n s + \omega_n^2)}$$

$$\omega_n^2 = A(s^2 + 2\xi_2 \omega_n s + \omega_n^2) + s(Bs + C)$$

$$\boxed{\omega_n^2 = A(s^2 + 2\xi_2 \omega_n s + \omega_n^2) + Bs^2 + C.s}$$

Comparing s^2 term

$0 = A + B$

$\boxed{A = -B}$

Comparing s term

$0 = 2A\xi_2 \omega_n + C$

$\boxed{C = 2A\xi_2 \omega_n}$

Comparing constant term

$\omega_n^2 = A\omega_n^2$

$A = 1$

$B = -1$

$C = -2 \cdot 1\, \xi_2 \omega_n$

$\boxed{C = -2\xi_2 \omega_n}$

$$C(s) = \frac{1}{s} + \frac{[-s + (-2\xi_2 \omega_n)]}{s^2 + 2\xi_2 \omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + 2\xi_2 \omega_n}{s^2 + 2\xi_2 s + \omega_n^2}$$

$$C(s) = \frac{1}{s} + \frac{[s + (-2\xi_2\omega_n)]}{s^2 + 2\xi_2\omega_n s + \omega_n^2}$$

$$= \frac{s + 2\xi_2\omega_n}{s^2 + 2\xi_2\omega_n s + \omega_n^2 + (\xi_2\omega_n)^2 - (\xi_2\omega_n)^2}$$

$$= \frac{1}{s} - \frac{s + 2\xi_2\omega_n}{(s + \xi_2\omega_n)^2 + \omega_n^2(1-\xi_2^2)}$$

$$C(s) = \frac{1}{s} - \frac{s + \xi_2\omega_n + \xi_2\omega_n}{(s+\xi_2\omega_n)^2 + \omega_n^2(1-\xi_2^2)}$$

$$C(s) = \frac{1}{s} - \frac{s + \xi_2\omega_n}{(s+\xi_2\omega_n)^2 + \omega_n^2(1-\xi_2^2)} + \frac{\xi_2\omega_n}{(s+\xi_2\omega_n)^2 + \omega_n^2(1-\xi_2^2)}$$

let $\omega_n^2(1-\xi_2^2) = \omega_d^2$

$$\boxed{\omega_d = \omega_n\sqrt{1-\xi_2^2}}$$

$$C(s) = \frac{1}{s} - \frac{s + \xi_2\omega_n}{(s+\xi_2\omega_n)^2 + \omega_d^2} + \frac{\xi_2\omega_n}{(s+\xi_2\omega_n)^2 + \omega_d^2} \times \frac{\omega_d}{\omega_d}$$

$$= \frac{1}{s} - \frac{s + \xi_2\omega_n}{(s+\xi_2\omega_n)^2 + \omega_d^2} + \frac{\xi_2\omega_n\omega_d}{(s+\xi_2\omega_n)^2 + \omega_d^2\ \cancel{\omega_n}\sqrt{1-\xi_2^2}}$$

$$C(s) = \frac{1}{s} - \left(\frac{s + \xi_2\omega_n}{(s+\xi_2\omega_n)^2 + \omega_d^2} - \frac{\xi_2}{\sqrt{1-\xi_2^2}}\frac{\omega_d}{(s+\xi_2\omega_n)^2 + \omega_d^2}\right)$$

taking inverse laplace transform.

$$L^{-1}\left\{\frac{s+a}{(s+a)^2+b^2}\right\} = e^{-at}\cos bt$$

$$L^{-1}\left\{\frac{b}{(s+a)^2+b^2}\right\} = e^{-at}\sin bt$$

$$L^{-1}\{C(s)\} = L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{s+\xi_2\omega_n}{(s+\xi_2\omega_n)^2+\omega_d}\right\}$$

$$- \frac{\xi_2}{\sqrt{1-\xi_2^2}} \cdot L^{-1}\left\{\frac{\omega_d}{(s+\xi_2\omega_n)^2+\omega_d^2}\right\}$$

$$c(t) = 1 - e^{-\xi_2\omega_n t}\cos\omega_d t - \frac{\xi_2}{\sqrt{1-\xi_2^2}} \cdot e^{-\xi_2\omega_n t}\sin\omega_d t$$

$$c(t) = 1 - \frac{e^{-\xi_2\omega_n t}}{\sqrt{1-\xi_2^2}}\left(\cos\omega_d t\sqrt{1-\xi_2^2} + \xi_2\sin\omega_d t\right)$$

let $\xi_2 = \cos\phi$

$$\sqrt{1-\xi_2^2} = \sqrt{1-\cos^2\phi} = \sin\phi$$

$$c(t) = 1 - \frac{e^{-\xi_2\omega_n t}}{\sqrt{1-\xi_2^2}}\left(\cos\omega_d t\,\sin\phi + \cos\phi\,\sin\omega_d t\right)$$

~~c(t) = 1~~

$$\boxed{c(t) = 1 - \frac{e^{-\xi_2 \omega_n t}}{\sqrt{1-\xi_2^2}}\left[\sin(\phi + \omega_d t)\right]}$$

Time response specification:

*) Time response specification are

→ Delay time (t_d)

→ Rise time (t_r)

→ Peak time (t_p)

→ maximum or peak over shoot (M_p)

→ settling time (t_s)

→ steady state error (e_n)

Delay time:

*) It is the time required for the response to reach from 0 to 50% of the its final value in first time

Rise time (t_r):

*) It is the time required from the response to rise from 10% to 90% of its final value for over damped system and 0 to 100% for underdamped system.

Peak time (t_p)

*) The peak time is the time required from the response to reach the first peak of the time response on first peak over shoot.

Maximum over shoot (M_p)

*) It is the normalized difference between the peak of the time response and study o/p

*) The max percent over shoot is defined by = $\frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$

*) settling time:

*) The settling time is the time required for the response to reach required for the response to reach and stay with in the specified specified.

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Expression for peak time:

- $$c(t) = 1 - \frac{e^{-\xi_z\omega_n t}}{\sqrt{1-\xi_z^2}} \sin\left(\omega_n\sqrt{1-\xi_z^2})t + \phi\right)$$

the peak time (t_p) is found by differentiating $c(t)$

For maximum $\frac{d\,c(t)}{dt} = 0$

$$\frac{d}{dt}[1] - \frac{d}{dt}\left[\frac{e^{-\xi_z\omega_n t}}{\sqrt{1-\xi_z^2}}\left[\sin\left(\omega_n\sqrt{1-\xi_z^2}\right)t + \phi\right)\right] = 0$$

$$\Rightarrow -\left[\frac{e^{-\xi_z\omega_n t}}{\sqrt{1-\xi_z^2}} \cos\left(\omega_n\sqrt{1-\xi_z^2}\right)t \cdot \omega_n\sqrt{1-\xi_z^2} + \left[\sin\left(\omega_n\sqrt{1-\xi_z^2}\right)t + \phi\right] \frac{e^{-\xi_z\omega_n t}}{\sqrt{1-\xi_z^2}} \cdot (-\xi_z\omega_n)\right]$$

$$\Rightarrow \frac{\omega_n e^{-\xi_z\omega_n t}}{\sqrt{1-\xi_z^2}}\left[\cos\left[\omega_n\sqrt{(1-\xi_z^2)}t + \phi\right]\sqrt{1-\xi_z^2} - \sin\left[\left(\omega_n\sqrt{1-\xi_z^2}\right)t + \phi\right]\xi_z\right] = 0$$

$e^{-\xi_z\omega_n t} \neq 0$ and $\xi_z = \cos\phi$ $\sqrt{1-\xi_z^2} = \sin\phi$

~~Cos [ωn 1-~~

$$\frac{\sin\phi}{\cos\phi} = \frac{\sin\left[\left(\omega_n\sqrt{1-\xi_z^2}\right)t + \phi\right]}{\cos\left[\omega_n\left(\sqrt{1-\xi_z^2}\right)t + \phi\right]}$$

$$\tan\phi = \tan\left[\left(\omega_n\sqrt{1-\xi_z^2}\right)t + \phi\right]$$

$$\phi = \omega_n\sqrt{1-\xi_z^2}\,t + \phi$$

$$\omega_n\sqrt{1-\xi_z^2}\,t = 0$$

$$\Rightarrow n\pi = \left(\omega_n\sqrt{1-\xi_z^2}\right)t_p$$

$\boxed{\because n = 1 \quad \because t = t_p}$

$$\Rightarrow \boxed{\frac{\pi}{\omega_n\sqrt{1-\xi_z^2}} = t_p}$$

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

Dt - 18/03/19

Expression for maximum over shoot:

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\left(\omega_n\sqrt{1-\xi^2}\right)t + \phi\right]$$

Maximum over shoot occurs at peak time $t = t_p$

$$C(t) = 1 - \frac{e^{-\xi \omega_n t_p}}{\sqrt{1-\xi^2}} \sin\left[\left(\omega_n\sqrt{1-\xi^2}\right)t_p + \phi\right]$$

$$= 1 - \frac{e^{-\xi \omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2} \cdot \frac{\pi}{\omega_n\sqrt{1-\xi^2}}\right]$$

$$= 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} (\sin \pi + \phi)$$

$\boxed{\sin \pi + \phi = -\sin\phi}$

$$C(t) = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} - \sin\phi$$

$$= 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \sin\phi$$

$$C(t) = 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\cancel{\sqrt{1-\xi^2}}} \cdot \cancel{\sqrt{1-\xi^2}}$$

$$C(t) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\boxed{M_p = 1(t_{max}) - 1}$$

$$= 1 + e^{-\xi\pi/\sqrt{1-\xi^2}} - 1$$

$$\boxed{\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100}$$

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settling time (t_s)

*) To evaluate the settling time we must find the time for which c(t) reach and stay within ± 2% or ± 5%.

*) using the definition the settling time is the time it takes the amplitude of the decaying ~~should~~ sinusoidal to decay to 0.02

$$t_s = \frac{4}{\xi\omega_n} \quad (\text{for } \pm 2\%)$$

$$t_s = \frac{3}{\xi\omega_n} \quad (\text{for } \pm 5\%)$$

Formula:

*) $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\left(\omega_n\sqrt{1-\xi^2}\right)t + \phi\right]$

*) $\phi = \tan^{-1} \sqrt{1-\xi^2} / \xi$

*) $t_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi - \tan^{-1}\sqrt{1-\xi^2}/\xi^2}{\omega_n\sqrt{1-\xi^2}}$

*) $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

*) $\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

*) $M_p = \frac{\text{first Peak value} - \text{final value}}{\text{final value}} \times 100$

$= \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$

*) $\cos\phi = \xi \Rightarrow \phi = \cos^{-1}\xi$

*) $\sin\phi = \sqrt{1-\xi^2}$

*) $\tan\phi = \frac{\sqrt{1-\xi^2}}{\xi}$

Q) When a second order control system is subjected to a unit step i/p the value of $\xi = 0.5$ and $\omega_n = 6$ rad/sec

$\xi = 0.5$
$\omega_n = 6$ rad/sec

$$t_r = \frac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi - \cos^{-1}(0.5)}{6\sqrt{1-(0.5)^2}} = 0.403 \text{ sec}$$

$$t_p = \frac{\pi}{6\sqrt{1-(0.5)^2}} = 0.604 \text{ sec}$$

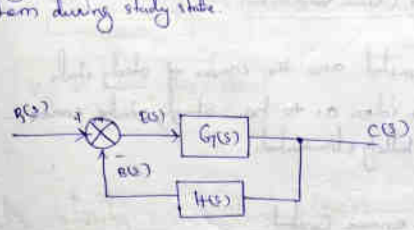
$$t_s = \frac{4}{0.5 \times 6} = 1.33 \text{ sec}$$

$$\% M_p = e^{-0.6\pi/\sqrt{1-(0.5)^2}} \times 100$$

$$= 16.3\%$$

<u>Study state error :-</u>

+) study state error is the difference between the i/p and o/p of the system during study state.



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

$E(S)/.G$ $\quad C(S) = E(S).G(S)$

$$\frac{E(S).G(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

$$\Rightarrow \frac{E(S)}{R(S)} = \frac{1}{1 + G(S)H(S)}$$

$$\Rightarrow E(S) = \frac{R(S)}{1 + G(S)H(S)}$$

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s.E(S)}$$

$$\boxed{e_{ss} = \lim_{s\to 0} s.\frac{R(S)}{1 + G(S)H(S)}}$$

*) steady state error depends on the i/p and openloop transfer function.

Static error Co-efficient (Error Constant)

*) Error Coefficient or constant are the measure of steady state error and give an idea as to how steady state error can be reduced or totally eliminated.

*) It is 3 type i) position error Constant
ii) static velocity error Constant
iii) accleration error Constant

Position error Constant (K_p):

*) position error Constant is define for a unitstep input
*) It has no dimension.
*) Laplace form unit step input is $R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s\to 0} s. \frac{R(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \cancel{s}. \frac{1/\cancel{s}}{1+G(s)H(s)}$$

$$= \frac{1}{1+G(s)H(s)}$$

$$e_{ss} = \frac{1}{1+G(s)H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}}$$

$$\boxed{K_p = \lim_{s\to 0} G(s)H(s)}$$

static velocity error Constant (K_v):

*) Velocity error Constant is defined for unit ramp input
*) Its unit is $1/sec$ or sec^{-1}

*) $$e_{ss} = \lim_{s\to 0} s. \frac{R(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \cancel{s}. \frac{1/s^{\cancel{2}}}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1/s}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1}{s+sG(s)H(s)}$$

$$e_{ss} = \frac{1}{sG(s)H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}} \qquad \boxed{K_v = \lim_{s\to 0} sG(s)H(s)}$$

acceleration error Constant (K_a):

*) acceleration error Const is defined for unit parabolic input

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} s \frac{1/s^3}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1/s^2}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \frac{1}{s^2 G(s) H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

$$\boxed{K_a = \lim_{s\to 0} s^2 G(s) H(s)}$$

$$K_p = \lim_{s\to 0} G(s) H(s) \qquad e_{ss} = \frac{1}{1+K_p} \qquad \text{No dimension}$$

$$K_v = \lim_{s\to 0} s \cdot G(s) H(s) \qquad e_{ss} = \frac{1}{K_v} \qquad 1/\text{sec}$$

$$K_a = \lim_{s\to 0} s^2 G(s) H(s) \qquad e_{ss} = \frac{1}{K_a} \qquad 1/\text{sec}^2$$

study state error for diff types of system :

For unit step input

a) type zero

$$G(s)\, H(s) = \frac{K(1+sT_1)(1+sT_2)}{(1+sT_a)(1+sT_b)}$$

$$K_p = \lim_{s\to 0} G(s) H(s)$$

$$= \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)}{(1+sT_a)(1+sT_b)}$$

$$K_p = \frac{K}{1} = K$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} = 0$$

For type 0, K_p = finite
e_{ss} = finite

Ramp input:

$$K_v = \lim_{s\to 0} s.\,G(s)\,H(s)$$

$$= \lim_{s\to 0} s.\,\frac{K(1+sT_1)(1+sT_2)}{(1+sT_a)(1+sT_b)}$$

$$\boxed{K_v = 0}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

$K_v = 0$
$e_{ss} = \infty$

Parabolic input:

$$K_a = \lim_{s\to 0} s^2\,G(s)\,H(s)$$

$$= \lim_{s\to 0} s^2.\,\frac{K(1+sT_1)(1+sT_2)}{(1+sT_a)(1+sT_2)}$$

$$\boxed{K_a = 0}$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

$K_a = 0$
$e_{ss} = \infty$

Type - 1 with unit step input:

$$K_p = \lim_{s\to 0} G(s)\,H(s)$$

$$= \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)}{s(1+sT_a)(1+sT_b)}$$

$$K_p = K/0 = \infty$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

Ramp.

$$K_v = \lim_{s\to 0} s.\,G(s)\,H(s)$$

$$= \lim_{s\to 0} \cancel{s}.\,\frac{K(1+sT_1)(1+sT_2)}{\cancel{s}(1+sT_a)(1+sT_b)}$$

$$= K$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}$$

K_v = finite
e_{ss} = finite.

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Parabolic :

$K_a = \lim_{s\to 0} s^2\, G(s)\, H(s)$

$= \lim_{s\to 0} s^2 \dfrac{K(1+sT_1)(1+sT_2)}{s(1+sT_a)(1+sT_b)}$

$= 0$

$e_{ss} = \dfrac{1}{K_a} = \dfrac{1}{0} = \infty$ $\quad K_a = 0$

$e_{ss} = \infty$

Type - 2 with unit step :

$K_p = \lim_{s\to 0} G(s)\, H(s)$

$= \lim_{s\to 0} \dfrac{K(1+sT_1)(1+sT_2)}{s^2(1+sT_a)(1+sT_b)}$

$K_p = K/0$

$e_{ss} = \dfrac{1}{1+K_p} = \dfrac{1}{1+\infty} = \dfrac{1}{\infty} = 0$

$K_p = \infty$

$e_{ss} = 0$

Ramp

$K_v = \lim_{s\to 0} s\, G(s)\, H(s)$

$= \lim_{s\to 0} s \dfrac{K(1+sT_1)(1+sT_2)}{s^2(1+sT_a)(1+sT_b)}$

$K_v = \dfrac{K}{0} = \infty$

$e_{ss} = \dfrac{1}{K_v} = \dfrac{1}{\infty} = 0$ $\quad K_v = \infty$

$e_{ss} = 0$

Parabolic

$K_a = \lim_{s\to 0} s^2\, G(s)\, H(s)$

$= \lim_{s\to 0} s^2 \dfrac{K(1+sT_1)(1+sT_2)}{s^2(1+sT_a)(1+sT_b)}$

$K_a = K$

$e_{ss} = 1/K_a = 1/K$

$K_a =$ finite

$e_{ss} =$ finite.

	type 0	type 1	type 2
unit step input (K_p)	$\frac{1}{1+K}$	0	0
unit ramp input (K_v)	∞	$\frac{1}{K}$	0
unit parabolic input (K_a)	∞	∞	$\frac{1}{K}$

Q) The open loop transfer function of a unit feed back system is given by

$$G(s) = \frac{50}{(1+0.1s)(s+10)}$$

determine the static error Coefficient K_p, K_v, K_a

$$K_p = \lim_{s \to 0} G(s)\,H(s)$$

$$= \lim_{s \to 0} \frac{50}{(1+0.1s)(s+10)}$$

$$= \frac{50}{10} = 5$$

$K_p = 5$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+5} = \frac{1}{6}$$

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$$

$$= \lim_{s \to 0} s\left(\frac{50}{(1+0.1s)(s+10)}\right)$$

$K_v = 0$ $\quad e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$

$$K_a = \lim_{s \to 0} s^2\,G(s)\,H(s)$$

$$= \lim_{s \to 0} s^2\left(\frac{50}{(1+0.1s)(s+10)}\right)$$

$K_a = 0$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Q) The forward path T.F of a unit feedback Control system is given by

$$G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

determine the step, Ramp, Parabolic error Co-efficient also determine the type of the system.

$$K_p = \lim_{s \to 0} G(s)\,H(s)$$

$$= \lim_{s \to 0} \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)} = \frac{500}{0}$$

$$= \infty$$

$$\therefore\ e_{ss} = \frac{1}{1+\infty} = 0$$

$K_v = \lim_{s \to 0} s\, G(s)\, H(s)$

$= \lim_{s \to 0} s \cdot \dfrac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$

$K_v = \infty$

$e_{ss} = 0$

$K_a = \lim_{s \to 0} s^2\, G(s)\, H(s)$

$= \lim_{s \to 0} s^2 \cdot \dfrac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$

$K_a = \dfrac{500}{5(10)} = \dfrac{500}{50} = 10$

$K_a = 10$

$e_{ss} = \dfrac{1}{10} = 0.1$

Dt - 12/01/19

8) A second order system has a transfer function as given by

$G(s) = \dfrac{25}{s^2+8s+25}$ Find ξ and ω_n if the system is subjected to a unit step input at t=0. the second peak in the response will occur at ________.

$R(s) = \dfrac{1}{s}$

$C(s) = \dfrac{G(s)}{1+G(s)H(s)} \cdot R(s)$

$C(s) = \dfrac{25}{s(s^2+8s+25)}$

$\dfrac{C(s)}{R(s)} = G(s)$

$C(s) = G(s)\, R(s)$

$C(s) = \dfrac{25}{s^2+8s+25} \cdot \dfrac{1}{s}$

Characteristic equation

$$\boxed{s^2+8s+25=0}$$

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$$s^2 + 2\xi_2 \omega_n s + \omega_n^2 = 0$$

$$s^2 + 8s + 25 = 0$$

$2\xi_2 \omega_n s = 8s$ $\qquad \omega_n^2 = 25$

$\qquad\qquad\qquad \omega_n = \sqrt{25} = 5$

$2\xi_2 5 = 8$

$\xi_2 = 8/10 = 0.8$

*) 2nd peak response $= 3\pi / \omega_n\sqrt{1-\xi_2^2}$

$$= \frac{3\pi}{\omega_n\sqrt{1-\xi_2^2}}$$

$$= \pi \text{ sec.}$$

Q) The open loop t.f of a servo system ~~of~~ with unit feedback is given by $G(s) = \frac{10}{(s+2)(s+5)}$ determine the damping ratio undamped natural frequency & overshoot of the response to a unit step input

Characteristic equation

$$1 + G(s)H(s) = 0$$

$$1 + \frac{10}{(s+2)(s+5)} = 0$$

$$\Rightarrow \frac{(s+2)(s+5)+10}{(s+2)(s+5)} = 0$$

$$\Rightarrow (s+2)(s+5)+10 = 0$$

$$s(s+5) + 2(s+5) + 10 = 0$$

$$\Rightarrow s^2 + 5s + 2s + 10 + 10 = 0$$

$$\Rightarrow s^2 + 7s + 20 = 0$$

$$s^2 + 2\xi_2\omega_n + \omega_n^2 = 0$$

$$\omega_n^2 = 20$$

$$\omega_n = \sqrt{20} = 4.472 \text{ Rad/sec}$$

$$2\xi_2\omega_n = 7$$

$$\xi_2 = 7/2\omega_n = 0.7826$$

$$M_p = e^{-\pi \xi_2/\sqrt{1-\xi_2^2}} \times 100$$

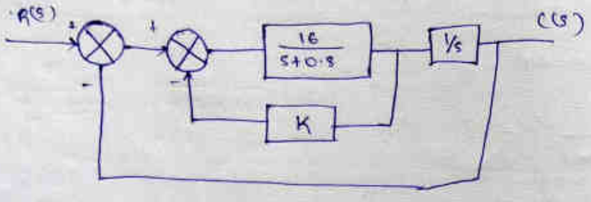
$$= e^{-\pi(0.7826)/\sqrt{1-(0.7825)^2}} \times 100$$

$$M_p = 1.92\ \%$$

Q) A feedback system is described by follwing T.F

$G(s) = \dfrac{12}{4s^2+4s+16}$ $H(s) = Ks$, the damping factor of the system is $= 0.8$ determine the overshoot of the system and the value of K

Q) Consider the system shown in fig determine the value of K such that damping ratio ($\xi_2 = 0.5$) then obtain rise time, peak time, maximum overshoot in the unit step response.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{16}{s+0.8}}{1+\frac{16}{s+0.8}\cdot K}$$

$$= \frac{\frac{16}{s+0.8}}{\frac{s+0.8+16K}{s+0.8}} = \frac{16}{s+0.8+16K}$$

$$= \frac{16}{(s+0.8+16K)s} = \frac{16}{s^2+(0.8+16K)s}$$

$$= \frac{\frac{16}{s^2+(0.8+16K)s}}{1+\frac{16}{s^2+(0.8+16K)s}} = \frac{\cancel{\frac{16}{s^2+(0.8+16K)s}}}{\frac{s^2+(0.8+16K)s+16}{\cancel{s^2+(0.8+16K)s}}} = \frac{16}{s^2+(0.8+16K)s+16}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2+(0.8+16K)s+16}$$

characteristic eqn $\Rightarrow s^2+2\xi_2\omega_n s+\omega_n^2=0$

$s^2+(0.8+16K)s+16=0$

$\omega_n^2 = 16$

$\Rightarrow \omega_n = \sqrt{16} = 4$

$\xi_2 = 0.5$

$2\xi_2\omega_n = 0.8+16K$

$\Rightarrow 2\times0.5\times4 = 0.8+16K$

$\Rightarrow \frac{2\times0.5\times4-0.8}{16} = K = 0.2$

$$t_r = \frac{\pi-\tan^{-1}\sqrt{1-\xi_2^2}/\xi_2}{\omega_n\sqrt{1-\xi_2^2}}$$

$$= \frac{\pi-\tan^{-1}\sqrt{1-0.5^2}/0.5}{4\sqrt{1-0.5^2}} = 0.605 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi_2^2}} = \frac{\pi}{4\sqrt{1-0.5^2}} = 0.906 \text{ sec}$$

$$M_p = e^{-\pi \xi_2/\sqrt{1-\xi_2^2}} \times 100$$

$$= e^{-\pi\, 0.5/\sqrt{1-0.5^2}} \times 100 = 10.3\,\%$$

$$t_s = \frac{4}{\xi_2 \omega_n} = \frac{4}{0.5 \times 4} = 2\,sec.$$

Polar plot.

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*) It is a frequency domain analysis it is drawn in G-H plane with polar co-ordinates that is magnitude vs phase angle by varying ω from '0' to '∞'

*) Polar plot travel from $\omega = 0$ to $\omega = \infty$ in the clockwise direction

→ It is drawn for open loop transfer function

$$z = x + iy$$

$$z = \text{Real} + \text{Imaginary}$$

Magnitude $|z| = \sqrt{(\text{Real})^2 + (\text{Img})^2}$

$$|z| = (x)^2 + (iy)^2$$

Phase angle $= \angle\theta = \tan^{-1}\left[\frac{y}{x}\right]$

$$\angle\theta = \tan^{-1}\left[\frac{\text{Img}}{\text{Real}}\right]$$

$\omega = 0$
$\omega = \infty$

Q) Find polar plot $G(s) = \frac{1}{s}$

Put $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{0^2 + \omega^2}} = \frac{1}{\sqrt{\omega^2}} = \frac{1}{\omega}$$

$\tan^{-1}(\infty) = 90$
$\frac{1}{0} = \infty$
$0/1 = 0$
$\frac{1}{\infty} = 0$

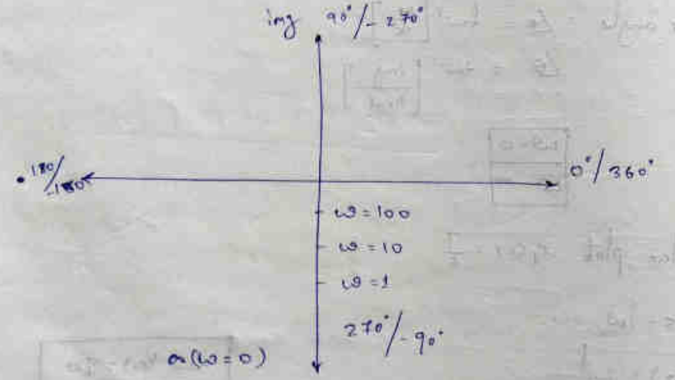
Scanned with CamScanner

$\theta = -\tan^{-1}\left(\frac{\omega}{0}\right)$

$= -\tan^{-1}(\infty)$

$= -90^\circ$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	-90°
1	1	-90°
10	0.1	-90°
⋮	⋮	
∞	0	-90°



Q) Find polar plot

$$G(s) = \frac{1}{1+sa}$$

Put $s = j\omega$

$$G(j\omega) = \frac{1}{1+(j\omega)a}$$

$$\lvert G(j\omega)\rvert = \frac{1}{\sqrt{(1)^2+(\omega a)^2}} = \frac{1}{\sqrt{1+\omega^2a^2}} = \frac{1}{\sqrt{1+\omega^2a^2}}$$

$$\angle G(j\omega) = -\tan^{-1}\left[\frac{\omega a}{1}\right] = -\tan^{-1}\omega a$$

$\omega = 0$,

$\angle G(j\omega) = -\tan^{-1}(0)a = 0$

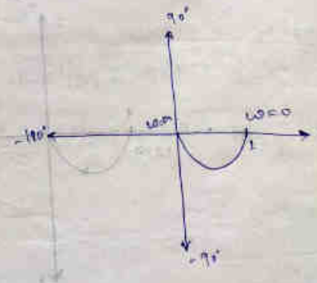
$G(j\omega) = \frac{1}{\sqrt{1}} = 1$

$\omega = \infty$

$G(j\omega) = \frac{1}{\infty} = 0$

$\angle G(j\omega) = -\tan^{-1}(\infty)a = -90^\circ$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	1	0°
∞	0	-90°



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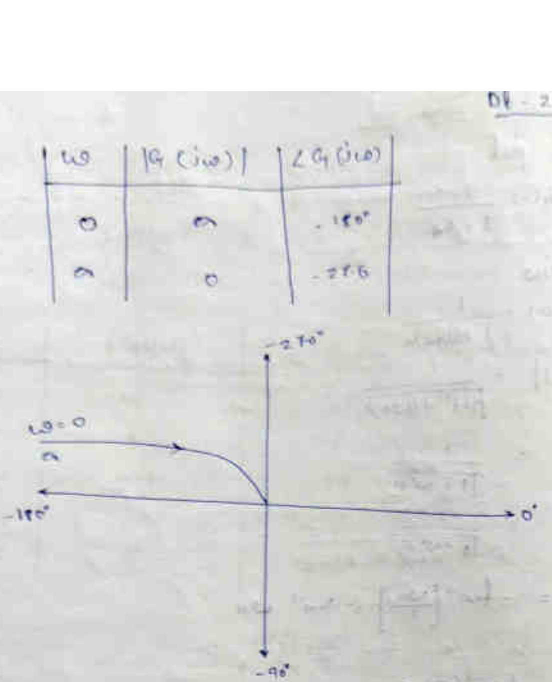
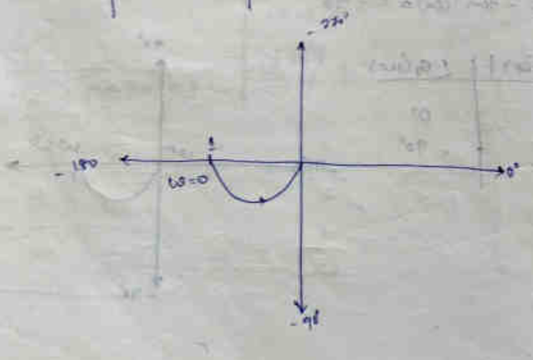
ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	$-180°$
∞	0	-270



ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	1	-180
∞	0	-90



$G(s) = \frac{1}{s(s+1)}$ sketch polar plot

Put $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{0^2+\omega^2}\sqrt{1^2+\omega^2}}$$

$$= \frac{1}{\sqrt{\omega^2}\sqrt{1+\omega^2}}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2}}$$

$\underline{\omega = 0}$

$$|G(j\omega)| = \frac{1}{0\sqrt{1+0}} = \frac{1}{0} = \infty$$

$\underline{\omega = \infty}$

$$|G(j\omega)| = \frac{1}{\infty\sqrt{1+\infty}} = \frac{1}{\infty} = 0$$

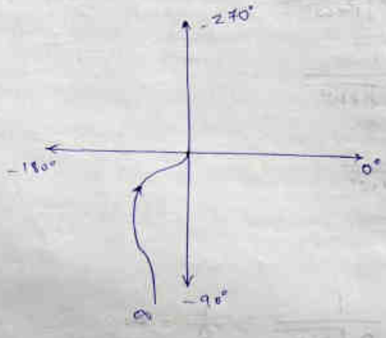
$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{1}\right)$$

$$= -90° - \tan^{-1}\omega$$

$\underline{\omega = 0}$ $\angle G(j\omega) = -90 - \tan^{-1}(0) = -90°$

$\underline{\omega = \infty}$ $\angle G(j\omega) = -90° - \tan^{-1}(\infty) = -90° - 90° = -180°$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	-90°
∞	0	-180°



*) $G(s) = \dfrac{20}{s(s+1)(s+2)}$

Put $s = j\omega$

$$G(j\omega) = \frac{20}{j\omega(j\omega+1)(j\omega+2)}$$

$$|G(j\omega)| = \frac{20}{\sqrt{0^2+\omega^2}\sqrt{1+\omega^2}\sqrt{2^2+\omega^2}}$$

$$= \frac{20}{\sqrt{\omega^2}\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$|G(j\omega)| = \frac{20}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$\underline{\omega=0}$ $|G(j\omega)| = \dfrac{20}{0\sqrt{1+0}\sqrt{4+0}} = \dfrac{20}{0} = \infty$

$\omega=\infty$ $|G(j\omega)| = \dfrac{20}{\infty\sqrt{1+\infty}\sqrt{4+\infty}} = \dfrac{20}{\infty} = 0$

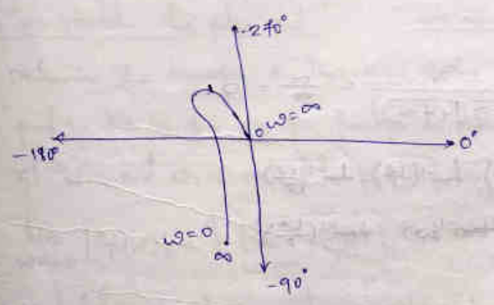
$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$\angle G(j\omega) = -\tan^{-1}(\infty) - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$\underline{\omega=0}$ $= -90^\circ - \tan^{-1}(0) - \tan^{-1}(0) = -90^\circ$

$= -90^\circ - \tan^{-1}(\infty) - \tan^{-1}(\infty) = -270^\circ$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	-90°
∞	0	-270°



4) $G(s) = \dfrac{s}{(s+1)(s+2)}$

Put $s = j\omega$

$$G(j\omega) = \frac{j\omega}{(j\omega+1)(j\omega+2)}$$

$$|G(j\omega)| = \frac{\sqrt{0^2+\omega^2}}{\sqrt{1^2+\omega^2}\sqrt{2^2+\omega^2}}$$

$$= \frac{\sqrt{\omega^2}}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$|G(j\omega)| = \frac{\omega}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$\underline{\omega = 0}$

$$|G(j\omega)| = \frac{0}{\sqrt{1+0}\sqrt{4+0}} = 0$$

$\underline{\omega = \infty}$

$$|G(j\omega)| = \frac{\infty}{\sqrt{1+\infty}\sqrt{4+\infty}} = \frac{\infty}{\infty} = 0$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$= 90^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

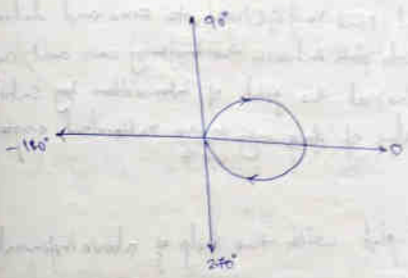
$\underline{\omega = 0}$

$$\angle G(j\omega) = 90^\circ - \tan^{-1}(0) - \tan^{-1}(0)$$
$$= 90^\circ$$

$\omega = \infty$

$$\angle G(j\omega) = 90^\circ - \tan^{-1}(\infty) - \tan^{-1}(\infty)$$
$$= -90^\circ$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	0	90°
∞	0	-90°



<u>Procedure to sketch the polar plot:</u>

<u>step-1</u>: Determine the transfer function $G(s)$ of the system.

<u>step-2</u>: put $s = j\omega$ in the transfer function to obtain $G(j\omega)$

<u>step-3</u>: At $\omega = 0$ and $\omega = \infty$ calculate $|G(j\omega)|$ by

$$\lim_{\omega \to 0} |G(j\omega)| \text{ and } \lim_{\omega \to \infty} |G(j\omega)|$$

<u>step-4</u>: calculate the phase angle of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$

$$\lim_{\omega \to 0} \angle G(j\omega) \text{ and } \lim_{\omega \to \infty} \angle G(j\omega)$$

Step 5: Rationalise the function $G(j\omega)$ and separate the real and imaginary parts

Step 6: Equate the imaginary part $Im|G(j\omega)|$ to ~~zero~~ zero and determine the frequency at which plots intersect the real axis and calculate the value $G(j\omega)$ at the point of ~~.....~~ intersects substituting the determined value of frequency in the expression of $G(j\omega)$

Step 7: Equating the real part ie $|G(j\omega)|$ to zero and determine the frequencies at which plots intersect the imaginary axis and calculate the value of $G(j\omega)$ at the point of intersection by substituting the determined value of frequency in the rationalised expression of $G(j\omega)$.

Step 8: sketch the polar plot with the help of above information.

Find or sketch polar plot:

$$G(s) = \frac{K}{(1+jT_1)(1+jT_2)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$|G(j\omega)| = \frac{K}{\sqrt{1^2+(\omega T_1)^2}\ \sqrt{1^2+(\omega T_2)^2}}$$

step-2

taking the limit for the magnitude of $G(j\omega)$

$$\lim_{\omega\to 0} G(j\omega) = \lim_{x\to 0} \frac{K}{\sqrt{1^2+(\omega T_1)^2}\ \sqrt{(1^2+\omega T_2)^2}} = K$$

$$\lim_{\omega\to \infty} G(j\omega) = \lim_{\omega\to \infty} \frac{K}{\sqrt{1^2+(\omega T_1)^2}\ \sqrt{(1^2+\omega T_2)^2}} = 0$$

step-3

$$\lim_{\omega\to 0} G(j\omega) = \lim_{\omega\to 0} -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = 0^\circ$$

$\omega = \infty$

$$\angle G(j\omega) = -\tan^{-1}(\infty) - \tan^{-1}(\infty)$$
$$= -90 - 90$$
$$= -180^\circ$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	K	0°
∞	0	−180°

*) $G(s) = \dfrac{K}{s(1+sT_1)(1+sT_2)}$

Put $s = j\omega$

$$G(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$|G(j\omega)| = \frac{K}{\sqrt{0^2 + j\omega^2}\,\sqrt{1^2 + (\omega T_1)^2}\,\sqrt{1^2 + (\omega T_2)}}$$

$$= \frac{K}{\sqrt{\omega^2}\,\sqrt{1 + (\omega T_1)^2}\,\sqrt{1 + (\omega T_2)^2}}$$

$\omega = 0$

$$\frac{K}{0\sqrt{1+0}\,\sqrt{1+0}} = \infty$$

$\omega = \infty$

$$\frac{K}{\infty\sqrt{1+\infty}\,\sqrt{1+\infty}} = 0^\circ$$

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

$$= -\tan^{-1}(\infty) - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

$$= -90 - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

$\omega = 0$

$$= -90^\circ - \tan^{-1}(0) - \tan^{-1}(0)$$

$$= -90^\circ$$

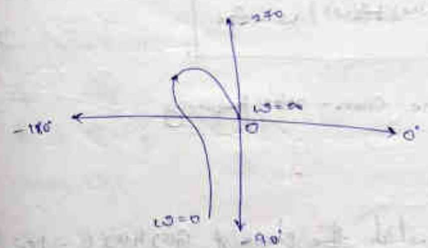
$\omega = \infty$

$$= -90^\circ - \tan^{-1}(\infty) - \tan^{-1}(\infty)$$

$$= -90^\circ - 90^\circ - 90^\circ$$

$$= -270^\circ$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	-90°
∞	0	-270°



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G.m) Gain margin:

*) It is the gain which can be varied before the system becomes just stable.

*) If the gain of the system increased, G + decreases

*) If the gain is double G_m become half

$$\boxed{G_m = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}}$$

ω_{pc} = phase cross-over frequency

Phase cross-over frequency:

*) It is the frequency at which the phase of $G(s)H(s)$ is $-180°$

*) $\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}} = -180°$

*) Img. of $G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}} = 0$

Phase Margin: (P.M)

$$\boxed{P_m = 180° + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}}$$

ω_{gc} = Gain ~~cross~~ cross over frequency.

ω_{gc} is the frequency at which the magnitude of $G(s)H(s)$ is unity or 0 dB.

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1$$

*) if ω_{pc} and ω_{gc} does not exist; gain and phase margins are ∞

*) if $\left.\begin{matrix} G_m = +ve \\ P_m = +ve \end{matrix}\right\}$ stable system

Then $\omega_{gc} < \omega_{pc}$

*) If $\left.\begin{matrix} G_m = 0\,dB \\ P_m = 0° \end{matrix}\right\}$ just stable

term ω_{gc}

*) If $\left.\begin{matrix} G_m = -ve \\ P_m = -ve \end{matrix}\right\}$ unstable system

then $\omega_{gc} > \omega_{pc}$

*) Find gain margin

$$G(s)H(s) = \frac{10}{s(s^2+s+1)}$$

$$G_m = \frac{1}{|G(j\omega)H(j\omega)|}\Bigg|_{\omega=\omega_{pc}}$$

Step-1

Find ω_{pc}

$$\angle G(j\omega)\,H(j\omega) = -180^\circ$$

$$\angle \frac{10}{j\omega((j\omega)^2+j\omega+1)} = -180^\circ \Bigg|_{\omega=\omega_{pc}}$$

$$\angle \frac{10}{j\omega(-\omega^2+1+j\omega)} \Bigg|_{\omega=\omega_{pc}}$$

$$= -\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left[\frac{\omega}{1-\omega^2}\right] \Bigg|_{\omega=\omega_{pc}} = -180^\circ$$

$$= -90^\circ - \tan^{-1}\left[\frac{\omega}{1-\omega^2}\right] \Bigg|_{\omega=\omega_{pc}} = -180^\circ$$

$$= \cancel{-}\left[90^\circ + \tan^{-1}\left[\frac{\omega}{1-\omega^2}\right] \Bigg|_{\omega=\omega_{pc}}\right] = \cancel{-}180^\circ$$

$$= 90 + \tan^{-1}\left[\frac{\omega_{pc}}{1-\omega_{pc}^2}\right] = 180^\circ$$

$$= \tan^{-1} \frac{\omega_{pc}}{1-\omega_{pc}^2} = 180^\circ - 90^\circ$$

$$\Rightarrow \tan^{-1} \frac{\omega_{pc}}{1-\omega_{pc}^2} = 90^\circ$$

$$\Rightarrow \frac{\omega_{pc}}{1-\omega_{pc}^2} = \tan 90^\circ$$

$$\Rightarrow \frac{\omega_{pc}}{1-\omega_{pc}^2} = \infty$$

$$\Rightarrow 1-\omega_{pc}^2 = 0$$

$$\Rightarrow \cancel{-}\omega_{pc}^2 = \cancel{-}1$$

$$\Rightarrow \omega_{pc} = \sqrt{1}$$

$$\Rightarrow \omega_{pc} = 1 \text{ rad/sec}$$

$$= \frac{10}{j\omega(j\omega^2 + j\omega + 1)}$$

$$= \frac{10}{j\omega(-\omega^2 + j\omega + 1)}$$

$$= \left|\frac{10}{\sqrt{0^2+\omega^2}\sqrt{(-\omega^2+1)+\omega^2}}\right|$$

$$= \left|\frac{10}{\sqrt{\omega^2}\sqrt{(1+\omega^2)^2+\omega^2}}\right|_{\omega_{pc}=1}$$

$$= \left|\frac{10}{\omega\sqrt{(1-\omega^2)^2+\omega^2}}\right|_{\omega_{pc}=1}$$

$$= \frac{10}{\omega_{pc}\sqrt{(1-\omega_{pc}^2)^2+\omega_{pc}^2}}$$

$$= \frac{10}{1\sqrt{(1-1)^2+1^2}}$$

$$= \frac{10}{1\sqrt{0+1}}$$

$$= \frac{10}{1} = 10$$

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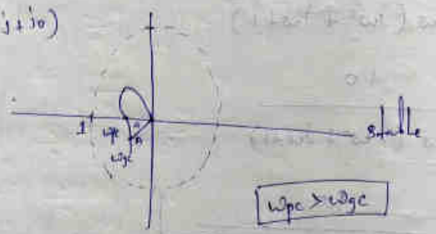
$(-1+j0)$



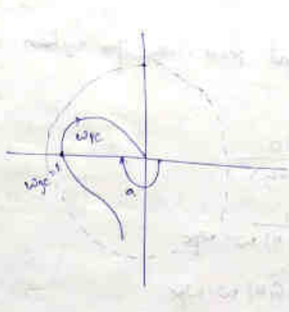
stable

$\boxed{\omega_{pc} > \omega_{gc}}$

$a < 1$

$G_m = \frac{1}{a} > 1$

$= 20 \log \frac{1}{a}$

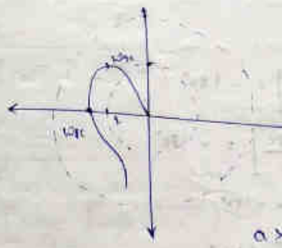


Marginal stable

$a = 1$

$G_m = 1$

$G_m = 20 \log 1 = 0$



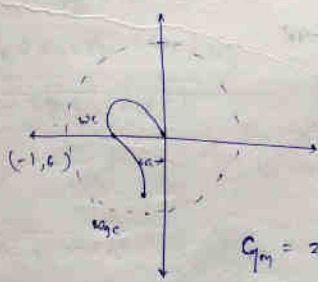
$a > 1$

$G_m = \frac{1}{a} < a$

$= 20 \log \frac{1}{a}$

$\boxed{\omega_{gc} > \omega_{pc}}$

unstable



$$G_m = 20 \log \frac{1}{|GH|_{\omega=\omega_{pc}}}$$

$$P_m = 180^\circ + \angle GH \,\Big|_{\omega=\omega_{gc}}$$

Q) Find the gain margin and phase margin for system having open loop transfer function

$$G(s)\,H(s) = \frac{10}{s(s+5)}$$

$$G_m = 20\log \frac{1}{|GH|}\Big|_{\omega=\omega_{pc}}$$

$$= -20\log |GH|\Big|_{\omega=\omega_{pc}}$$

at $\omega = \omega_{pc}$

$$\angle GH = -180^\circ$$

$\Rightarrow -\tan^{-1}\left[\frac{\omega}{0}\right] - \tan^{-1}\left[\frac{\omega}{5}\right] = -180^\circ$

$\Rightarrow -\tan^{-1}(\infty) - \tan^{-1}\left[\frac{\omega_{pc}}{5}\right] = -180^\circ$

$\Rightarrow -90^\circ - \tan^{-1}\left[\frac{\omega_{pc}}{5}\right] = -180^\circ$

$\Rightarrow \cancel{-}\left[90^\circ + \tan^{-1}\left[\frac{\omega_{pc}}{5}\right]\right] = \cancel{-}180^\circ$

$\Rightarrow 90^\circ + \tan^{-1}\left[\frac{\omega_{pc}}{5}\right] = 180^\circ$

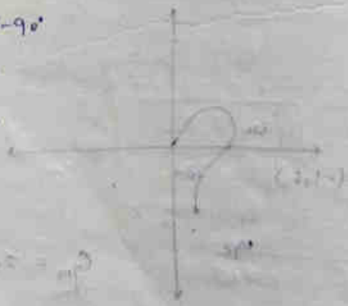
$\Rightarrow \tan^{-1}\left[\frac{\omega_{pc}}{5}\right] = 180^\circ - 90^\circ$

$\Rightarrow \tan^{-1}\left[\frac{\omega_{pc}}{5}\right] = 90^\circ$

$\Rightarrow \frac{\omega_{pc}}{5} = \tan 90^\circ$

$\Rightarrow \omega_{pc} = \infty$

$\Rightarrow \omega_{pc} = \infty$



$$G_m = 20\log\left|\frac{1}{\frac{10}{\sqrt{0^2+\omega^2}\sqrt{\omega^2+25}}}\right|_{\omega=\omega_{pc}=\infty}$$

$$= 20\log\left|\frac{1}{0}\right|$$

$$\boxed{G_m = \infty}$$

$$P_m = 180^\circ + \angle GH\Big|_{\omega=\omega_{gc}}$$

$\omega = \omega_{gc}$

$$|GH| = 1$$

$\Rightarrow \frac{10}{\sqrt{0^2+\omega^2}\sqrt{\omega^2+25}} = 1$

$\Rightarrow \frac{10}{\omega_{gc}\sqrt{\omega_{gc}^2+25}} = 1$

$\Rightarrow (10)^2 = \left(\omega_{gc}\sqrt{\omega_{gc}^2+25}\right)^2$

$\Rightarrow 100 = \omega_{gc}^2(\omega_{gc}^2+25)$

$\Rightarrow -\omega_{gc}^4 - 25\omega_{gc}^2 + 100 = 0$

$\Rightarrow -(\omega_{gc}^4 + 25\omega_{gc}^2 - 100) = 0$

$\Rightarrow \omega_{gc}^4 + 25\omega_{gc}^2 - 100 = 0$

(c)

$x = \omega_{gc}^2 \Rightarrow \omega_{gc} = \sqrt{x}$

$-x^2 + 25x - 100 = 0$

$x = 3.50$

$\omega_{gc} = \sqrt{x}$

$= \sqrt{3.50}$

$= 1.87 \text{ rad/sec}$

$\angle GH / _{\omega = \omega_{gc}} = -\tan^{-1}\left[\frac{\omega}{0}\right] - \tan^{-1}\left[\frac{\omega}{5}\right]$

$= -\tan^{-1}(\infty) - \tan^{-1}\left[\frac{\omega_{gc}}{5}\right]$

$= -\tan^{-1}(\infty) - \tan^{-1}\left[\frac{1.87}{5}\right]$

$= -90° - 20.50$

$= -110$

$P_m = 180° + \angle GH|_{\omega = \omega_{gc}}$

$= 180° + (-110) = 70°$ Ans

Umesh 30/1/19

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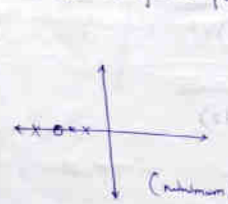
Classification of the system

minimum phase system

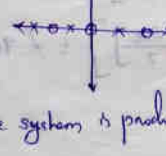
- A system in which all the finite pole and finite zeroes lies in the left of s-plane.



(minimum phase system)

All Pass system:

- All pole lies on left side and all zeroes lies on Right side
- Magnitude independent of frequency general magnitude = 1



Non-minimum phase system:

- A system in which one or more pole lies in the right of s-plane then it is called non-minimum phase system

- Non-minimum phase system is product of minimum phase system or all pass system

Problem:

Find Gm and Pm:

$$G(s)\,H(s) = \frac{10}{s(s+1)(s+2)}$$

$$|GH| = \frac{10}{j\omega(j\omega+1)(j\omega+2)}$$

$$= \frac{10}{\sqrt{(0^2+\omega^2)}\sqrt{(\omega)^2+(1)^2}\sqrt{\omega^2+(2)^2}}$$

$$= \frac{10}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$\angle GH = -\tan^{-1}\left[\frac{\omega}{0}\right] - \tan^{-1}\left[\frac{\omega}{1}\right] - \tan^{-1}\left[\frac{\omega}{2}\right]$$

$$\angle GH = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$\omega = \omega_{pc}$

$$\angle GH\Big|_{\omega=\omega_{pc}} = -180^\circ$$

$$\Rightarrow -90 - \tan^{-1}\left[\frac{\omega_{pc}}{1}\right] - \tan^{-1}\left[\frac{\omega_{pc}}{2}\right] = -180^\circ$$

$$\Rightarrow \cancel{-}\left(\tan^{-1}(\omega_{pc}) + \tan^{-1}\left[\frac{\omega_{pc}}{2}\right]\right) = \cancel{-}\,90^\circ$$

$$\tan^{-1}a + \tan^{-1}b = \tan^{-1}\left[\frac{a+b}{1-ab}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\omega_{pc} + \omega_{pc}/2}{1 - \omega_{pc}\frac{\omega_{pc}}{2}}\right] = 90$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{3\omega_{pc}}{2}}{1 - \frac{\omega_{pc}^2}{2}}\right] = 90^\circ$$

$$\left[\frac{\frac{3\omega_{pc}}{2}}{1 - \frac{\omega_{pc}^2}{2}}\right] = \tan 90$$

$$\frac{\frac{3\omega_{pc}}{2}}{1 - \frac{\omega_{pc}^2}{2}} = \infty$$

$$\frac{\frac{3\omega_{pc}}{2}}{1 - \frac{\omega_{pc}^2}{2}} = \frac{1}{0}$$

$$1 - \frac{\omega_{pc}^2}{2} = 0$$

$$\frac{\omega_{pc}^2}{2} = 1$$

$$\omega_{pc}^2 = 1 \times 2$$

$$\omega_{pc}^2 = 2$$

$$\boxed{\omega_{pc} = \sqrt{2}}$$

$$|GH|_{\omega=\omega_{pc}} = \frac{10}{\omega_{pc}\sqrt{1+\omega_{pc}^2}\sqrt{4+\omega_{pc}^2}}$$

$$= \frac{10}{\sqrt{2}\sqrt{1+(\sqrt{2})^2}\sqrt{4+(\sqrt{2})^2}}$$

$$= \frac{10}{\sqrt{2}\sqrt{3}\sqrt{6}}$$

$$= \frac{10}{6}$$

$$= 5/3$$

$$G_m = 20\log\left|\frac{1}{GH}\right|$$

$$= 20\log\left(\frac{6}{10}\right)$$

$$= -4.43 \text{ dB}$$

Phase margin $= 180° + \angle GH|_{\omega=\omega_{gc}}$

$|GH|_{\omega_{gc}}$

$\underline{\omega = \omega_{gc}}$ $|GH| = 1$

$$\Rightarrow \frac{10}{\omega_{gc}\sqrt{1+\omega_{gc}^2}\sqrt{4+\omega_{gc}^2}} = 1$$

$$\Rightarrow \left(\omega_{gc}\sqrt{1+\omega_{gc}^2}\sqrt{4+\omega_{gc}^2}\right)^2 = (10)^2$$

$$\Rightarrow \omega_{gc}^2(1+\omega_{gc}^2)(4+\omega_{gc}^2) = 100$$

Let $x = \omega_{gc}^2$

$$\Rightarrow x(1+x)(4+x) = 100$$

$$\Rightarrow (x+x^2)(4+x) = 100$$

$$\Rightarrow x^3 + 5x^2 + 4x - 100 = 0$$

$$\Rightarrow x = 3.24$$

$$\Rightarrow x = \omega_{gc}^2$$

$$\Rightarrow 3.24 = \omega_{gc}^2$$

$$\Rightarrow \omega_{gc} = \sqrt{3.24}$$

$$= 1.8 \text{ rad/sec}$$

$$\angle GH|_{\omega=\omega_{gc}} = -90° - \tan^{-1}[\omega_{gc}] - \tan^{-1}\left[\frac{\omega_{gc}}{2}\right]$$

$$= -90° - \tan^{-1}[1.8] - \tan^{-1}\left[\frac{1.8}{2}\right]$$

$$= -193°$$

Phase margin $= 180° + \angle GH|_{\omega=\omega_{gc}}$

$$= 180° - 193$$

$$= -13° \text{ Ans}$$

01 - 31/01/19

Root locus:

*) It is defined as the locus of roots of characteristic equation in the s-plane as the open loop gain is varied from 0 - ∞

*) The locus of roots of characteristic equation in the s-plane as K varies from -∞ - 0 is called inverse root locus.

*) The complete root locus is combination of rootlocus and inverse locus.

Q) Explain step for drawing root locus.

i) The root locus start from the open loop poles where the value of 'K' = '0' and end at ∞ where the value of K = ∞ if the open loop T.F has finite zeros the no. of root loci ending at ∞ = (P-Z) when P is equal to no. of pole and Z is equal to no. zeros.

ii) The number of asymptotes = (P-Z) then the angle of asymptote $\phi A = \frac{180(2K+1)}{P-Z}$ where K = 0, 1, 2, ...

Centroid of asymptotes

$$\sigma A = \frac{\Sigma(\text{Real part of pole}) - \Sigma(\text{Real part of zeros})}{(P-Z)}$$

3) Root locus is always symmetrical to real axis.

4) imaginary intersection of root loci is got from routh array.

5) The location of break away point is determined using equation $\frac{dK}{ds} = 0$

6) If complex poles are present angle of departure is calculated using using relation $\phi d = \pm(180+\phi)$. where ϕ = Net angle contribution at the pole by remaining pole and zeros.

= Σ (angle contribution by zeros) - Σ (angle contributed by pole)

7) if complex zeros are present in transfer function angle of arrival is calculated using relation $\phi A = \pm(180-\phi)$ where ϕ is the net angle contributed at the zeros by the remaining poles and zeros. Σ (angle contributed by zero) - Σ (angle contributed by poles)

8) The value of K on root locus is given by

$$\frac{\text{Product of phasor length of poles}}{\text{Product of phasor length of zeros}}$$

9) That segment on s-plane containing root locus if the number of poles and zero to the right of the segment is odd.

Q) For the open loop transfer function $G(s) = \frac{K}{s(s+2)(s+4)}$

Pole

$\boxed{S=0}$

$s+2=0$

$\Rightarrow \boxed{S=-2}$

$s+4=0$

$\Rightarrow \boxed{S=-4}$

No. of $P = 3$

Zero

No. of $Z = 0$

Centroid (σ)

$= \frac{\Sigma P - \Sigma Z}{P-Z}$

$= \frac{[0+(-2)+(-4)] - [0]}{3-0}$

$= \frac{-6}{3}$

$= -2$

Angle of Asymptote.

$= \frac{(2K+1)180}{P-Z}$

$K=0 \quad \phi_0 = \frac{(2\times0+1)}{3-0}\times180° = \frac{1}{3}\times180° = 60°$

$K=1 \quad \phi_1 = \frac{(2\times1+1)}{3-0}\times180° = \frac{3}{3}\times180° = 180°$

$K=2 \quad \phi_2 = \frac{(2\times2+1)}{3-0}\times180° = \frac{5}{3}\times180° = 300°$

Characteristics eq^n $1+G(s)H(s)=0$

$\Rightarrow 1 + \frac{K}{s(s+2)(s+4)} = 0$

$\Rightarrow \frac{s(s+2)(s+4)+K}{s(s+2)(s+4)} = 0$

$\Rightarrow s(s+2)(s+4)+K=0$

$\Rightarrow (s^2+2s)(s+4)+K=0$

$\Rightarrow s^2(s+4)+2s(s+4)+K=0$

$\Rightarrow s^3+4s^2+2s^2+8s+K=0$

$\Rightarrow s^3+6s^2+8s+K=0$

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Routh array

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	
s^0	K	

A.E

$\frac{48-K}{6}=0$

$\Rightarrow 48-K=0$

$\Rightarrow -K=-48$

$\Rightarrow \boxed{K=48}$

$A.E=0$

$6s^2+K=0$

$\Rightarrow 6s^2+48=0$

$\Rightarrow 6s^2=-48$

$s^2=\frac{-48}{6}$

$s^2=-8$

$s=\sqrt{-8}$

$=\pm j2.82$ (Imaginary crossover point)

breakaway point

$\frac{d(K)}{ds}=0$

$\frac{dK}{ds}=\frac{d}{ds}-(s^3+6s^2+8s)$

$\Rightarrow -(3s^2+12s+8)=0$

$\Rightarrow 3s^2+12s+8=0$

$s=-0.845,-3.154$

Right side odd number of pole → Root locus exist.

$s=-0.845$ (Exist)

$s=-3.154$ (does not exist)

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Q) $G(s) = \frac{K}{s(s+4)(s+5)}$

Pole :

$\boxed{s=0}$

$s+4=0$

$\Rightarrow \boxed{s=-4}$

$s+5=0$

$\Rightarrow \boxed{s=-5}$

Number of pole = 3

Zero

No of z = 0

Centroid (σ)

$\frac{\sum P - \sum Z}{P-Z}$

$= \frac{[0+(-4)+(-5)]}{3-0}$

$= -3$

Angle of Asymptote :

$= \frac{(2K+1)180}{P-Z}$

$K=0 \quad \phi_0 = \frac{(2\times 0+1)}{3-0}\times 180 = \frac{1}{3}\times 180° = 60°$

$K=1 \quad \phi_1 = \frac{(2\times 1+1)}{3-0}\times 180° = \frac{3}{3}\times 180° = 180°$

$K=2 \quad \phi_2 = \frac{(2\times 2+1)}{3-0}\times 180° = \frac{5}{3}\times 180 = 300°$

characteristic eq^n $1+G(s)H(s)=0$

$\Rightarrow 1 + \frac{K}{s(s+4)(s+5)} = 0$

$\Rightarrow \frac{s(s+4)(s+5)+K}{s(s+4)(s+5)} = 0$

$\Rightarrow s(s+4)(s+5)+K=0$

$\Rightarrow (s^2+4s)(s+5)+K=0$

$\Rightarrow s^2(s+5)+4s(s+5)+K=0$

$\Rightarrow s^3+5s^2+4s^2+20s+K=0$

$\Rightarrow \boxed{s^3+9s^2+20s+K=0}$

$K = -s^3-9s^2-20s$

$= -(s^3+9s^2+20s)$

Routh array

s^3	1	20
s^2	9	K
s^1	$\frac{180-K}{9}$	0
s^0	K	

$\frac{180-K}{9} = 0$

$\Rightarrow 180 - K = 0$

$\Rightarrow -K = -180$

$\Rightarrow \boxed{K = 180}$

$\boxed{A.E = 0}$

$9s^2 + k = 0$

$9s^2 + 180 = 0$

$9s^2 = -180$

$s^2 = \frac{-180}{9}$

$s = \sqrt{\frac{-180}{9}}$

$= \pm j4.47$

$= j4.47, -j4.47$

Breakaway point:

$\frac{dK}{ds} = 0$

$\frac{dK}{ds} = \frac{d}{ds}$

$K = -s^3 - 9s^2 - 20s$

$\Rightarrow -(3s^2 + 18s + 20) = 0$

$-(3s^2 + 18s + 20) = 0$

$s = -1.4$

$s = -4.52$

since

-4 to -5 is not the segment of root locus therefore we consider -1.4725 as a breakaway point

Q) For a unit feedback system the open loop transfer function is given by

$$G(s) = \frac{K}{s(s+4)(s^2+2s+2)}$$

pole!

$\boxed{s_1 = 0}$

$\boxed{s_2 = -4}$

$s_3 = -1 + i$

$s_4 = -1 - i$

No. of pole = 4

No. of zero = 0

Centroid(σ) = $\dfrac{\Sigma P - \Sigma Z}{P - Z}$

$$= \frac{0 + (-4) + (-1 + i) + (-1 - i) - 0}{4 - 0}$$

$$= \frac{0 - 4 - 1 - \cancel{i} - 1 + \cancel{i} - 0}{4}$$

$$= \frac{-6}{4} = \frac{-3}{2} = -1.5$$

angle of asmptote:

$$\frac{(2K+1)180}{P - Z}$$

$K=0$ $\quad \dfrac{(2.0+1)180}{4} = \dfrac{1}{4} \times 180 = 45°$

$K=1$ $\quad \dfrac{(2.1+1)180}{4} = \dfrac{3}{\cancel{4}} \times \cancel{180}^{45} = 135°$

$K=2$ $\quad \dfrac{(2.2+1)180}{4} = \dfrac{5}{4} \times 180° = 225°$

$K=3$ $\quad \dfrac{(2.3+1)180}{4} = \dfrac{7}{4} \times 180 = 315°$

characteristic equation:

$$1 + \frac{K}{s(s+4)(s^2+2s+2)} = 0$$

$$\frac{s(s+4)(s^2+2s+2) + K}{s(s+4)(s^2+2s+2)} = 0$$

$\Rightarrow s(s+4)(s^2+2s+2) + K = 0$

$\Rightarrow (s^2 + 4s)(s^2+2s+2) + K = 0$

$\Rightarrow s^2(s^2+2s+2) + 4s(s^2+2s+2) + K = 0$

$\Rightarrow s^4 + 2s^3 + 2s^2 + 4s^3 + 8s^2 + 8s + K = 0$

$\Rightarrow \cancel{s^4 + 6s^3 + 10s^2 + 8s + K = 0}$

$\Rightarrow s^4 + 6s^3 + 10s^2 + 8s + K = 0$

Routh array:

s^4	1	10	K
s^3	6	8	0
s^2	8.6	K	
s^1	$\dfrac{68.6 - 6K}{8.6}$		
s^0			

$$\frac{(6 \times 10) - (1 \times 8)}{6} = \frac{60 - 8}{6} = 8.6$$

$$\frac{68.6 - 6K}{8.6} = 0$$

$\Rightarrow 68.6 - 6K = 0$

$\Rightarrow -6K = -68.6$

$$K = \frac{-68.6}{-6} = 11.43$$

~~Bisdaly/axbay point:~~

$\cancel{\frac{dK}{ds}} = 0$

$\boxed{D.E = 0}$

$8.6s^2 + K = 0$

$8.6s^2 + 11.43 = 0$

$8.6s^2 = -11.43$

$s^2 = \frac{-11.43}{8.6}$

$s = \sqrt{\frac{-11.43}{8.6}}$

$= \pm j1.15$

Breakaway point

$\frac{dK}{ds} = 0$

$K = -(s^4 + 6s^3 + 10s^2 + 8s)$

$\Rightarrow \frac{dK}{ds} = \frac{d}{ds} - (s^4 + 6s^3 + 10s^2 + 8s)$

$\Rightarrow (4s^3 + 18s^2 + 20s + 8) = 0$

$\Rightarrow 4s^3 + 18s^2 + 20s + 8 = 0$

$x_1 = -3.09$

$x_2 = -0.70$

$x_3 = -0.70$

Angle of departure

$\phi_d = 180 - \left[\begin{array}{l} \text{Sum of angle of vector drawn to the pole from} \\ \text{Other pole} \\ + \text{Sum of angle of vector drawn to the pole from the zero} \end{array} \right]$

$\phi_1 = 180^\circ - \tan^{-1}\left(\frac{1}{1}\right)$

$= 180^\circ - 45^\circ$

$= 135^\circ$

$\phi_2 = \tan^{-1}(1/3) = 18.4^\circ$

$\phi_3 = 90^\circ$

Q) $G(s) = \frac{K}{s(s+2)(s^2+6s+25)}$

Poles

$\boxed{S_1 = 0}$

$S+2=0$

$\boxed{S_2 = -2}$

$s^2+6s+25$

$\boxed{S_3 = -0.35}$

$\boxed{S_4 = -5.64}$

No. of $\boxed{\text{pole}(P) = 4}$

Zero

No. of zero $Z = 0$

Centroid: (σ)

$$\frac{\Sigma P - \Sigma Z}{P - Z}$$

$$= \frac{(0-2-0.35-5.64)-(0)}{4-0}$$

$$= \frac{-8}{4} = -2$$

Angle of asymptote:

$$\phi_0 = \frac{(2K+1)180}{P-Z}$$

$K=0$ $\quad \phi_1 = \frac{(2\times0+1)180^\circ}{4-0} = 45^\circ$

$K=1$ $\quad \phi_2 = \frac{(2\times1+1)180^\circ}{4-0} = 135^\circ$

$K=2$ $\quad \phi_3 = \frac{(2\times2+1)}{4-0}\times180^\circ = 225^\circ$

$K=3$ $\quad \phi_4 = \frac{(2\times3+1)}{4-0}\times180^\circ = 315^\circ$

characteristic eq^n:

$$1 + G(s)H(s) = 0$$

$$= 1 + \frac{K}{s(s+2)(s^2+6s+25)} = 0$$

$$\Rightarrow \frac{(s^2+2s)(s^2+6s+25)+K}{s(s+2)(s^2+6s+25)} = 0$$

$$\Rightarrow (s^2+2s)(s^2+6s+25)+K=0$$

$$\Rightarrow s^4+8s^3+37s^2+50s+K=0$$

Scanned with CamScanner

Routh array

$$\begin{array}{llll} s^4 & 1 & 37 & K \\ s^3 & 8 & 50 & 0 \\ s^2 & \boxed{30.75 \quad K} & & A.E \\ s^1 & \frac{1537.5 - 8K}{30.75} & & \\ s^0 & & & \end{array}$$

$$\frac{1537.5 - 8K}{30.75} = 0$$

$\Rightarrow 1537.5 = 8K$

$\Rightarrow K = \frac{1537.5}{8} = 192.18$

$A.E = 0$

$30.75 s^2 + K = 0$

$\Rightarrow 30.75 s^2 + 192.18 = 0$

$\Rightarrow 30.75 s^2 = -192.18$

$\Rightarrow s^2 = \frac{-192.18}{30.75}$

$\Rightarrow s = \sqrt{-6.24} = \pm j2.5$

Breakaway point

$$\frac{dK}{ds} = 0$$

$$\frac{dK}{ds} = -\frac{d}{ds}(s^4 + 8s^3 + 37s^2 + 50s) = 0$$

$\Rightarrow -(4s^3 + 24s^2 + 74s + 50) = 0$

$\Rightarrow 4s^3 + 24s^2 + 74s + 50 = 0$

$S_1 = -0.9$

$\left.\begin{array}{l} S_2 = -2.5 \\ S_3 = -2.5 \end{array}\right\}$ doesn't exist

$\phi_1 = 180° - \tan^{-1}\left(\frac{4}{3}\right)$
$= 127°$

$\phi_2 = 180° - \tan^{-1}\left(\frac{4}{1}\right)$
$= 104°$

$\phi_3 = 90°$

Explain effect of addition of pole and zero on root locus!

Q) The effect of addition of poles are as follows:

→ There is change in shape of the root locus and it shifts towards the imaginary axis.

→ The intercept on the $j\omega$ axis occurs for a lower value of K because of asymptote angle being lower down

→ System becomes oscillatory
→ Transient response deteriorates
→ Gain margin and relative stability decrease

→ There is reduction in the range of K

→ A sluggish response can be changed to a quick response for artful introduction of a pole

→ Settling time increases.
→ damping ratio (ξ) decrease

Effect of Addition of Zero:

The effect of addition of zero are as follows:

→ There is change in shape of the root locus and it shifts towards the left of the s-plane

→ stability of the system is enhanced.
→ Range of K increases
→ settling time speed up.
→ damping ratio (ξ) increases
→ Transient response improves.

Q) $G(s) = \dfrac{K}{s(s^2+6s+10)}$

Pole

$\boxed{s_1 = 0}$

Pole $\boxed{P = 3}$

$s_1 = -3+i$

$s_2 = -3-i$

Zero

$\boxed{z = 0}$

Centroid (σ)

$$= \frac{\Sigma P - \Sigma Z}{P - Z}$$

$$= \frac{0+(-3)+(-3)-0}{3-0}$$

$$= \frac{-3-3}{3} = \frac{-6}{3} = -2$$

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$K=0 \quad \phi_0 = \frac{(2 \cdot 0+1)}{3} \times 180^\circ = 60^\circ$

$K=1 \quad \phi_1 = \frac{(2 \cdot 1+1)}{3} \times 180^\circ = 180^\circ$

$K=2 \quad \phi_2 = \frac{(2 \cdot 2+1)}{3} \times 180^\circ = 300^\circ$

characteristic equation!

$1 + G(s)H(s) = 0$

$\Rightarrow 1 + \frac{K}{s(s^2+6s+10)} = 0$

$\Rightarrow \frac{s(s^2+6s+10)+K}{s(s^2+6s+10)} = 0$

$\Rightarrow s^3+6s^2+10s+K = 0$

s^3	1	10
s^2	6	K
s^1	$\frac{60-K}{6}$	0
s^0	K	

A.E

$\frac{(6 \times 10) - K}{6}$

$\frac{60-K}{6} = 0$

$\Rightarrow \boxed{K = 60}$

A.E $= 0$

$6s^2 + K = 0$

$6s^2 + 60 = 0$

$s^2 = \frac{-60}{6} = -10$

$s = \pm j3.16$

break away point

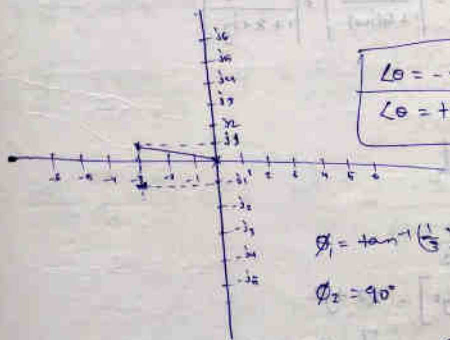
$\frac{dK}{ds} = \frac{d}{ds}(-s^3 - 6s^2 - 10s)$

$= -(s^3 + 6s^2 + 10s)$

$\Rightarrow K = 3s^2 + 12s + 10$

$x_1 = -1.18$

$x_2 = -2.81$



$\angle\theta = -ve \Rightarrow$

$\angle\theta = +ve \Rightarrow$

$\phi_1 = \tan^{-1}\left(\frac{1}{3}\right) = 161^\circ$

$\phi_2 = 90^\circ$

$\phi_A = [180 - (90 + 161)]$

$= -71^\circ$

Close loop frequency responce:

*) The various method use for finding frequency responce of closed loop system are

→ M circle
→ N circle
→ Nichol's chart.

Constant M circle or Constant magnitude circle:

*) M. circle are also called Constant gain circle for unit feedback system.

*) Consider any Point

$$G(j\omega) = x + jy$$

$$M = \left|\frac{C(j\omega)}{R(j\omega)}\right| = \left|\frac{G(j\omega)}{1+G(j\omega)}\right| = \left|\frac{x+jy}{1+x+jy}\right|$$

$$M = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

Squaring both side

$$M^2 = \frac{x^2+y^2}{(1+x)^2+y^2}$$

$$\Rightarrow M^2\left[(1+x)^2+y^2\right] = x^2+y^2$$

$$\Rightarrow M^2\left[1+x^2+2x+y^2\right] = x^2+y^2$$

$$\Rightarrow M^2 + M^2x^2 + 2xM^2 + M^2y^2 - x^2 - y^2 = 0$$

$$\Rightarrow M^2x^2 - x^2 + M^2y^2 - y^2 + 2M^2x + M^2 = 0$$

$$\Rightarrow x^2(M^2-1) + y^2(M^2-1) + 2M^2x + M^2 = 0$$

$$\Rightarrow x^2(M^2-1) + y^2(M^2-1) + 2M^2x = -M^2$$

$$\Rightarrow x^2(1-M^2) + y^2(1-M^2) - 2M^2x = M^2$$

devide both side by $(1-M^2)$

$$\frac{x^2(1-M^2)}{(1-M^2)} + \frac{y^2(1-M^2)}{(1-M^2)} - \frac{2M^2x}{(1-M^2)} = \frac{M^2}{1-M^2}$$

$$\Rightarrow x^2 + y^2 - \frac{2M^2x}{(1-M^2)} = \frac{M^2}{(1-M^2)}$$

$\left(\frac{M^2}{1-M^2}\right)^2$ add a both side

$$x^2 + \left(\frac{M^2}{1-M^2}\right)^2 - \frac{2x\cdot M^2}{(1-M^2)} + y^2 = \frac{M^2}{1-M^2} + \left(\frac{M^2}{1-M^2}\right)^2$$

$$\left(x - \frac{M^2}{1-M^2}\right)^2 + (y-0)^2 = \frac{M^2}{1-M^2} + \frac{M^4}{(1-M^2)^2}$$

$$= \frac{M^2(1-M^2) + M^4}{(1-M^2)^2}$$

$$= \frac{M^2 - M^4 + M^4}{(1-M^2)^2}$$

$$\left(x-\frac{m^2}{1-m^2}\right)^2+(y-0)^2=\frac{m^2}{(1-m^2)^2}=\cancel{\left[\frac{M}{(1-m^2)}\right]^2}$$

equation of circle

$$(x-a)^2+(y-b)^2=R^2$$

where a, b = Center
R = Radius.

$$\boxed{\left(x-\frac{m^2}{1-m^2}\right)^2+(y-0)^2=\cancel{\frac{m^2}{(1-m^2)^2}}\left[\frac{M}{(1-m^2)}\right]^2} \quad -(i)$$

Center $\left(\frac{M^2}{1-m^2}, 0\right)$

Radius $\left(\frac{M}{1-m^2}\right)$

M	Centre $\left(\frac{M^2}{1-m^2}, 0\right)$	Radius $\left(\frac{M}{1-m^2}\right)$
0.3	(0.098, 0)	0.329
0.5	(0.33, 0)	0.666
0.67	(0.814, 0)	1.215
1.2	(−3.27, 0)	−2.72
1.3	(−2.45, 0)	−1.88
1.5	(−1.8, 0)	1.2
2.0	(−1.33, 0)	−0.66
3.0	(−1.125, 0)	−0.375