LECTURE NOTES
ON
FLUID MECHANICS
4TH SEMESTER,

(TH-3)

(MECHANICAL ENGINEERING)

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** FLUID ** MECHANICS **

PROPERTIES OF FLUIDS :-

(1) Density or Mass Density :-

→ Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (rho). The unit of mass density in S.I unit is kg per cubic metre i.e. kg/m^3.

→ The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

→ Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \frac{m}{V}$$

→ The value of density of water is $1\,gm/cm^3$ or $1000\,kg/m^3$.

(2) Specific Weight or Weight Density :-

→ Specific weight or weight Density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically, $w = \frac{\text{weight of fluid}}{\text{Volume of fluid}}$

Here, weight of fluid = mass of fluid × Acceleration due to gravity

$= m \times g$

$\Rightarrow$ Weight Density $(w) = \frac{mg}{V} = \frac{\text{mass of fluid} \times g}{\text{Volume of fluid}}$

$= \rho \times g$

$$\Rightarrow \boxed{w = \rho g}$$

→ The unit of weight density in S.I unit is $Newton/m^3$.

→ The value of specific weight or weight density (w) for water is 9.81 X 1000 Newton /m³ in S.I units.

[3] Specific Volume :—

→ Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. Mathematically, it is expressed as

$$\text{Specific Volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}}$$

$$= \frac{1}{\frac{\text{Mass of fluid}}{\text{volume of fluid}}} = \frac{1}{\rho}$$

→ Thus specific volume is the reciprocal of mass density.
→ It is expressed as m^3/kg.
→ It is commonly applied to gases.

[4] Specific Gravity :—

→ Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S.

Mathematically,
$$S(\text{for liquids}) = \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

$$S(\text{for gases}) = \frac{\text{weight density of gas}}{\text{weight density of air}}$$

Thus weight density of a liquid = S × weight density of water
= S × 1000 × 9.81 N/m^3

The density of a liquid = S × Density of water
= S × 1000 kg/m^3

Density of standard liquid $= \frac{\rho_l g}{\rho_w g} = \frac{\rho_l}{\rho_w}$

$$\Rightarrow S = \frac{\rho_l}{\rho_w}$$

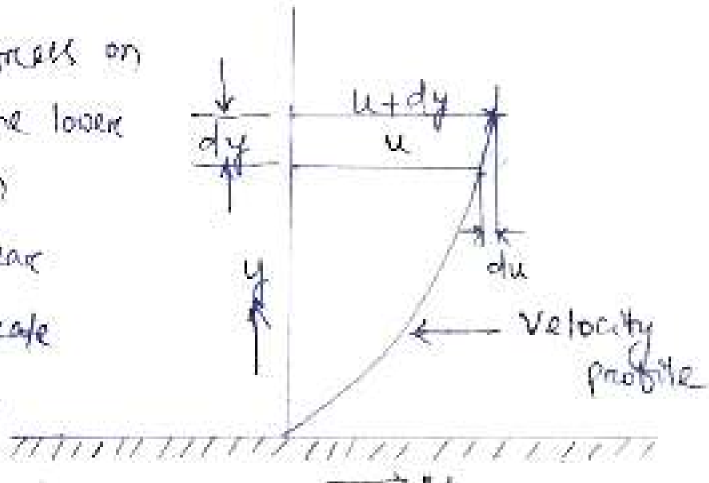
$$\Rightarrow \rho_l = S \times \rho_w$$

If the specific gravity of fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water. For example, the specific gravity of mercury is 13.6, hence density of mercury $= 13.6 \times 1000$

$\Rightarrow S_{Hg}$ $= 13600\ kg/m^3$.

:- VISCOSITY :-

- Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.
- When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and u+du as shown in figure, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.
- The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by symbol τ (Tau).



(Velocity variation Near a solid boundary)

Mathematically, $\tau \propto \frac{du}{dy} \Rightarrow \tau = \mu \frac{du}{dy}$ ——(1)

where μ is the constant of proportionality and is known as the coefficient of dynamic viscosity or only viscosity.

$\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From eqn (1) we have, $\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain

Here $\tau = \frac{Force}{Area} = N/m^2$

$$\mu = \frac{Force}{Area} \div \frac{du}{dy}$$

$$= N/m^2 \times \frac{dy}{du} = \frac{N}{m^2} \times \frac{m}{m/s} = \frac{N.s}{m^2}$$

SI unit of viscosity $= NS/m^2 = Pa\,s$

$$= \frac{Newton.sec}{m^2}$$

Acceleration $= \frac{Speed}{sec}$

or speed $= m/sec$ & $g = \frac{m/sec}{sec}$

$\Rightarrow g = m/sec^2$

then, $\mu = \frac{N.s}{m^2}$ (Newton = mass x acceleration)

$$\Rightarrow \mu = \frac{kg\frac{m}{s^2} \times s}{m^2}$$

$$\Rightarrow \boxed{\mu = kg/m.s}$$

The unit of viscosity in CGS is also called poise which is equal to $\frac{dyne-sec}{cm^2}$.

The numerical conversion of the unit of viscosity from MKS unit to CGS unit is given below.

$$\frac{One\ kgf-sec}{m^2} = \frac{9.81\ N-sec}{m^2} \quad (\because 1\ kgf = 9.81\ Newton)$$

But 1 Newton = one kg (mass) × one $\frac{m}{sec^2}$ (acceleration)

$$= \frac{1000\ gm \times 100\ cm}{sec^2} = 1000 \times 100\ \frac{gm\text{-}cm}{sec^2}$$

$$= 1000 \times 100\ dyne$$

$$\text{one } \frac{kgf\text{-}sec}{m^2} = 9.81 \times 100000\ \frac{dyne\text{-}sec}{m^2}$$

$$= 9.81 \times 100000\ \frac{dyne\text{-}sec}{100 \times 100 \times cm^2}$$

$$= 98.1\ \frac{dyne\text{-}sec}{cm^2} = 98.1 \text{ poise}$$

Thus for solving numerical problems, if viscosity is given in poise, it must be divided by 98.1 to get its equivalent value in MKS.

$$\text{But one } \frac{kgf\text{-}sec}{m^2} = \frac{9.81\ NS}{m^2} = 98.1 \text{ poise}$$

$$\text{one } \frac{NS}{m^2} = \frac{98.1}{9.81} \text{ poise} = 10 \text{ poise}$$

$$\rightarrow 1 \text{ poise} = \frac{1}{10} \frac{NS}{m^2}$$

KINEMATIC VISCOSITY :-

It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by Greek symbol (ν) called 'nu'. Thus mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)}$$

$$= \frac{(\text{Length})^2}{\text{Time}}$$

$\left\{ \because \text{Force} = \text{Mass} \times \text{Acc} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \right\}$

In MKS and SI, the unit of kinematic viscosity is metre²/sec or m²/sec while in CGS units it is written as cm²/s.

In CGS units, Kinematic Viscosity is also known as stroke.

Thus, One Stroke $= cm^2/s = \left(\frac{1}{100}\right)^2 m^2/s = 10^{-4} m^2/s$

Centistoke means $= \frac{1}{100}$ stoke

SURFACE TENSION AND CAPILLARITY ⇒

Surface Tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as Kgf/m while in SI units as N/m.

SURFACE TENSION ON LIQUID DROPLET ⇒

Consider a small spherical droplet of a liquid of radius 'r'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

d = Dia of droplet

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

(i) Tensile force due to Surface tension acting around the circumference of the cut portion as shown in figure. and this is equal to $= \sigma \times$ circumference

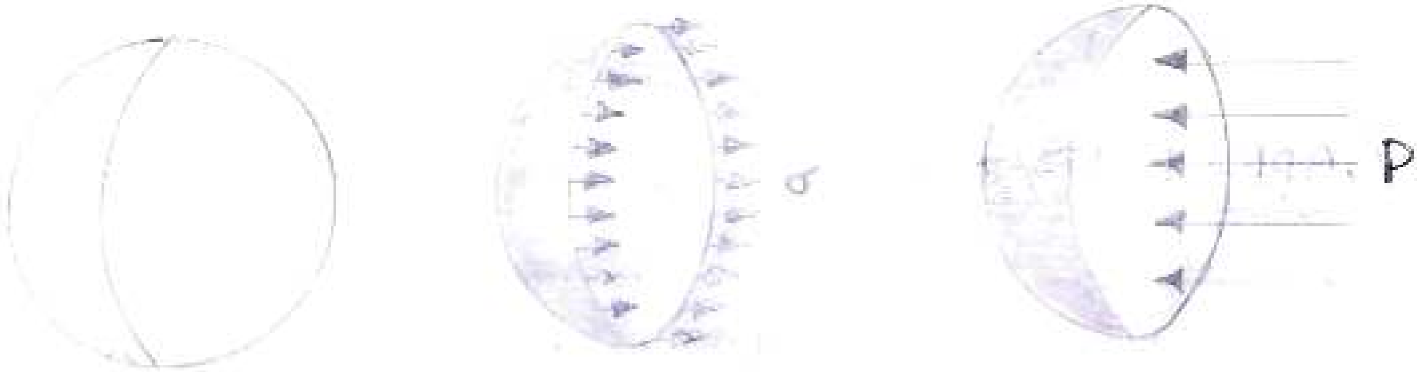
$= \sigma \times \pi d$

(ii) Pressure force on the area $\frac{\pi}{4}d^2 = p \times \frac{\pi}{4}d^2$ as shown in figure. These two forces will be equal and opposite under equilibrium conditions, i.e.

$$p \times \frac{\pi}{4}d^2 = \sigma \times \pi d$$

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d}$$

The above equation shows that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.



(a) Droplet (b) Surface Tension (c) Pressure Forces

(FORCES ON DROPLET)

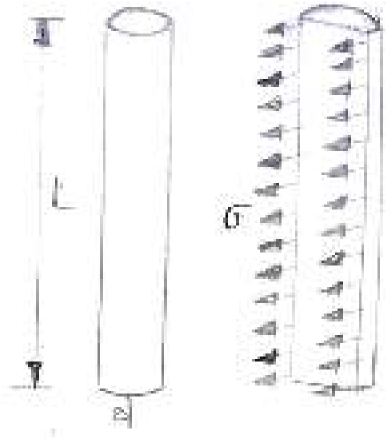
SURFACE TENSION ON A LIQUID JET :-

Consider a liquid jet of diameter 'd' and length 'L' as shown in figure.

Let p = Pressure intensity inside the liquid jet above the outside pressure.

σ = Surface tension of the liquid

Consider the equilibrium of the semi jet, we have force due to pressure = p × area of semi jet = p × L × d



(a) (b)

(FORCES ON LIQUID JET)

Force due to surface tension = σ × 2L

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$\Rightarrow p = \frac{\sigma \times 2L}{L \times d}$$

SURFACE TENSION ON A HOLLOW BUBBLE ⇒

A Hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$\Rightarrow p = \frac{2\sigma \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$

⇒ CAPILLARITY :-

→ Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

→ The rise of liquid surface is known as Capillary rise while the fall of the liquid surface is known as Capillary depression.

→ It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

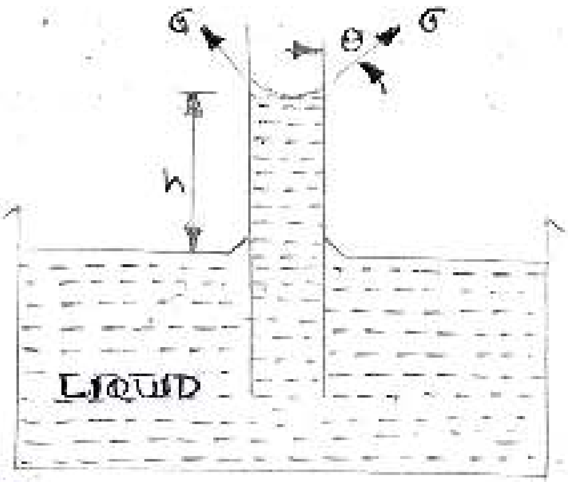
Expression for Capillary Rise :-

Consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube.

But the force at the surface of the liquid in the tube is due to surface tension.

Let σ = surface tension of liquid
θ = Angle of contact between liquid and glass tube



The weight of liquid of height h in the tube = (Area of tube × h) × ρ × g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \quad —(1)$$

(CAPILLARY RISE)

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$
$$= \sigma \times \pi d \times \cos\theta \quad —(2)$$

For equilibrium, equating (1) & (2), we get

$$\frac{\pi}{4} d^2 h \rho g = \sigma \times \pi d \times \cos\theta$$

$$\Rightarrow h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d} \quad —(3)$$

or, The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d} \quad —(4)$$

Expression for Capillary Fall :-

If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in the figure.

Let h = Height of depression in tube

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' x Area

$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2$

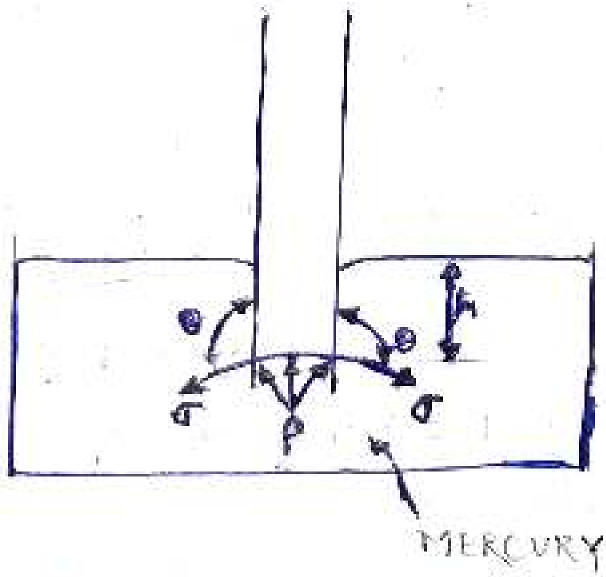
$(\because p = \rho g h)$

Equating the two, we get

$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$

$\Rightarrow h = \frac{4\sigma \cos\theta}{\rho g d}$

∴ Value of θ for mercury and glass tube is 128°.



(CAPILLARY FALL)

FLUID PRESSURE AT A POINT :-

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Consider a small Area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by p. Hence mathematically the pressure at a point in a fluid at rest is

$$p = \frac{dF}{dA}$$

If the Force (F) is uniformly distributed over the Area (A), then pressure at any point is given by

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$$

∴ Force or pressure force, $F = p \times A$.

The units of Pressure are (i) kgf/m^2 and kgf/cm^2 in MKS unit.

(ii) $Newton/m^2$ or N/m^2 and N/mm^2 in SI unit.

N/m^2 is known as Pascal and is represented by Pa.

Other Commonly used units of pressure are :-

kPa = kilo pascal = $1000\ N/m^2$

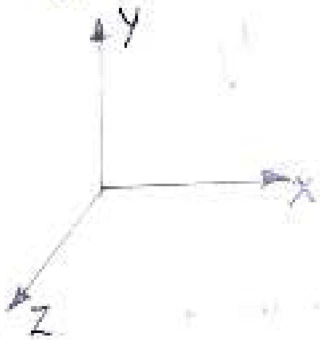
$bar = 100\ kPa = 10^5\ N/m^2$

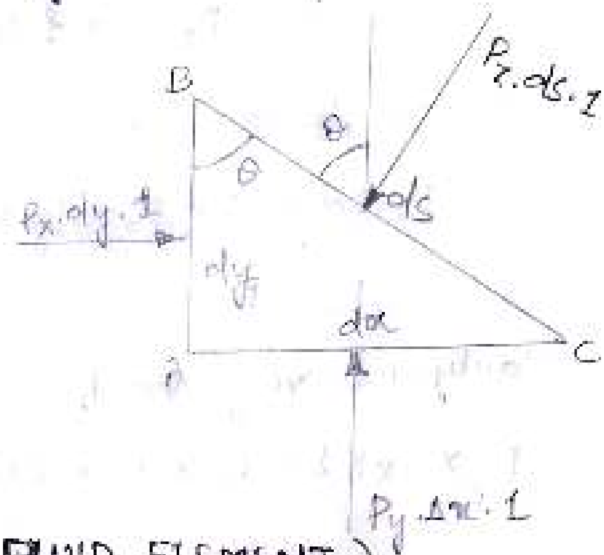
PASCAL'S LAW →

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

This is proved as:

The fluid element is of very small dimensions. i.e. dx, dy and ds.





(FORCES ON A FLUID ELEMENT)

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in figure. Let the width of the element perpendicular to the plane of paper is unity and p_x, p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively.

Let $\angle ABC = \theta$, then the forces acting on the element are:

(1) Pressure forces normal to the surfaces and

(2) Weight of element in the vertical direction.

The forces on the faces are :-

Force on the face AB = P_x × Area of face AB

$= P_x \times dy \times 1$

Similarly force on the face AC $= P_y \times dx \times 1$

force on the face BC $= P_z \times ds \times 1$

weight of element = (Mass of element) $\times g$

$= (\text{Volume} \times \rho) \times g$

$= \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$

where ρ = density of fluid

Resolving the forces in x-direction, we have

$$P_x \times dy \times 1 - P(ds \times 1)\sin(90^\circ - \theta) = 0$$

or $$P_x \times dy \times 1 - P_z ds \times 1 \cos\theta = 0$$

But from figure, $ds \cos\theta = AB = dy$

$$P_x \times dy \times 1 - P_z \times dy \times 1 = 0$$

$$P_x = P_z$$

or Similarly, resolving the forces in y-direction, we get

$$P_y \times dx \times 1 - P_z \times ds \times 1 \cos(90^\circ - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

$$\Rightarrow P_y\, dx - P_z\, ds \sin\theta - \frac{dx\, dy}{2} \times \rho \times g = 0$$

OR, Let the width of the elements is 1. Hence

the area of force on face AB $= dy \times 1$ ($F_{AB} = P_y \times dy \times 1$)

area of force on face AC $= dx \times 1$ ($F_{AC} = P_x \times dx \times 1$)

area of force on face BC $= ds \times 1$ ($F_{BC} = P_z \times ds \times 1$)

Weight of element = (mass of element) $\times g$

$= \rho \times \text{Volume} \times g$

$= \rho \times (\frac{1}{2} AC \times AB \times 1) \times g$

For equilibrium

Considering the body at equilibrium

Resolving the left and right forces.

$F_{AB} = F_{BC} \cos\theta$

$\Rightarrow P_y \cdot dy \cdot 1 = P_z \cdot ds \cdot 1 \cos\theta$

$\Rightarrow P_y \cdot dy = P_z \cdot ds \cos\theta$

$\Rightarrow \cos\theta = \frac{AB}{AC} \times \frac{dy}{ds}$

$\Rightarrow ds \cos\theta = dy$

Applying this in the equation

$P_y \cdot dy = P_z \cdot dy$

$\Rightarrow P_y = P_z$ ——— ①

Resolving the up forces and down forces

$F_{AC} = F_{BC} \cdot \sin\theta + W$

$\Rightarrow P_x \cdot dx \cdot 1 = P_z \cdot ds \cdot 1 \cdot \sin\theta + \rho \times \left(\frac{1}{2} \cdot dx \cdot dy \cdot 1\right) \times g$

$\Rightarrow P_x \cdot dx = P_z \, ds \sin\theta + \rho g \frac{dx\,dy}{2}$

as dx.dy will be very small, Hence it can be neglected.

Applying in the equation

$P_x \cdot dx = P_z \cdot dx$

$\Rightarrow P_x = P_z$

$\therefore P_x = P_y = P_z$ ——— ②

PRESSURE HEAD & HYDROSTATIC LAW :-

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertical downward direction must be equal to the weight density of the fluid at the point.

Let ΔA = Cross sectional Area

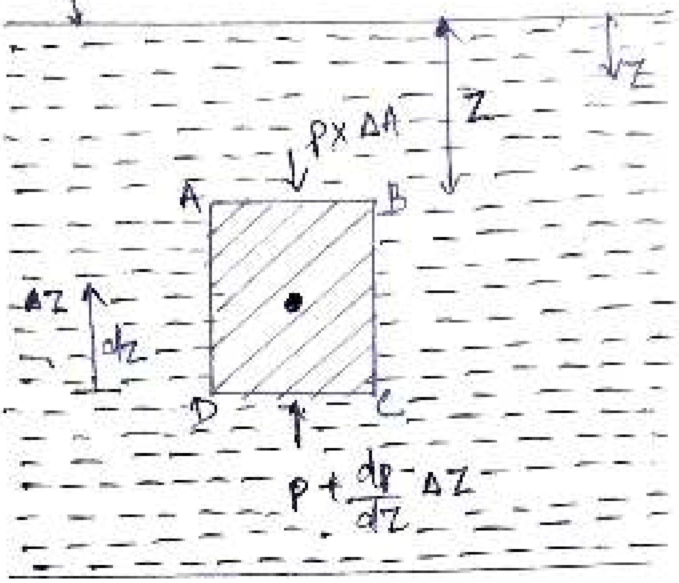
ΔZ = Height of fluid element

P = pressure on face AB

Z = Distance of fluid element from free surface

W = weight density of fluid

Free surface of fluid



(Forces on a fluid element)

For equilibrium

$$w + (p \times \Delta A) = \left(p + \frac{dp}{dz}\Delta Z\right)\Delta A$$

$$\Rightarrow \left[\rho(\Delta A + \Delta Z)g\right] + p * \Delta A = \left(p + \frac{dp}{dz}\right)\Delta Z \cdot \Delta A$$

$$\Rightarrow \rho\,\Delta A \cdot \Delta Z g + p \cdot \Delta A = p\,\Delta A + \frac{dp}{dz} \cdot \Delta Z \cdot \Delta A$$

$$\Rightarrow \rho \cdot \Delta A \cdot \Delta Z g = \frac{dp}{dz} \cdot \Delta Z \cdot \Delta A$$

$$\Rightarrow \rho g = \frac{dp}{dz}$$

This equation is known as Hydrostatic law.

$$\frac{dp}{dz} = \rho g$$

$$\Rightarrow \int dp = \int \rho g\, dz \qquad \left(\because Z = \frac{p}{\rho g}\right)$$

$$\Rightarrow p = \rho g Z$$

Types of Pressures :-

→ The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system pressure is measured above the atmospheric pressure and it is called gauge pressure.

→ There are different types of pressure in this system.

1. Absolute pressure
2. Gauge pressure
3. Vacuum pressure

① Absolute Pressure ⇒

It is defined as the pressure which is measured with ~~the help of a pre~~ reference to absolute vacuum pressure.

② Gauge Pressure ⇒

It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

③ Vacuum Pressure ⇒

It is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure gauge pressure and vacuum pressure are shown in figure below.

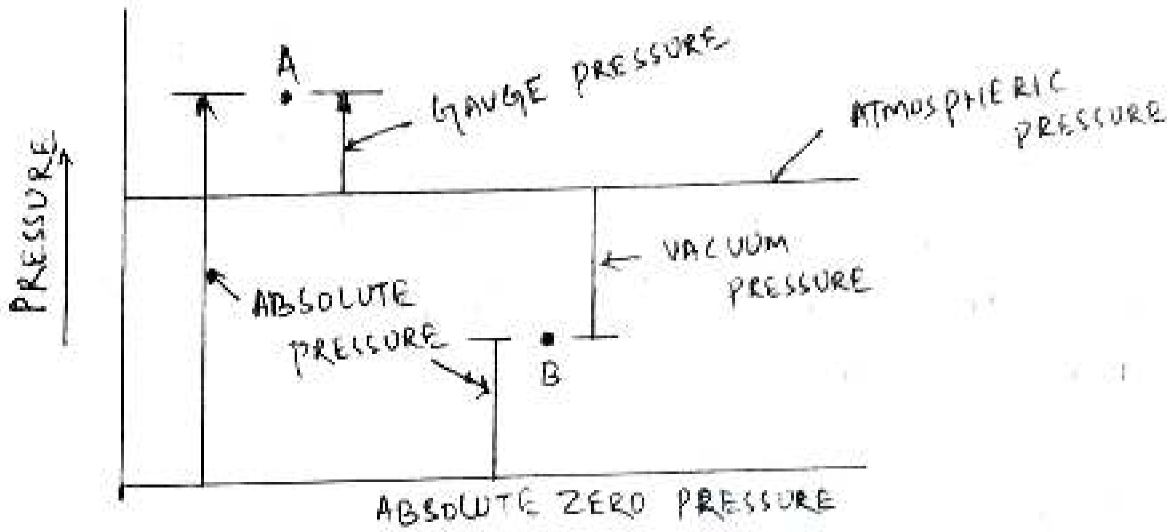
Mathematically,

(i) Absolute pressure = Gauge pressure + Atmospheric pressure

or $P_{ab} = P_{atm} + P_{gauge}$

(ii) Vacuum pressure = Atmospheric pressure − Absolute pressure

$\Rightarrow P_{vac} = P_{atm} - P_{abs}$



[Relationship between pressures]

MEASUREMENT OF PRESSURE ⇒

The pressure of a fluid is measured by the following devices:

1. Manometers
2. Mechanical Gauges

(1) MANOMETERS ⇒

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as:-

(a) Simple manometers (b) Differential manometers

(2) MECHANICAL GAUGES ⇒

Mechanical Gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are:-

(a) Diaphragm pressure gauge
(b) Bourdon tube pressure gauge
(c) Dead-weight pressure gauge
(d) Bellows pressure gauge

PROBLEMS :-

Q.1 Calculate the density, specific weight and weight of 1 lit of petrol of specific gravity 0.7 ?

Ans) $\rho = ?$

$w = ?$

$W = ?$

$V = 1 \text{ lit} = 10^{-3} m^3 / 10^3 cm^3$

$S = 0.7$

$$S = \frac{w_{liq}}{w_{std\ liq}} = \frac{\rho_{liq}}{\rho_{std\ liq}}$$

$$\Rightarrow S = \frac{\rho_{liq}}{1000 kg/m^3}$$

$$\Rightarrow 0.7 = \frac{\rho_{liq}}{1000 kg/m^3}$$

$\Rightarrow \rho_{liq} = 0.7 \times 1000 = 700$

$w = \rho \times g = 700 \times 9.81 = 6867 \ N/m^3$

$W = mg$

$= \rho \times V \times g$

$= 700 \times V \times 9.8 = 700 \times 10^{-3} \times 9.81$

$W = w \times Vol$

$= 6867 \times 10^{-3} = 6.867 \ N$ (Ans)

Q.2 Two horizontal plate are place 1.25 cm apart from each other & the space between them is filled with oil of viscosity 14 poise. Calculate the shear stress in oil if the upper plate is moving with a velocity of 2.5 m/s.

Ans: $dy = 1.25 \ cm = 1.25 \times 10^{-2} \ m$

$\mu = 14 \text{ poise} = 14/10 = 1.4 \ N/m^2$

$V_2 = 2.5 \ m/s$

$y_1 = 0$

$$\tau = \mu \frac{du}{dy} = 1.4 \times \frac{2.5}{1.25 \times 10^{-2}}$$

$= 280 \ N/m^2$ (Ans)

Q.3 Find the kinematic viscosity & specific gravity of an oil having density of 981 kg/m3. The shear stress at a point in oil is 0.2452 N/m2 & velocity gradient is given by 0.2/sec.

(Ans) $\rho = 981\ kg/m^3$ $\quad$ $\nu = ?$

$\tau = 0.2452\ N/m^2$ $\quad$ $S = ?$

$\frac{du}{dy} = 0.2/sec$

$\nu = \frac{\mu}{\rho}$

$\Rightarrow \mu = \tau / \frac{du}{dy} = \frac{0.2452}{0.2} = 1.226\ Ns/m^2$

$\nu = \mu/\rho$

$= \frac{1.226}{981} = 0.001249\ m^2/s$

$= 12.49$ stroke

$S = \frac{\rho_{oil}}{\rho_{water}} = \frac{981}{1000} = 0.981$ (Ans)

Q.4 The velocity distribution for flow over a flat plate is given by $U = 3/4\,y - y^2$ in which U is the velocity in m/s & y is the distance in metre above the plate. Determine the shear stress at y = 1.5 m & the dynamic viscosity at 8.6 poise.

(Ans) $U = \frac{3}{4}y - y^2$ $\quad$ $\mu = 8.6$ poise $= \frac{8.6}{10} = 0.86\ Ns/m^2$

$y = 1.5\ m$

$\frac{du}{dy} = \frac{3}{4} - 2y$

$\frac{du}{dy}$ at $y = 1.5\ m$

then, $\frac{du}{dy} = \frac{3}{4} - 2(1.5) = \frac{3}{4} - 3 = \frac{3-12}{4} = \frac{-9}{4} = -2.25$

$\mu = 8.6$ poise

$\tau = \mu \times \left(\frac{du}{dy}\right) = \frac{8.6}{10} \times -2.25 = 0.86 \times (-2.25)$

$= -1.935\ N/m^2$ (Ans)

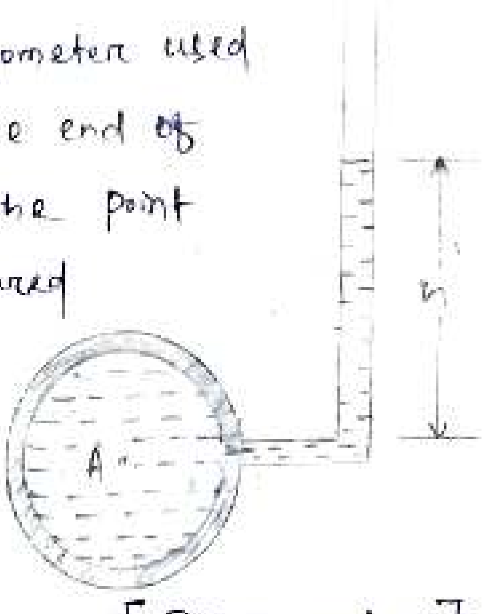
SIMPLE MANOMETERS :-

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :-

(1) piezometer,
(2) U-tube manometer and
(3) Single column manometer.

(1) PIEZOMETER ⇒

It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in figure. The rise of liquid gives the pressure head at that point.

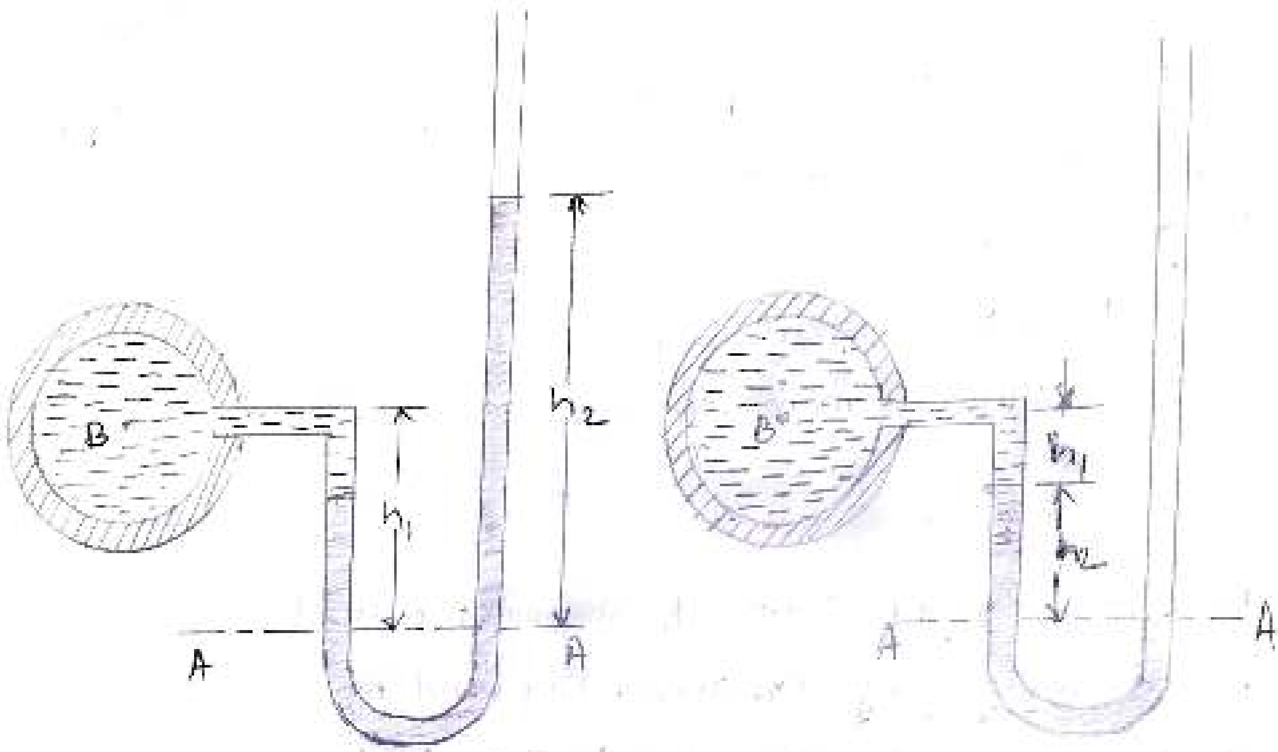


[Piezometer]

If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A = $\rho \times g \times h \; \frac{N}{m^2}$

(2) U-TUBE MANOMETER ⇒

It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured, and other end remains open to the atmosphere as shown in the figure. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



(a) For Gauge pressure (b) For Vacuum pressure

(A) FOR GAUGE PRESSURE ⇒

Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

Let h_1 = height of light liquid above the datum line

h_2 = height of heavy liquid above the datum line

S_1 = Specific gravity of light liquid

ρ_1 = Density of light liquid $= 1000 \times S_1$

S_2 = Specific gravity of heavy liquid

ρ_2 = Density of heavy liquid $= 1000 \times S_2$

As the pressure is the same for the horizontal surface, Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures,

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$\Rightarrow p = \rho_2 g h_2 - \rho_1 g h_1 \quad \text{—— (1)}$$

(B) FOR VACUUM PRESSURE ⇒

For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in the above figure.

Then pressure above A-A in the left column = $\rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A = 0

∴ Hence $\rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$$\Rightarrow p = -(\rho_2 g h_2 + \rho_1 g h_1) \quad \text{—— (2)}$$

[3] SINGLE COLUMN MANOMETER ⇒

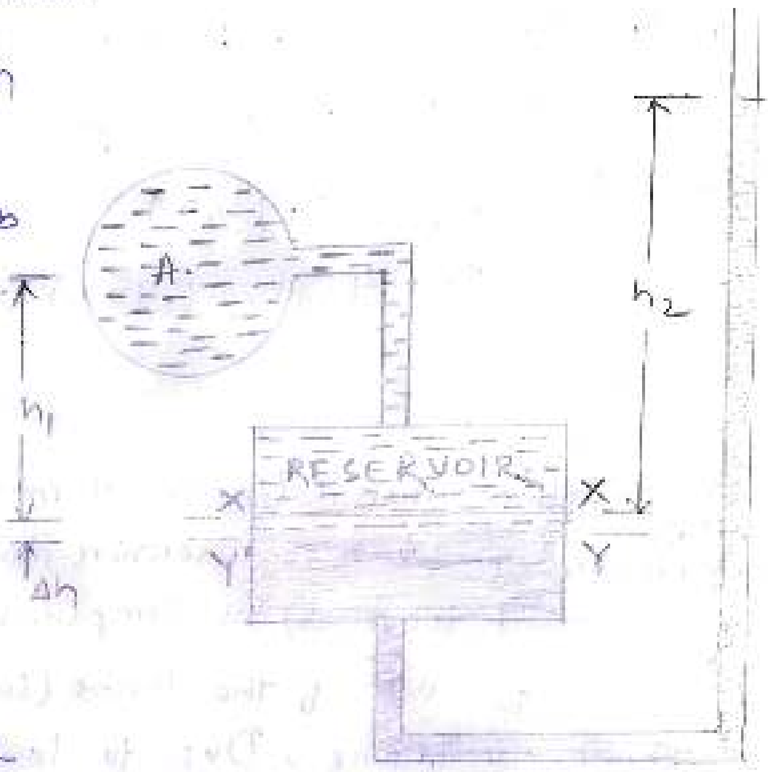
Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in figure. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

(1) Vertical single column manometer

(2) Inclined single column manometer

(1) VERTICAL SINGLE COLUMN MANOMETER :-

The figure shows the vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. when the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.



Let Δh = fall of heavy liquid in reservoir.

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

P_A = Pressure at A, which is to be measured

A = Cross-sectional Area of the reservoir

a = Cross-sectional area of the right limb

S_1 = Sp. gravity of liquid in pipe

S_2 = Sp. gravity of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir

Fall of heavy liquid in reservior will cause a rise of heavy liquid level in the right limb,

$$A \times \Delta h = a \times h_2$$

$$\Rightarrow \Delta h = \frac{a \times h_2}{A} \quad \text{—— (1)}$$

Now consider the datum line Y-Y as shown in figure, then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y-Y = $\rho_1 \times g \times (\Delta h + h_1) + P_A$

Equating the pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

$$\Rightarrow p_A = \rho_2 g (\Delta h + h_2) - \rho_1 g (\Delta h + h_1)$$

$$= \Delta h (\rho_2 g - \rho_1 g) + h_2 \rho_2 g - h_1 \rho_1 g$$

But from equation (i); $\Delta h = \frac{a \times h_2}{A}$

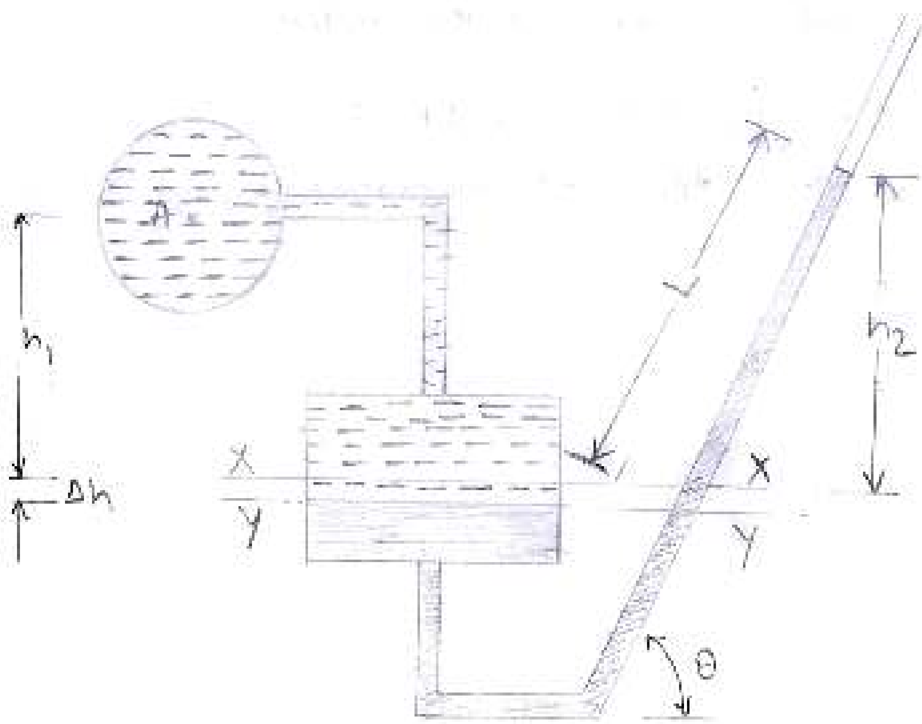
$$\Rightarrow p_A = \frac{a \times h_2}{A} [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected.

Then $p_A = h_2 \rho_2 g - h_1 \rho_1 g$

from this equation it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be ~~neglected~~ calculated.

[2] INCLINED SINGLE COLUMN MANOMETER ⇒



The figure shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

Let L = Length of heavy liquid moved in right limb from X–X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X–X $= L \times \sin\theta$

from equation, the pressure at A is

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

substituting the value of h_2, we get

$$P_A = \sin\theta \times \rho_2 g - h_1 \rho_1 g$$

DIFFERENTIAL MANOMETERS :-

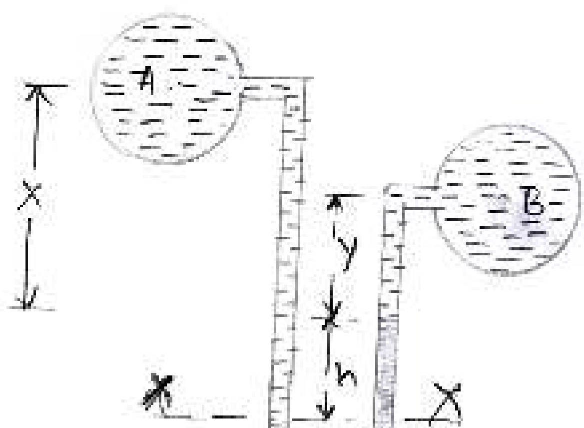
Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :-

(1) U-tube differential manometer and

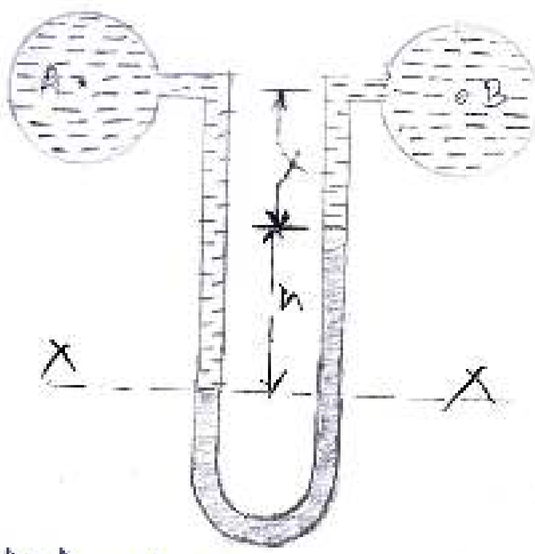
(2) Inverted U-tube differential manometer

(1) U-TUBE DIFFERENTIAL MANOMETER :-

The figures shows the differential manometers of U-tube type.



(a) Two pipes at different levels



(b) A and B are at same level

In figure (a) the two points A and B are at different level and also contains liquids of different specific gravity. These points are connected to the U-tube differential manometer. Let the pressure at A and B are P_A and P_B

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb

x = Distance of the centre of A, from the mercury level in the right limb

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B

ρ_g = Density of heavy liquid of mercury

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h+x) + P_A$

where P_A = Pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + P_B$

where P_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h+x) + P_A = \rho_g \times g \times h + \rho_2 g y + P_B$$

$$\Rightarrow P_A - P_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h+x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

Difference of pressure at A and B =

$$h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

In figure (b), the two points A and B are at the same level and contains the same liquid of density ρ_1, then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + P_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h+x) + P_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + P_B = \rho_1 \times g \times (h+x) + P_A$$

$$\Rightarrow P_A - P_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g (h+x)$$

$$= g \times h (\rho_g - \rho_1)$$

[2] INVERTED U-TUBE DIFFERENTIAL MANOMETER ⇒

It consists of an inverted U-tube containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. The figure shows an inverted U-tube differential manometer connected to the two points A and B.

Let the pressure at A is more than the pressure at B.

Let h_1 = height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = Difference of light liquid

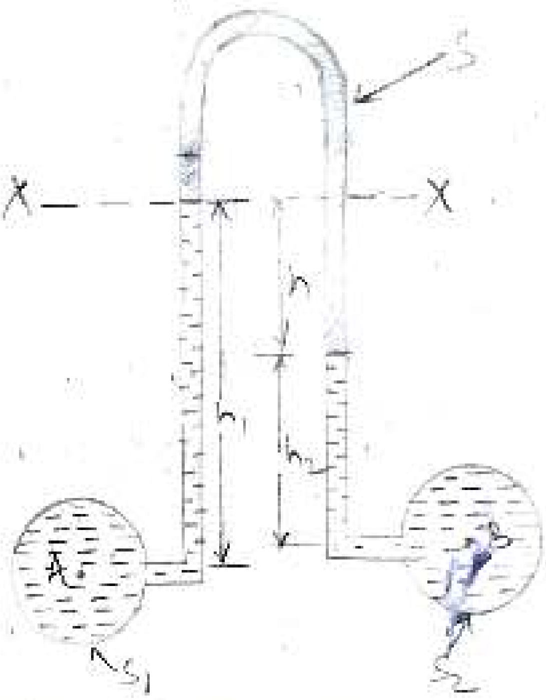
ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

P_A = pressure at A

P_B = pressure at B



Taking X-X as datum line, then pressure in the left limb below X-X

$$= P_A - \rho_1 \times g \times h_1$$

Pressure in the right limb below X-X

$$= P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure,

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$\Rightarrow P_A - P_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Questions ⇒

(1) A simple U-tube monometer is used to measure the pressure of water in a pipe line which is above the atmospheric pressure. the contact between the big. the right limb of the manometer contains mercury & is open to the atm, mercury & water is in the left limb as shown in the fig. determine the pressure of H_2O in the main line if the difference in the level of Hg in the limb of U-tube is 10cm and the free surface of the Hg is at the same level as the center of the pipe?

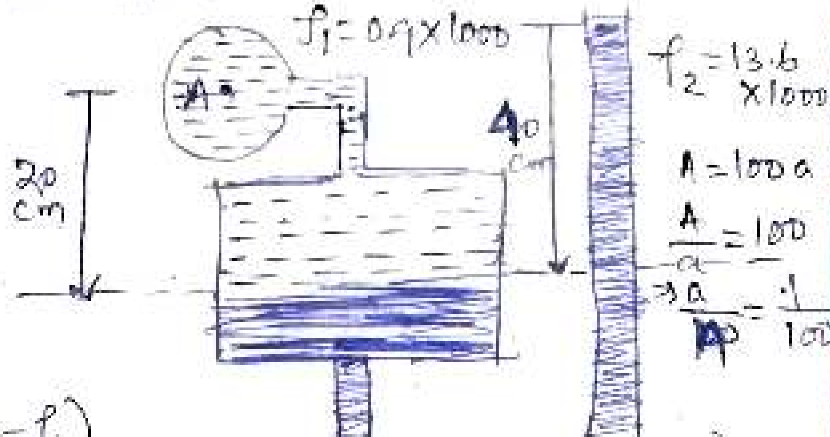
Ans) $P_A + \rho_1 g h_1 = \rho_2 g h_2$

$\Rightarrow P_A + (1000 \times 9.81 \times 10 \times 10^{-2}) = (13.6 \times 1000 \times 9.81 \times 100 \times 10^{-2})$

$\Rightarrow P_A = (13.6 \times 1000 \times 9.81 \times 10 \times 10^{-2}) - (1000 \times 9.81 \times 10 \times 10^{-2})$

$\Rightarrow P_A = 133416 - 981 = 12360.6\ N/m^2$ (Ans)

(2) A single column monometer is connected to a pipe containing a liquid of specific gravity 0.9 as shown in figure. find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube of manometer?



(Ans) $h_1 = 20\ cm = 0.20\ m$

$h_2 = 40\ cm = 0.4\ m$

$\rho_1 = 0.9 \times 1000 = 900$

$\rho_2 = 13.6 \times 1000 = 13600$

$g = 9.81$

$P_A = \rho_2 g h_2 - \rho_1 g h_1 + g \frac{a}{A} \cdot h_2 (\rho_2 - \rho_1)$

$= 13600 \times 9.81 \times 0.4 - 900 \times 9.81 \times 0.2 + 9.81 \times \frac{1}{100} \times 0.4 (13600 - 900)$

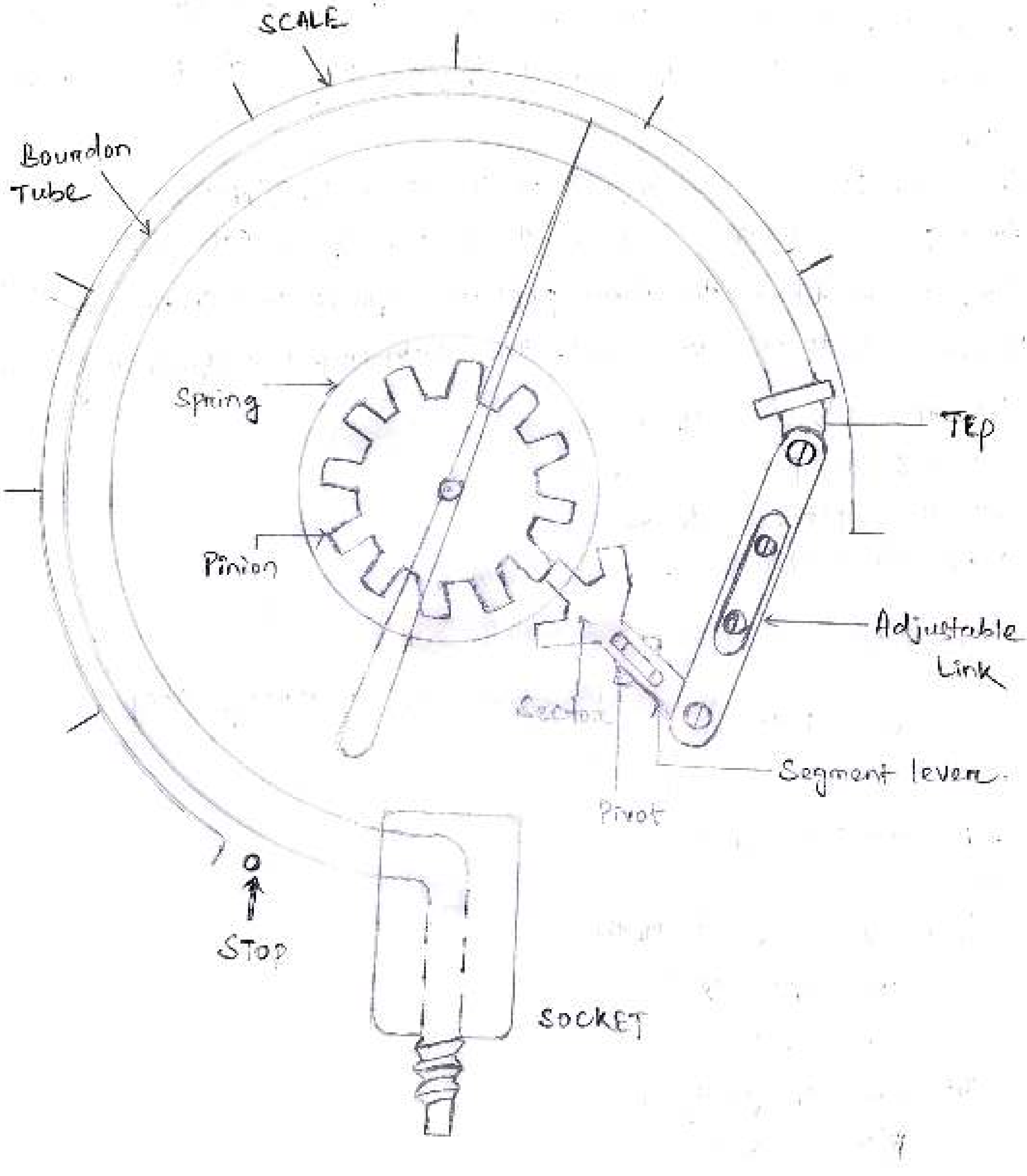
$= 53366.4 - 17658 + 0.03924 \times 12700$

$= 35708.4 + 498.348$

$= 36206.748\ N/m^2$ (Ans)

BOURDON TUBE PRESSURE GAUGE

- Bourdon tube pressure gauges are classified as mechanical pressure measuring instruments, and thus operate without any electrical power. This type of pressure gauges were first developed by E. Bourdon in 1849.
- Bourdon tubes are radially formed tubes with an oval cross-section.
- Bourdon tube pressure gauges can be used to measure over a wide range of pressure from vacuum to pressure as high as few thousand psi.
- It is basically consisted of a C-shaped hollow tube, whose one end is fixed and connected to the pressure tapping, the other end free.
- The cross section of the tube is elliptical. When pressure is applied, the elliptical tube (Bourdon tube) tries to acquire a circular cross-section, as a result, stress is developed and the tube tries to straighten up.
- Thus the free-end of the tube moves up, depending on magnitude of pressure.
- This motion is the measure of the pressure and is indicated via the movement of a deflecting and indicating mechanism is attached to the free end that rotates the pointer and indicates the pressure reading.
- The materials used are commonly phosphor bronze, brass and beryllium, copper.
- Though the C-type tubes are most common, other shapes of tubes, such as helical, twisted or spiral tubes are also in use.



[BOURDON TUBE PRESSURE GAUGE]

TOTAL PRESSURE AND CENTRE OF PRESSURE :-

→ Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

→ Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :-

(1) Vertical plane surface
(2) Horizontal plane surface
(3) Inclined plane surface
(4) Curved surface

(1) Vertical plane surface submerged in Liquid :-

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in figure.

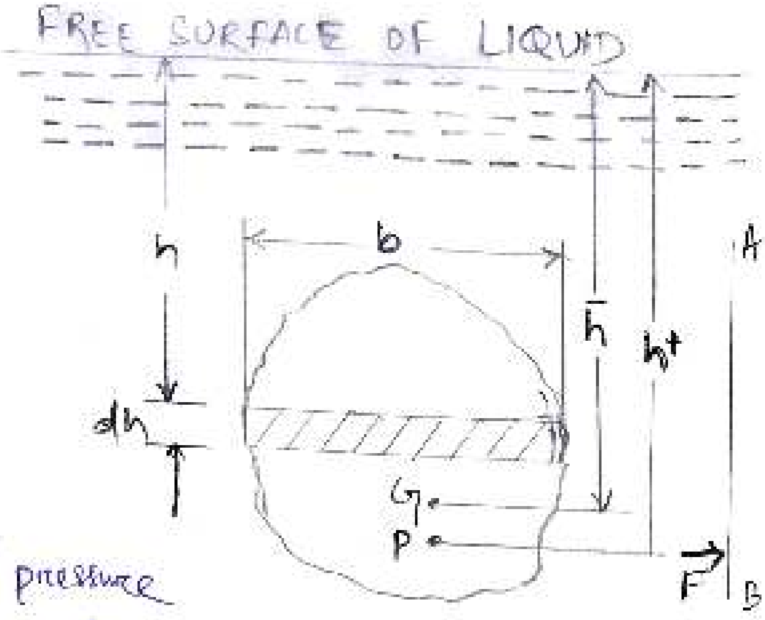
Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid



(a) TOTAL PRESSURE (F) :-

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in figure.

Pressure intensity on the strip, $p = \rho g h$

Area of the strip : $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$
$= \rho g h \times b \times dh$

∴ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= moment of surface area about the free surface of liquid

= Area of surface × Distance of c.g. from the free surface

$= A \times \bar{h}$

$$\therefore \quad F = \rho g A \bar{h}$$

For water the value of $\rho = 1000\ kg/m^3$ and $g = 9.81\ m/s^2$. The force will be in Newton.

(b) Centre of Pressure (h') :-

Centre of pressure is calculated by using the 'principle of moments' which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h' from free surface of the liquid as shown in figure. Hence moment of the force F about free surface of the liquid $= F \times h'$ —— (1)

Moment of force dF, acting on a strip about free surface of liquid

$$= dF \times h \qquad [\because dF = \rho g h \times b \times dh]$$

$$= \rho g h \times b \times dh \times h$$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h$$

$$= \rho g \int b \times h \times h\, dh$$

$$= \rho g \int b h^2 dh$$

$$= \rho g \int h^2 dA \qquad (\because b\,dh = dA)$$

But $\int h^2 dA = \int b h^2 dh$

= moment of Inertia of the surface about free surface of liquid $= I_0$

$\therefore$ Sum of moments about free surface $= \rho g I_0$ —— (2)

Equating (1) and (2), we get

$$F \times h' = \rho g I_0$$

But $F = \rho g A \bar{h}$

$$\therefore \rho g A\bar{h} \times h' = \rho g I_0$$

$$\Rightarrow h' = \frac{\rho g I_0}{\rho g A\bar{h}} = \frac{I_0}{A\bar{h}} \qquad (3)$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3), we get

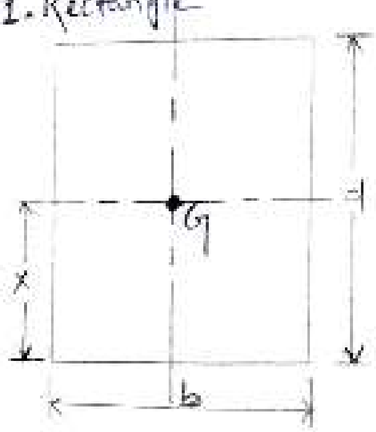
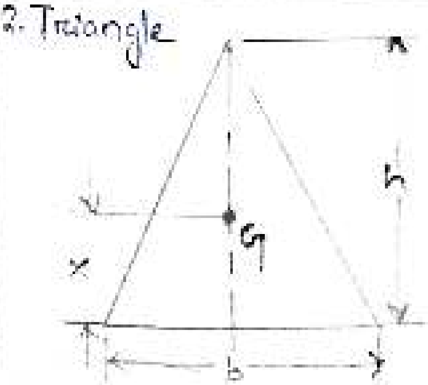
$$h' = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \quad \text{--- (4)}$$

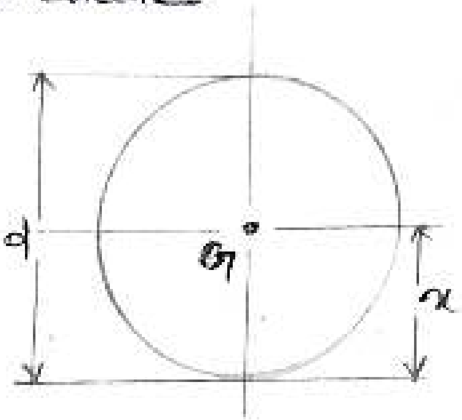
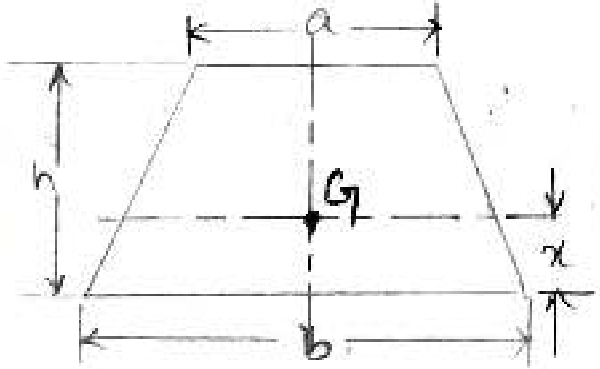
In eqn (4), $\bar{h}$ is the distance of C.G of the area of the vertical surface from the surface of the liquid. Hence from equation (4), it is clear that,

(i) Centre of pressure (i.e. h') lies below the centre of gravity of the vertical surface.

(ii) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

The Moments of Inertia and other geometric properties of some important plane surfaces :—

Plane Surface	C.G. from the base	Area	Moment of Inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane Surface	C.G. from the Base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

ARCHIMEDES' PRINCIPLE

→ When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid displaced by the object.

→ When a solid object is wholly or partly immersed in a fluid, the fluid molecules are continually striking the submerged surface of the object. The force due to these impacts can be combined into a single force the "buoyant force". The immersed object will be "lighter" i.e. It will be buoyed up by an amount equal to the weight of the fluid it displaces.

BUOYANCY →

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

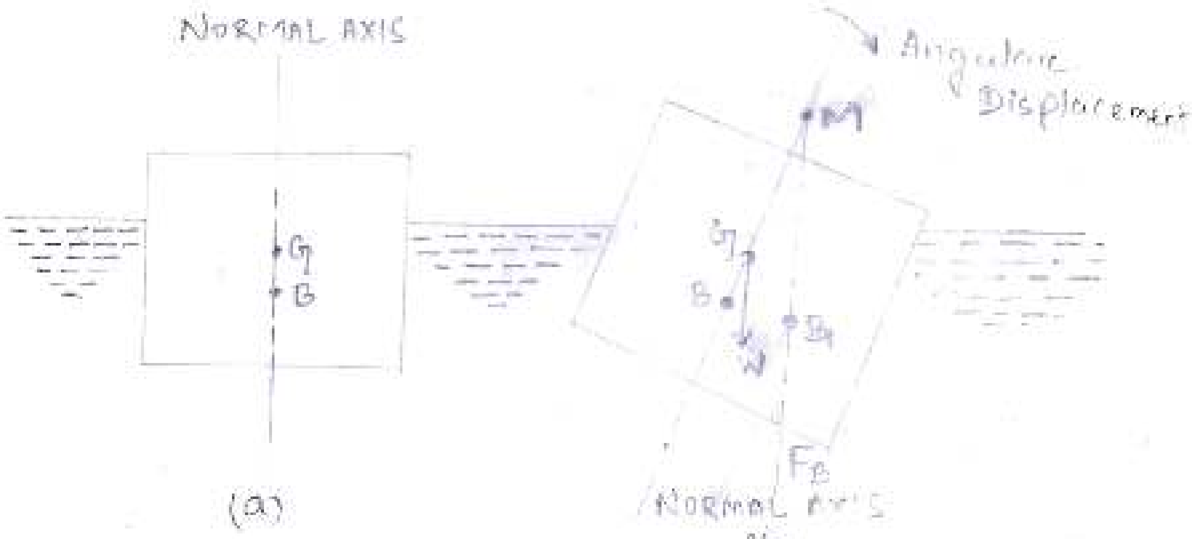
CENTRE OF BUOYANCY :-

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

META-CENTRE :→

→ It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

→ Consider a body floating in a liquid as shown in figure. Let the body is in equilibrium and 'G' is the centre of gravity and 'B' the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

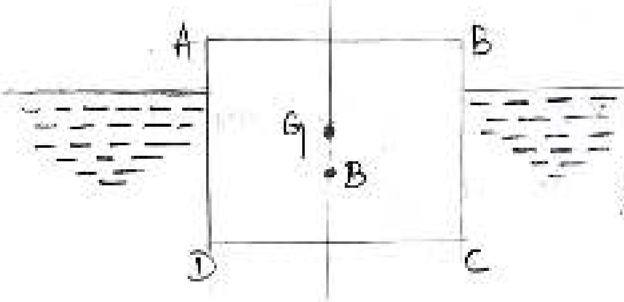


(a) (b)

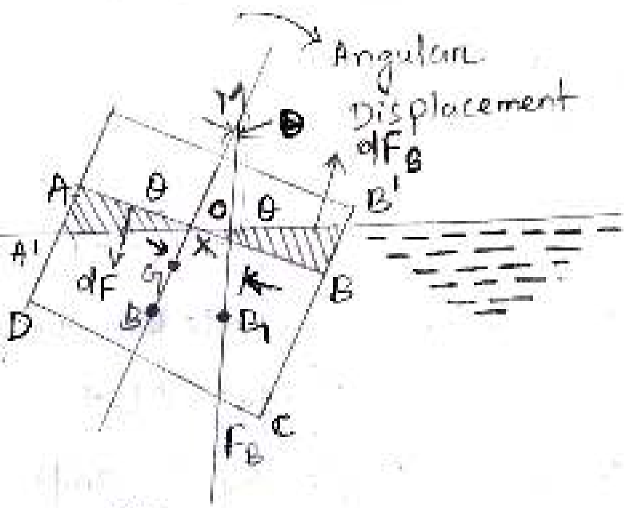
Let the body is given a small angular displacement in the clockwise direction as shown in figure (a). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid will now be shifted towards right from the normal axis, let it is at B_1 as shown in figure (b). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called "Meta-Centre".

META-CENTRIC HEIGHT :-

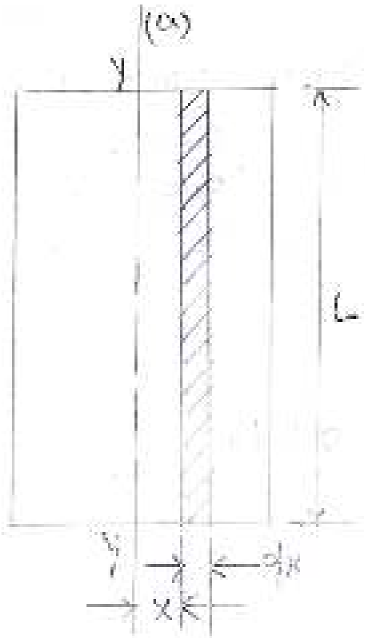
The distance MG, i.e. the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.



(a)



(b)



(C) PLAN OF BODY AT WATER LINE

(Meta-centre height of floating body.)

Couple Due to Wedges :-

Consider towards the right of the axis a small strip of thickness dx at a distance x from O as shown in fig (b).

The height of strip $x \times \angle BOB' = x \times \theta$ $\quad (\because \angle BOB' = \angle AOA' = \angle BMB_1 = \theta)$

$\therefore$ Area of strip = Height × Thickness $= x \times \theta \times dx$

If L is the length of the floating body, then

Volume of strip = Area × L

$= x \times \theta \times L \times dx$

∴ Weight of strip = $\rho g \times$ Volume

$= \rho g x \theta L dx$

Similarly, if a small strip of thickness dx at a distance x from O towards the left of the axis is considered, the weight of strip will be $\rho g x \theta L dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = Weight of each strip × Distance between these two weights

$= \rho g x \theta L dx (x + x)$

$= \rho g x \theta L dx \times 2x = 2\rho g x^2 \theta L dx$

∴ Moment of the couple for the whole wedge

$= \int 2\rho g x^2 \theta L dx$ —(1)

Moment of couple due to shifting of centre of buoyancy from B to B_1

$= F_B \times BB_1$

$= F_B \times BM \times \theta$ (∵ $BB_1 = BM \times \theta$ if θ is very small.)

$= W \times BM \times \theta$ —(2) ($F_B = W$)

But these two couples are the same. Hence equating equations (1) & (2), we get

$W \times BM \times \theta = \int 2\rho g x^2 \theta L dx$

$\Rightarrow W \times BM \times \theta = 2\rho g \theta \int x^2 L dx$

$\Rightarrow W \times BM = 2\rho g \int x^2 L dx$

Now $L dx$ = Elemental area on the water line shown in figure (c) and $= dA$

∴ $W \times BM = 2\rho g \int x^2 dA$

But from figure (c), it is clear that $2\int x^2 dA$ is the second moment of area of the plan of the body at water surface about the axis Y-Y. Therefore

$W \times BM = \rho g I$ (where $I = 2\int x^2 dA$)

$\Rightarrow BM = \frac{\rho g I}{W}$

But W = weight of the body
= weight of the fluid displaced by the body
= $\rho g \times$ Volume of the fluid displaced by the body
= $\rho g \times$ Volume of the body sub-merged in water
= $\rho g \times \forall$

$$\therefore BM = \frac{\rho g \times I}{\rho g \times \forall} = \frac{I}{\forall} \quad \text{--- (3)}$$

$$GM = BM - BG = \frac{I}{\forall} - BG$$

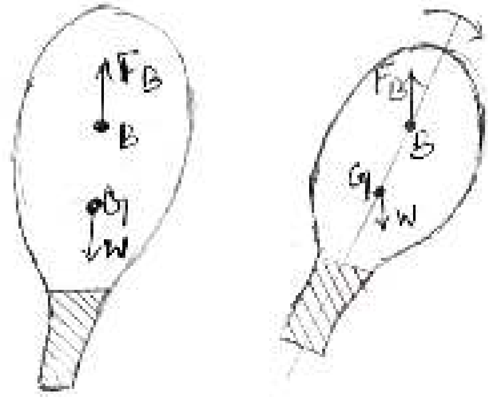
$$\therefore \text{Metacentric height} = GM = \frac{I}{\forall} - BG \quad \text{--- (4)}$$

CONDITIONS OF EQUILIBRIUM OF A FLOATING AND SUB-MERGED BODIES

A sub-merged or a floating body is said to be stable if it comes back to its original position after a slight disturbance. The relative position of the centre of gravity (G) and centre of buoyancy (B_1) of a body determines the stability of a submerged body.

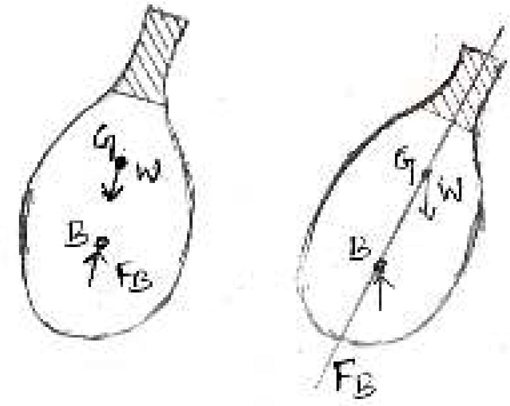
* Stability of a Sub-merged body :-

The position of centre of gravity and centre of buoyancy in case of a completely submerged body are fixed. Consider a balloon, which is completely submerged in air. Let the lower portion of the ~~buoyancy~~ balloon contains heavier material. So that its centre of gravity is lower than its centre of buoyancy as shown in figure (a). Let the weight of the balloon is W. The weight W is acting through G, vertically in the downward direction, while the buoyant force F_B is acting vertically up, through B. For the equilibrium of the balloon $W = F_B$. If the balloon is given an angular displacement in the clockwise direction as shown in figure (a), then W and F_B constitute a couple acting in the anti-clockwise direction and brings the balloon in the original position. Thus the balloon in the position, shown by figure (a) is in stable equilibrium.



(a)

STABLE EQUILIBRIUM



(b)

UNSTABLE EQUILIBRIUM

(c)

NEUTRAL EQUILIBRIUM

(Stabilities of sub-merged bodies)

(a) Stable Equilibrium :-

When $W = F_B$ and point B is above G, the body is said to be in stable equilibrium.

(b) Unstable Equilibrium :-

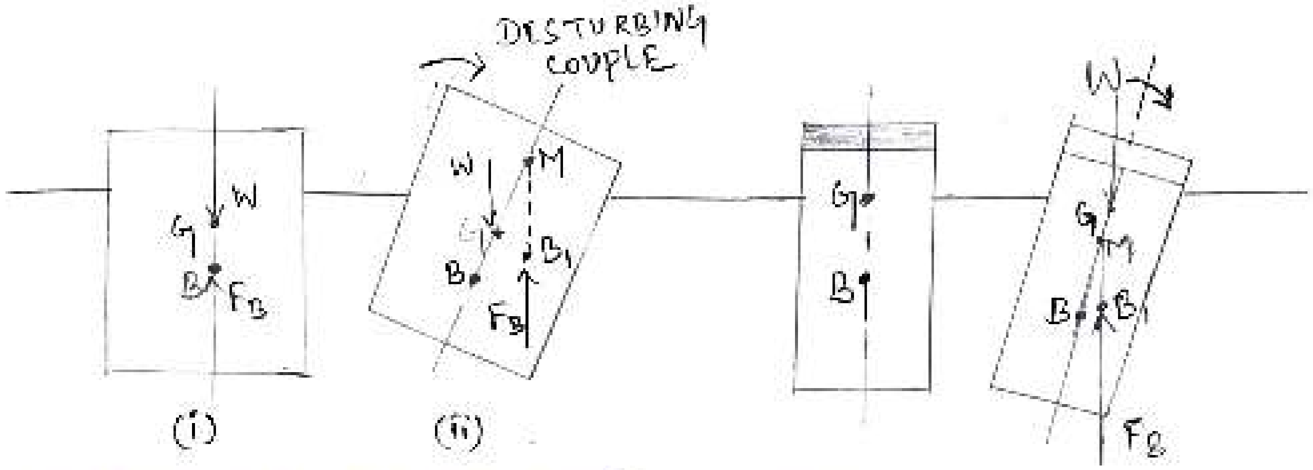
If $W = F_B$, but the centre of buoyancy (B) is below centre of gravity (G), the body is in unstable equilibrium as shown in fig (b). A slight displacement to the body in the clockwise direction to the body, in the clockwise direction, gives the couple due to W and F_B also in the clockwise direction. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

(C) Neutral equilibrium :-

If $F_B = W$ and B and G are at the same point, as shown in fig (c), the body is said to be in neutral equilibrium.

*Stability of Floating Body =>

The stability of a floating body is determined from the position of Meta-centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.



(a) Stable equilibrium M is above G

(b) Unstable equilibrium M is below G.

(Stability of floating bodies)

(a) Stable Equilibrium :-

If the point M is above M is above G, the floating body will be in stable equilibrium as shown in fig (a). If a slight angular displacement is given to the floating body is the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating the floating body in the original position.

(b) Unstable Equilibrium :-

If the point M is below G, the floating body will be in unstable equilibrium as shown in (b). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(c) Neutral Equilibrium :-

If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

TYPES OF FLUID FLOW :-

The fluid flow is classified as:

(i) Steady and unsteady flows

(ii) Uniform and non-uniform flows

(iii) Laminar and turbulent flows

(iv) Compressible and incompressible flows

(v) Rotational and irrotational flows and

(vi) One, two or three dimensional flows

(i) Steady and Unsteady flows →

→ Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0,y_0,z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field

→ Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Thus mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0,y_0,z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} \neq 0 \text{ etc}$$

(ii) Uniform and Non-Uniform flows →

→ Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e. length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t=\text{constant}} = 0$$

where ∂V = change of velocity

∂s = length of flow in the direction s

→ Non-Uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0$$

(iii) Laminar and Turbulent Flow →

→ Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or steam line and all the steam-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called steam-line flow or viscous flow.

→ Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number called the Reynold number.

where D = Diameter of pipe

V = mean velocity of flow in pipe

ν = Kinematic viscosity of fluid

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, then it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

(iv) Compressible and Incompressible Flows ⇒

→ Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow,

$$\rho \neq \text{constant}$$

→ Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow,

$$\rho = \text{constant}$$

(v) Rotational and Irrotational Flows ⇒

Rotational flow is that type of flow in which the fluid particles while flowing along steam lines, also rotates about their own axis. And if the fluid particles while flowing along steam lines, do not rotate about their own axis then that type of flow is called irrotational flow.

(vi) One-, Two-, and Three-Dimensional Flows :-

→ One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one space co-ordinate only. The variation of velocities in other two mutually perpendicular direction is assumed negligible. Hence, mathematically, for one-dimensional flow,

$$u = f(x), \; v = 0 \text{ and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively,

→ Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two-space co-ordinate only. The variation of velocity in the third direction is negligible. Thus, mathematically for two-dimensional flow,

$$u = f_1(x, y), v = f_2(x, y), \text{and } w = 0$$

→ Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow,

$$u = f_1(x, y, z), v = f_2(x, y, z) \text{ and } w = f_3(x, y, z)$$

RATE OF FLOW OR DISCHARGE (Q)

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(i) For liquids the units of Q are m^3/s or litres/s

(ii) For gases the units of Q is kgf/s or Newton/s

Consider a liquid flowing through a pipe in which

A = Cross-sectional area of pipe

V = Average velocity of fluid across the section

Then Discharge $Q = A \times V$.

CONTINUITY EQUATION →

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

Consider two cross-sections of a pipe as shown in figure;

Let V_1 = Average velocity at cross-section 1-1

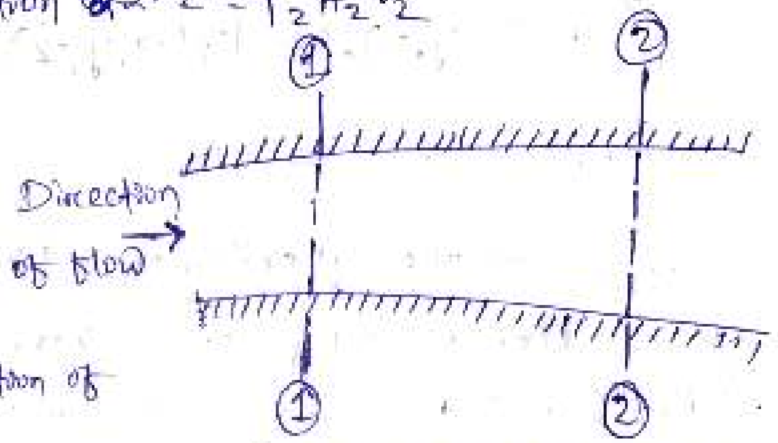
ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

and V_2, ρ_2, A_2 are corresponding value at section 2-2.

Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$



(Fluid flowing through a pipe)

According to law of conservation of mass,

Rate of flow at section 1-1

= Rate of flow at section 2-2

or
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

The above equation is applicable to the compressible as well as incompressible fluids and is called Continuity Equation.

If the fluid is incompressible,

then $\rho_1 = \rho_2$ and continuity equation reduces to

$$A_1 V_1 = A_2 V_2$$

EQUATIONS OF MOTION ⇒

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction.

Thus mathematically, $F_x = m \cdot a_x$

In the fluid flow, the following forces are present,

(i) F_g, gravity force
(ii) F_p, the pressure force
(iii) F_v, force due to viscosity
(iv) F_t, force due to turbulence
(v) F_c, force due to compressibility

Thus in equation, the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

(i) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and equation of motions are called Reynold's equations of motion.

(ii) For flow, where (F_t) is negligible, the ~~requed equatn~~ resulting equations of motion are known as Navier-stokes Equation.

(iii) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as Euler's equation of motion.

EULER'S EQUATION OF MOTION :-

This is equation of motion in which the force due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in a direction as shown in figure. Consider a cylindrical element of cross-section dA and length ds. The force acting on the cylindrical element are

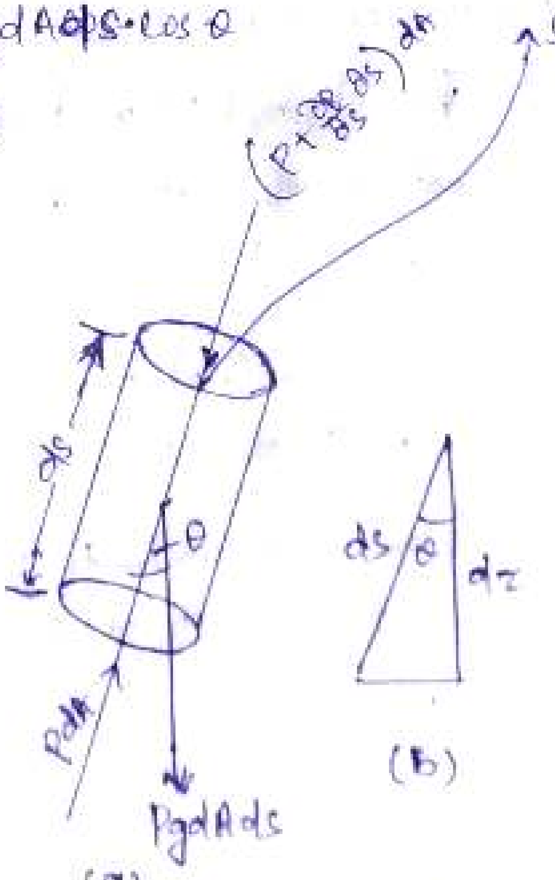
1. pressure force pdA in the direction of flow
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element $\times$ acceleration in the direction s.

$$\therefore \; pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cdot \cos\theta$$
$$= \rho dA ds \times a_s \quad \text{---} (1)$$

where a_s is the acceleration in the direction of s.



(Forces on a fluid element)

Now, $a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}$$
$$= \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left(\because \frac{ds}{dt} = v\right)$$

If the flow is steady,

$$\frac{\partial v}{\partial t} = 0$$

$$\therefore a_s = \frac{v\,dv}{\partial s}$$

Substituting the value of a_s in eqn (1) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds\,dA - \rho g\,dA\,ds\cos\theta = \rho\,dA\,ds \times \frac{\partial v}{\partial s}$$

Dividing by $\rho\,ds\,dA$, $\therefore -\frac{\partial p}{\rho\,\partial s} - g\cos\theta = \frac{v\,dv}{\partial s}$

or $\frac{\partial p}{\rho\,\partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$

But from fig(b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{v\,dv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + g\,dz + v\,dv = 0$$

$$\boxed{\frac{dp}{\rho} + g\,dz + v\,dv = 0} \quad \text{—— (2)}$$

Equation (2) is known as Euler's equation of motion.

BERNOULLI'S EQUATION FROM EULER'S EQUATION →

Bernoulli's equation is obtained by integrating the Euler's equation of motion as

$$\int\frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{Constant}$$

If flow is incompressible, ρ is constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\Rightarrow \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\Rightarrow \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad \text{—— (3)}$$

Equation (3) is a Bernoulli's equation in which,

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$V^2/2g$ = kinetic energy per unit weight or kinetic head

Z = potential energy per unit weight or potential head

ASSUMPTIONS :—

The following are the assumptions made in the derivation of Bernoulli's equation:

(i) The fluid is ideal, i.e. viscosity is zero.

(ii) The flow is steady.

(iii) The flow is incompressible.

(iv) The flow is irrotational.

PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION :—

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices:

1. Venturimeter
2. Orifice meter
3. Pitot-tube

(1) Venturimeter ⇒

→ A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts:

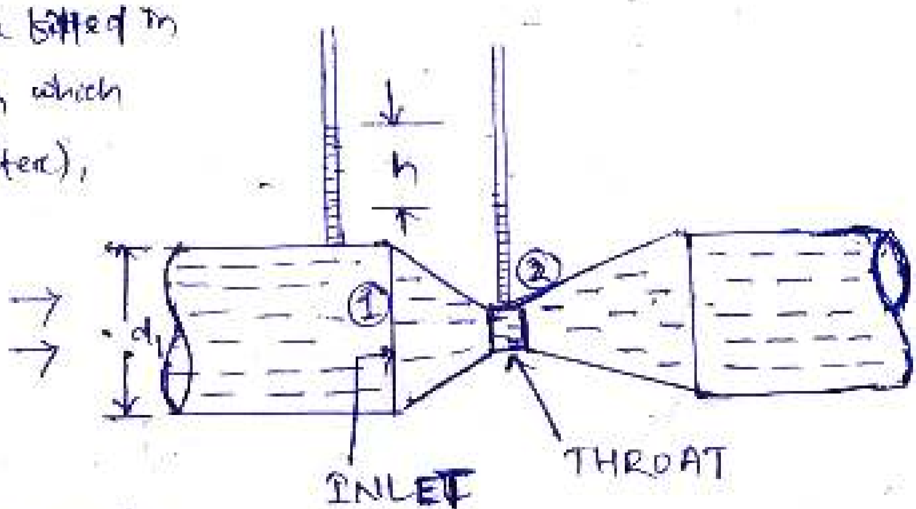
(i) A short converging part,

(ii) Throat and (iii) Diverging part.

→ It is based on the principle of Bernoulli's equation.

Expression for rate of flow through Venturimeter :-

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in figure.



[VENTURIMETER]

Let P_1 = pressure at section (1)

d_1 = diameter at inlet or at section (1),

v_1 = velocity of fluid at section (1),

a = Area at section (1) $= \frac{\pi}{4} d_1^2$

and d_2, P_2, V_2, a_2 are corresponding values at section (2).

Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 \qquad \text{---(4)}$$

As pipe is horizontal, hence $Z_1 = Z_2$

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad \text{or} \quad \frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{P_1 - P_2}{\rho g} = h$

Substituting this value of $\frac{P_1 - P_2}{\rho g}$ in the above equation, we get

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad \text{---(5)}$$

Now applying continuity equation at section 1 & 2

$$a_1 V_1 = a_2 V_2 \quad \text{or} \quad V_1 = \frac{a_2 V_2}{a_1}$$

Substituting the value of V_1 in equation (5),

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{V_2^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

$$\Rightarrow V_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\Rightarrow V_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge, $Q = a_2 V_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$\Rightarrow Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{---(6)}$$

Equation (6) gives the discharge under ideal conditions and is called theoretical discharge. Actual discharge will be less than theoretical discharge.

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{——— (7)}$$

where C_d = Co-efficient of Venturimeter and its value is less than 1. (Co-efficient of discharge)

Value of 'h' given by different U-tube manometers:—

Case-1: Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe.

Let S_h = Specific gravity of the heavier liquid

S_0 = specific gravity of the liquid flowing through pipe

x = Difference of the heavier liquid column in U-tube

Then $h = x\left[\frac{S_h}{S_0} - 1\right]$

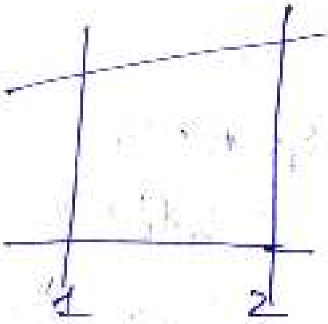
Case - II : If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by,

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

S_l = Specific gravity of lighter liquid in U-tube
S_o = specific gravity of fluid flowing through pipe
x = Difference of the lighter liquid columns in U-tube.

(1) **Question:-** The diameter of pipe at section 1 & 2 are 10 cm & 15 cm respectively. Find the distance through the pipe, if the velocity of water flowing through the pipe at section 1 is 5 m/s. Also determine the velocity at section 2.

Answer:-



$d_1 = 10\text{ cm} = 0.01\text{ m}$, $d_2 = 15\text{ cm} = 0.15\text{ m}$
$V_1 = 5\text{ m/s}$, $V_2 = ??$

$Q_1 = ??$

$Q_1 = A_1 \times V_1$
$= \frac{\pi}{4} d_1^2 \times V_1$
$= \frac{\pi}{4} \times (10)^2 \times 5 = \frac{\pi}{4} \times 100 \times 5 = 3.141 \times 125 = 392.62$

$Q_1 = A_1 \times V_1$
$= \frac{\pi}{4} \times (0.01)^2 \times 5$
$= \frac{\pi}{4} \times 0.0001 \times 5$
$= \frac{3.141 \times 0.0001 \times 5}{4}$
$= 0.0392\text{ m}^3/\text{s}$

Then, $A_1 V_1 = A_2 V_2$

$\Rightarrow V_2 = \frac{A_1 V_1}{A_2} \Rightarrow V_2 = \frac{0.0392}{\frac{\pi}{4}(15 \times 10^{-2})} = \frac{0.0392}{0.78 \times (0.15)^2} = \frac{0.0392}{0.0175}$

$= 2.24$ (Ans)

(2) A 30 cm. diameter pipe in which water is flowing branches into two pipes of diameter 20 cm. and 15 cm respectively. If the average velocity in the 30 cm. diameter pipe is 2.5 m/s, find out the discharge in the pipe? Also determine the velocity in 15 cm. pipe if the average velocity in 20 cm. diameter pipe is 2 m/s?

Ans Given, $d_1 = 30$ cm. $= 0.30$ m

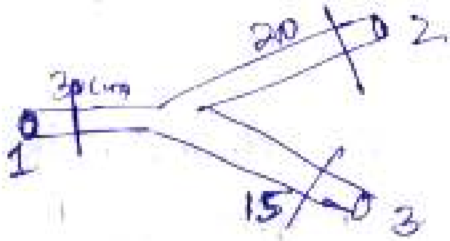
$d_2 = 20$ cm. $= 0.20$ m

$d_3 = 15$ cm. $= 0.15$ m

$v_1 = 2.5$ m/s , $Q = ?$

$v_2 = 2$ m/s

$v_3 = ??$



$Q_1 = A_1 \times v_1$

$= \frac{\pi}{4} d_1^2 \times v_1$

$= \frac{\pi}{4} \times (0.30)^2 \times 2.5$

$= \frac{\pi}{4} \times 0.09 \times 2.5 = \frac{3.141}{4} \times 0.09 \times 2.5$

$= 0.78 \times 0.09 \times 2.5$

$= 0.176$ m³/s

In figure,

$Q_1 = Q_2 + Q_3$

$\Rightarrow A_1 V_1 = A_2 V_2 + A_3 V_3$

$\Rightarrow 0.176 = \frac{\pi}{4} \times d_2^2 \times V_2 + \frac{\pi}{4} \times d_3^2 \times V_3$

$= \frac{\pi}{4} \times (0.20)^2 \times 2 + \frac{\pi}{4} \times (0.15)^2 \times V_3$

$= 0.78 \times 0.04 \times 2 + 0.78 \times 0.0225 \times V_3$

$\Rightarrow 0.176 = 0.0624 + 0.0175 \times V_3$

$\Rightarrow 0.176 - 0.0624 = 0.0175 \times V_3$

$\Rightarrow 0.1136 = 0.0175 \times V_3$

$\Rightarrow V_3 = \frac{0.1136}{0.0175}$

$\Rightarrow V_3 = 6.4 \ m/s$ (Ans)

or, $Q_1 = Q_2 + Q_2$

$\Rightarrow A_1V_1 = A_2V_2 + A_3V_3$

$\Rightarrow \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} \times d_2^2 \times V_2 + \frac{\pi}{4} \times d_3^2 \times V_3$

$\Rightarrow \frac{\pi}{4}\{(0.30)^2 \times 2.5\} = \frac{\pi}{4}\{(0.20)^2 \times 2 + (0.15)^2 \times V_3\}$

$\Rightarrow (30\times10^{-2})^2 \times 2.5 = (20\times10^{-2})^2 \times 2 + (15\times10^{-2})^2 \times V_3$

$\Rightarrow (30)^2 \times 2.5 = (20)^2 \times 2 + (15)^2 \times V_3$

$\Rightarrow 900 \times 2.5 = 400 \times 2 + 225 \times V_3$

$\Rightarrow 2250 = 800 + 225 V_3$

$\Rightarrow 2250 - 800 = 225 V_3$

$\Rightarrow V_3 = \frac{1450}{225} = 6.44 \ m/s$ (Ans)

(3) Water flows through a pipe 'AB' 1.2 m in diameter velocity of 3 m/s & then passes through a pipe BC 1.5 m in diameter. At C the pipe branches. Branch CD 0.8 m in diameter and carries 1/3 of the flow in AB. The velocity in the branch CE is 2.5 m/s. Find the discharge and AB, velocity in BC, velocity in CD and the diameter of CE ?

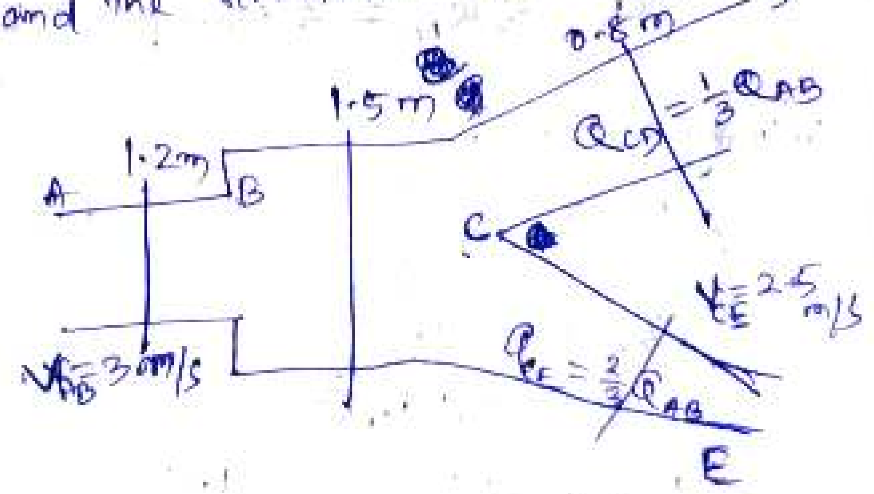
Ans Given,

$d_{AB} = 1.2 \ m$

$d_{BC} = 1.5 \ m$

$d_{CD} = 0.8 \ m$

$d_{CE} = ??$



$V_{AB} = 3 m/s$ $\qquad$ $Q_{AB} = ?$

$V_{BC} = ?$ $\qquad$ $Q_{BC} = ?$

$V_{CD} = ?$ $\qquad$ $Q_{CD} = \frac{1}{3} Q_{AB}$

$V_{CE} = 2.5 m/s$ $\qquad$ $Q_{CE} = \frac{2}{3} Q_{AB}$

Rate of discharge at AB,

$$Q_{AB} = A_{AB} \times V_{AB}$$
$$= \frac{\pi}{4}(d_{AB})^2 \times V_{AB}$$
$$= \frac{\pi}{4}(1.2)^2 \times 3 = \frac{\pi}{4} \times 1.44 \times 3 = 0.78 \times 3 \times 1.44$$
$$= 3.39 \ m^3/s$$

from figure,

$$Q_{AB} = Q_{BC}$$
$$\Rightarrow A_{AB} \times V_{AB} = A_{BC} \times V_{BC}$$
$$\Rightarrow \frac{\pi}{4}(d_{AB})^2 \times 3 = \frac{\pi}{4} \times (d_{BC})^2 \times V_{BC}$$
$$\Rightarrow \frac{\pi}{4} \times 3 \times (1.2)^2 = \frac{\pi}{4} \times (1.5)^2 \times V_{BC}$$
$$\Rightarrow 3.39 = 1.76 \times V_{BC}$$
$$\Rightarrow V_{BC} = \frac{3.39}{1.76} = 1.92 \ m/s$$

$\therefore$ Velocity in BC is 1.92 m/s.

Then, $Q_{CD} = \frac{1}{3} Q_{AB} = \frac{1}{3} \times 3.39 = 1.131 \ m^3/s$

$$Q_{CE} = Q_{AB} - Q_{CD} = 3.39 - 1.131 = 2.262 \ m^3/s$$

or $Q_{CE} = \frac{2}{3} Q_{AB} = \frac{2}{3} \times 3.39 = 2.262 \ m^3/s$

Velocity in CD:

$$V_{CD} = \frac{Q_{CD}}{A_{CD}} \qquad (\because Q_{CD} = A_{CD} \times V_{CD})$$

$$\Rightarrow V_{CD} = \frac{Q_{CD}}{\frac{\pi}{4}(d_{CD})^2}$$

$$\Rightarrow V_{CD} = \frac{1.131}{\frac{\pi}{4}(0.8)^2} = \frac{1.131}{\frac{\pi}{4} \times 0.64} = \frac{1.131}{0.502}$$

$\Rightarrow V_{CD} = 2.25\ m/s$

$\therefore$ velocity in CD is 2.25 m/s

diameter of CE can get from this expression,

we know, Discharge at CE,

$$Q_{CE} = A_{CE} \times V_{CE}$$

$$\Rightarrow Q_{CE} = \frac{\pi}{4} \times (d_{CE})^2 \times V_{CE}$$

$$\Rightarrow 2.262 = \frac{\pi}{4} \times (d_{CE})^2 \times 2.5$$

$$\Rightarrow 2.262 = (d_{CE})^2 \times 1.963$$

$$\Rightarrow (d_{CE})^2 = \frac{2.262}{1.963}$$

$$\Rightarrow (d_{CE})^2 = 1.152$$

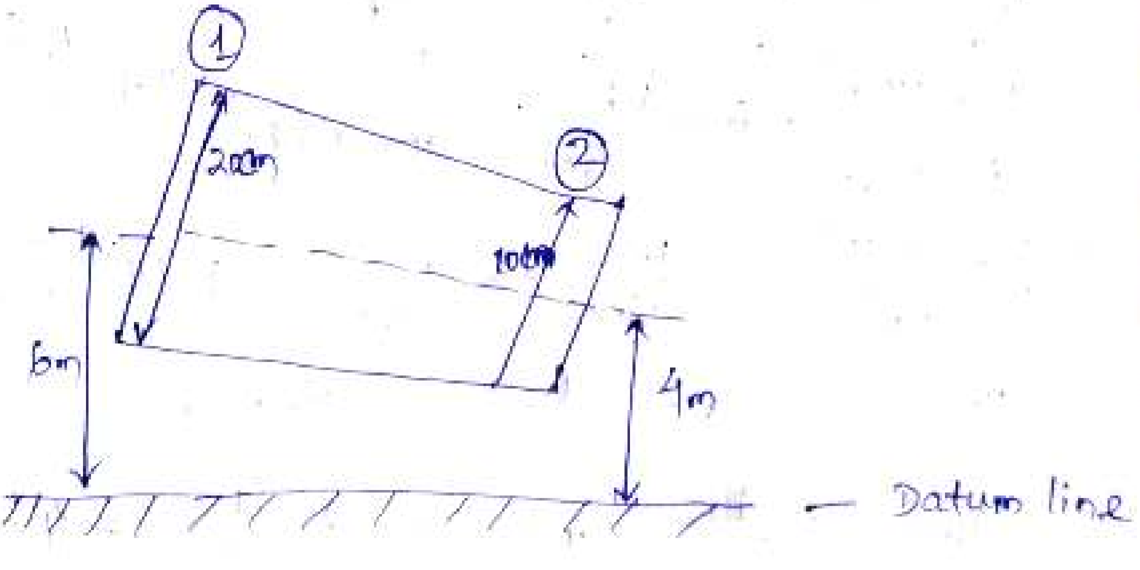
$$\Rightarrow d_{CE} = \sqrt{1.152}$$

$$= 1.073\ m.$$

$\therefore$ diameter of CE is 1.073 m. (Ans)

(4) Water is flowing through a pipe having diameter 20 cm. & 10 cm at section 1 & 2 respectively. The rate of flow through pipe is 35 litre/sec. The section 1 is 6 m. above the datum and section 2 is 4 m. above the datum. If the pressure at cross section 1 is 39.24 N/cm² then find out the intensity of pressure at section 2.

(Ans)



Given,

$d_1 = 20$ cm, $= 0.20$ m $Z_1 = 6$ m.

$d_2 = 10$ cm $= 0.10$ m $Z_2 = 4$ m.

$Q = 35$ l/s $g = 9.81$

$\Rightarrow Q = 35 \times 10^{-3}$ m³/s $\rho = 1000$ kg/m²

$1l = 10^{-3}$ m³/s

$P_1 = 39.24$ N/cm² $= 39.24 \times 10^4$ N/m²

$P_2 = ?$

According to Bernoulli's equation,

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$Q_1 = Q_2 = Q = 35$ l/s

$Q_1 = A_1 V_1$

$\Rightarrow 35 \times 10^{-3} = \frac{\pi}{4} \times (d_1)^2 \times V_1$

$= \frac{\pi}{4} \times (0.20)^2 \times V_1$

$\Rightarrow 35 \times 10^{-3} = 0.78 \times 0.04 \times V_1$

$\Rightarrow 35 \times 10^{-3} = 0.0312 \times V_1$

$\Rightarrow 0.035 = 0.0312 \times V_1$

$\Rightarrow V_1 = \frac{0.035}{0.0312} = 1.12$ m/s

$Q_2 = A_2 V_2$

$\Rightarrow 35 \times 10^{-3} = \frac{\pi}{4} \times (d_2)^2 \times V_2$

$\Rightarrow 0.035 = 0.78 \times (0.10)^2 \times V_2$

$= 0.78 \times 0.01 \times V_2$

$\Rightarrow 0.035 = 0.0078 \times V_2$

$\Rightarrow V_2 = \frac{0.035}{0.0078}$

$= 4.48$ m/s

Then according to Bernoulli's equation,

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{39.24 \times 10^4}{1000 \times 9.81} + 6 + \frac{(1.1)^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + 4 + \frac{(4.4)^2}{2 \times 9.81}$$

$$\Rightarrow \frac{39.24 \times 10^4}{9810} + 6 + \frac{1.21}{19.62} = \frac{P_2}{9810} + 4 + \frac{19.36}{19.62}$$

$$\Rightarrow 40 + 6 + 0.061 = \frac{P_2}{9810} + 4 + 0.986$$

$$\Rightarrow 46.061 = \frac{P_2}{9810} + 4.986 \quad \Rightarrow \frac{P_2}{9810} = 41.075$$

$\Rightarrow P_2 = 41.075 \times 9810$

$= 402945.75 \ N/m^2$

$= 40.29 \ N/cm^2$ (Ans)

(5) An oil of specific gravity 0.8 is flowing through a venturimeter having inlet diameter 20 cm. and throught diameter 10 cm. The oil mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through horizontal venturimeter taking Cd = 0.98 ?

(Ans) Given, $d_1 = 20 \ cm = 0.20 \ m$ $\quad C_d = 0.98$

$d_2 = 10 \ cm = 0.10 \ m$

S_x = specific gravity of oil = 0.8

S_h = Specific gravity of mercury = 13.6

x = Differential reading = 25 cm = 0.25 m

According to case - I,

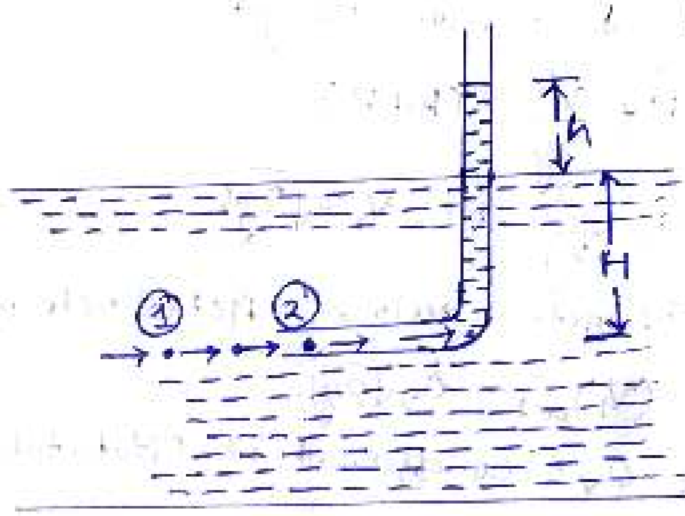
$$h = x\left[\frac{S_h}{S_x} - 1\right]$$

$$= 0.25 \times \left[\frac{13.6}{0.8} - 1\right] = 0.25 \times (17-1) = 0.25 \times 16$$

$$= 4 \ m = 400 \ cm$$

Pitot-Tube :-

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in figure.



Pitot-tube

The lower end, which is bent through 90° is directed in the up steam direction as shown in figure. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points ① and ② at the same level in such a way that point (2) is just as the inlet of the pitot-tube and point (1) is far away from the tube.

Let P_1 = intensity of pressure at point (1)
V_1 = velocity of flow at (1)
P_2 = pressure at point (2)
V_2 = velocity at point (2), which is Zero
H = depth of tube in the liquid
h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equation at points (1) and (2), we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But $Z_1 = Z_2$ as points (1) and (2) are on the same line and $V_2 = 0$

$$\frac{P_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{P_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \quad H + \frac{V_1^2}{2g} = (h + H)$$

$$\therefore \quad h = \frac{V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(V_1)_{act} = C_v \sqrt{2gh}$$

where C_v = co-efficient of pitot-tube

$\therefore$ velocity at any point

$$V = C_v \sqrt{2gh}$$

{CHAPTER - 05} {F.M}

ORIFICE

Introduction ⇒

Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing. A mouth piece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank or vessel containing the fluid. Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

Classification of Orifices ⇒

The orifices are classified on the basis of their size, shape, nature of discharge and shape of the upsteam edge. The following are the important classifications:-

(1) The orifices are classified as small orifice or large orifice depending upon the size of orifice and head of liquid from the centre of the orifice. If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice. And if the head of liquids is less than five times the depth of orifice, it is known as large orifice.

2) The orifices are classified as (i) Circular orifice, (ii) Triangular orifice, (iii) Rectangular orifice and (iv) Square orifice depending upon their cross-sectional areas.

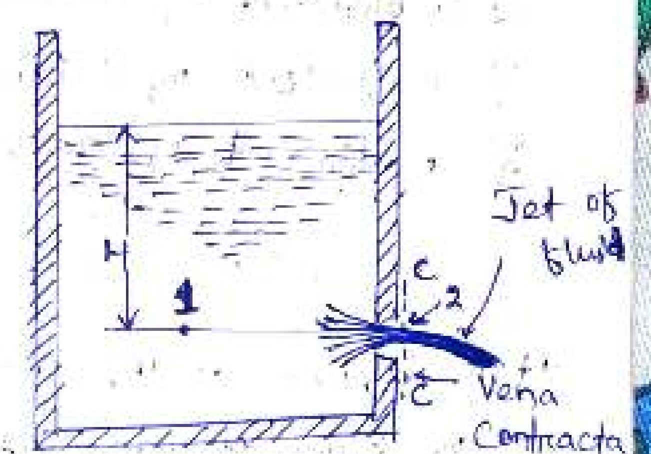
3) The orifices are classified as (i) Sharp-edged orifice and (ii) Bell-mouthed orifice depending upon the shape of upstream edge of the orifices.

(4) The orifices are classified as
(i) Free discharging orifices and (ii) Drowned or sub-merged orifices depending upon the nature of discharge.

The submerged orifices are further classified as (a) fully submerged orifices and (b) partially submerged orifices.

Flow through an Orifice :

Consider a tank fitted with a circular orifice in one of its sides as shown in figure.
Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice. The area of jet of fluid goes on decreasing and at a section C-C, the area is minimum. This section is approximately at a distance of half of diameter of the orifice. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called "Vena-Contracta." Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.



(Tank with an Orifice)

Consider two points 1 and 2 as shown in figure. Point 1 is inside the tank and point 2 at the vena-contracta. Let the flow is steady and at a constant head H. Applying Bernoulli's equation at point 1 and 2.

But

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But $Z_1 = Z_2$

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

Now $\frac{P_1}{\rho g} = H$ $\quad \frac{P_2}{\rho g} = 0$ (atmospheric pressure)

V_1 is very small in comparison to V_2 as area of tank is very large as compared to the area of the jet of liquid.

$$H + 0 = 0 + \frac{V_2^2}{2g}$$

$$\therefore V_2 = \sqrt{2gH}$$

This is theoretical velocity. Actual velocity will be less than this value.

HYDRAULIC CO-EFFICIENTS →

The hydraulic co-efficients are :-

1] Co-efficient of velocity, C_v

2] Co-efficient of contraction, C_c

3] Co-efficient of discharge, C_d

(1) Co-efficient of Velocity (C_v) :→

It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical of jet. It is denoted by C_v and mathematically C_v is given as

$$C_v = \frac{\text{Actual velocity of jet at vena contracta}}{\text{Theoretical velocity}}$$

$= \frac{V}{\sqrt{2gH}}$, where V = actual velocity,

$\sqrt{2gH}$ = Theoretical velocity

The value of C_v varies from 0.95 to 0.99 for different orifices, depending on the shape, size of the orifice and on the head under which flow takes place.

Generally, the value of $C_v = 0.98$ is taken for sharp-edged orifices.

(2) Co-efficient of Contraction (C_c):-

It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_c.

Let a = area of orifice and

a_c = area of jet at vena-contracta

$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a}$$

The value of C_c varies from 0.61 to 0.69 depending upon shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may be taken as 0.64.

(3) Co-efficient of Discharge (C_d):-

It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given as

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual Area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual Area}}{\text{Theoretical area}}$$

$C_d = C_v \times C_c$

the value of C_d varies from 0.61 to 0.65. for general purpose the value of C_d is taken as 0.62.

NOTCH

Introduction:-

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a Concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of Concrete or masonary structure.

1. Nappe or Vein :- The sheet of water flowing through a notch or over a weir is called Nappe or Vein.
2. Crest or Sill :- The bottom edge of a notch or a top of a weir over which the water flows, is known as the Sill or Crest.

Classification of Notes and weirs :

The notches are classified as :

(1) According to the shape of the opening :

(a) Rectangular notch
(b) Triangular notch
(c) Trapezoidal notch and
(d) Stepped notch

(2) According to the effect of the sides on the nappe:

(a) Notch with end contraction
(b) Notch without end contraction or suppressed notch

Weirs are classified according to the shape of the opening, the shape of the crest, the effect of the side on the nappe and nature of discharge. The following are important classification.

(a) According to the shape of the opening :

(i) Rectangular weir
(ii) Triangular weir and
(iii) Trapezoidal weir (Cipolletti weir)

(b) According to the shape of the crest :

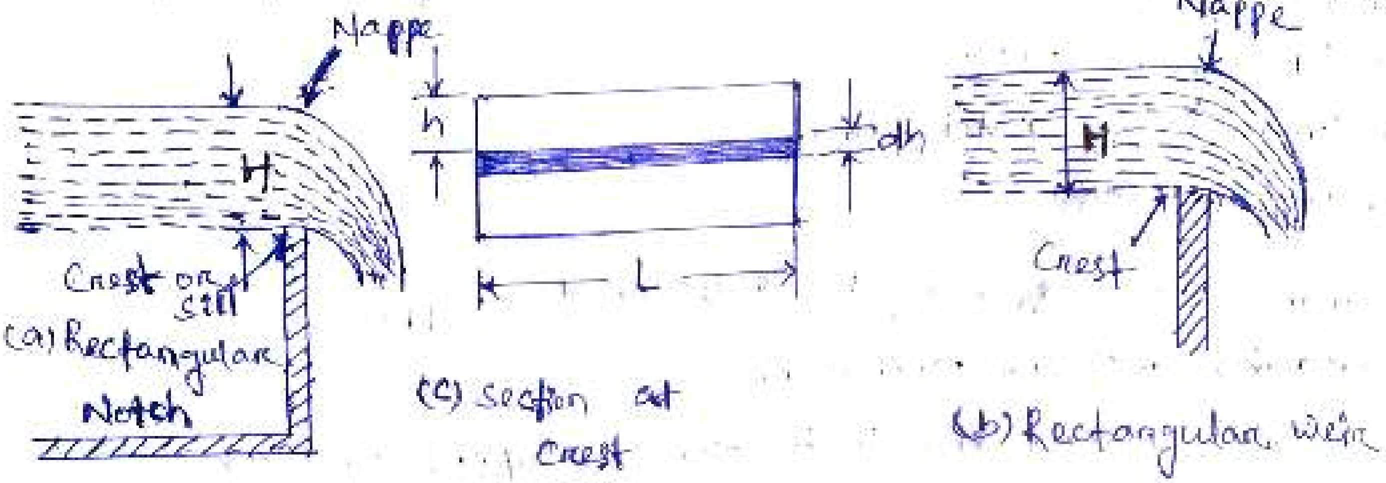
(i) Sharp-crested weir
(ii) Broad-crested weir
(iii) Narrow-crested weir and
(iv) Ogee-shaped weir.

(c) According to the effect on sides on the emerging nappe :

(i) Weir with end contraction and (ii) Weir without end contraction

Discharge over a Rectangular Notch or Weir:-

The expression for discharge over a rectangular notch or weir is the same.



(Rectangular notch and weir)

Consider a rectangular notch or weir provided in a channel carrying water as shown in figure.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at depth h from the free surface of water as shown in figure.

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical Velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh}$$

where C_d = Co-efficient of discharge

The total discharge, Q, for the whole notch or weir is determined by integrating equation (i) between the limits 0 and H.

$$\therefore Q = \int_0^H C_d \cdot L \cdot \sqrt{2gh}\, dh$$

$$= C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2}$$

Discharge Over a Triangular Notch or Weir :-

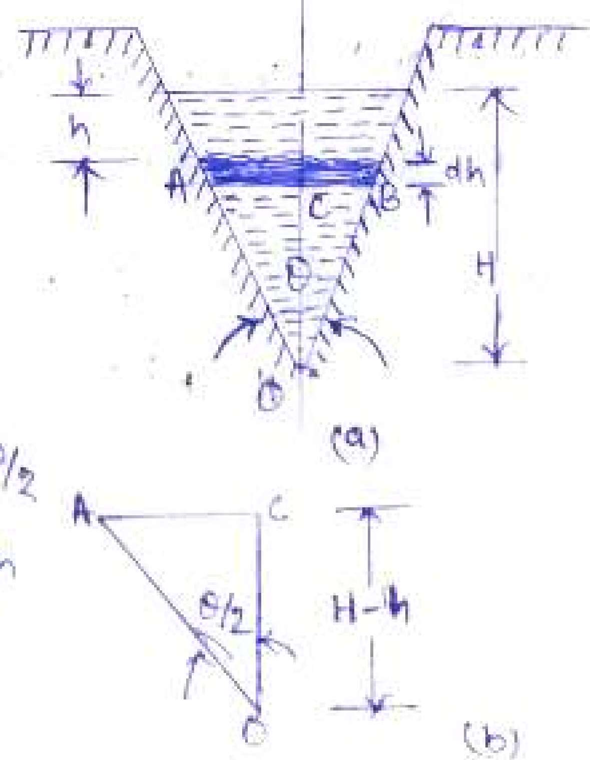
The expression for the discharge over a triangular notch or weir is the same. It is ~~[illegible]~~ derived as :

Let H = head of water above the V-notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in figure.

~~[illegible] figure~~.



From figure (b), we have

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$\therefore AC = (H-h)\tan\frac{\theta}{2}$$

Width of strip $= AB = 2AC$

$$= 2(H-h)\tan\theta/2$$

$\therefore$ Area of strip $= 2(H-h)\tan\theta/2 \times dh$

The theoretical velocity of water through strip $= \sqrt{2gh}$

$\therefore$ Discharge, through the strip,

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2(H-h)\tan\theta/2 \times dh \times \sqrt{2gh}$$

$$= 2C_d (H-h)\tan\theta/2 \times \sqrt{2gh} \times dh$$

$\therefore$ Total discharge,

$$Q = \int_0^H 2C_d (H-h)\tan\theta/2 \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\theta/2 \times \sqrt{2g}\int_0^H (H-h)\, h^{1/2}\, dh$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\, dh$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15} C_d \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}$$

For a right angled V-notch if $C_d = 0.6$

$\theta = 90°$, $\therefore \tan\theta/2 = 1$

Discharge, $Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$

$= 1.417\ H^{5/2}$

Loss of Energy In pipes →

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :-

Energy losses

1. Major Energy losses
   This is due to friction and it is calculated by the following formulae:
   (a) Darcy - Weisbach formula
   (b) Chezy's formula

2. Minor Energy losses
   This is due to
   (a) Sudden expansion of pipe
   (b) Sudden Contraction of pipe
   (c) Bend in pipe
   (d) pipe fittings etc.
   (e) An obstruction in pipe

Loss of Energy (OR, HEAD) DUE TO FRICTION →

(a) Darcy - Weisbach Formula :-

The loss of head (or energy) in ~~purpose~~ pipes due to friction is calculated from Darcy-weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g} \quad \text{—— (1)}$$

where h_f = loss of head due to friction

f = Co-efficient of friction which is a function of Reynold's number

$= \frac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \frac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = length of pipe

V = mean velocity of flow

d = diameter of pipe.

(b) Chezy's Formula for loss of head due to friction in pipes:-

Refer to chapter 10 article 10 in which expression for loss of head due to friction in ~~pipe~~ pipes is derived.

Equation (iii) of article 10 is

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{———— (2)}$$

where h_f = loss of head due to friction

P = wetted perimeter of pipe

A = Area of cross-section of pipe

L = length of pipe

V = mean velocity of flow

and

Now the ratio of $\frac{A}{P}\left(= \frac{\text{Area of flow}}{\text{Perimeter(wetted)}}\right)$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (2),

we get, $h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m}$ or $V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$

$$= \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\Rightarrow V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}}$$

$$= \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{h_f}{L}} \quad \text{———— (3)}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant

and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe

Substituting the values of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation —(3)

we get, $V = C\sqrt{mi}$ —— (4)

Equation (4) is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

MINOR ENERGY (HEAD) LOSSES ⇒

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases:

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance of a pipe.
4. Loss of head at the exit of a pipe
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

HYDRAULIC GRADIENT AND TOTAL ENERGY LINE ⇒

The Concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as:

Hydraulic Gradient Line ⇒

It is defined as the line which gives the sum of pressure head $\left(\frac{P}{W}\right)$ and datum head (Z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a ~~fluid~~ flowing fluid in a pipe from the Centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

Total Energy Line ⇒

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the Centre of the pipe. It is briefly written as T.E.L (Total Energy Line).

{CHAPTER-07}

Introduction

The liquid comes out in the form of a jet from the outlet of a nozzle, which is ~~fixe~~ fitted to a pipe through which the ~~fluid~~ liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's 2nd law of motion or from impulse-momentum equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving. In this chapter, the following cases of the impact of jet i.e. the force exerted by the jet on a plate, will be considered.

(1) Force exerted by the jet on a stationary plate when

(a) plate is vertical to jet

(b) plate is inclined to the jet, and

(c) plate is curved.

(2) Force exerted by the jet on a moving plate, when

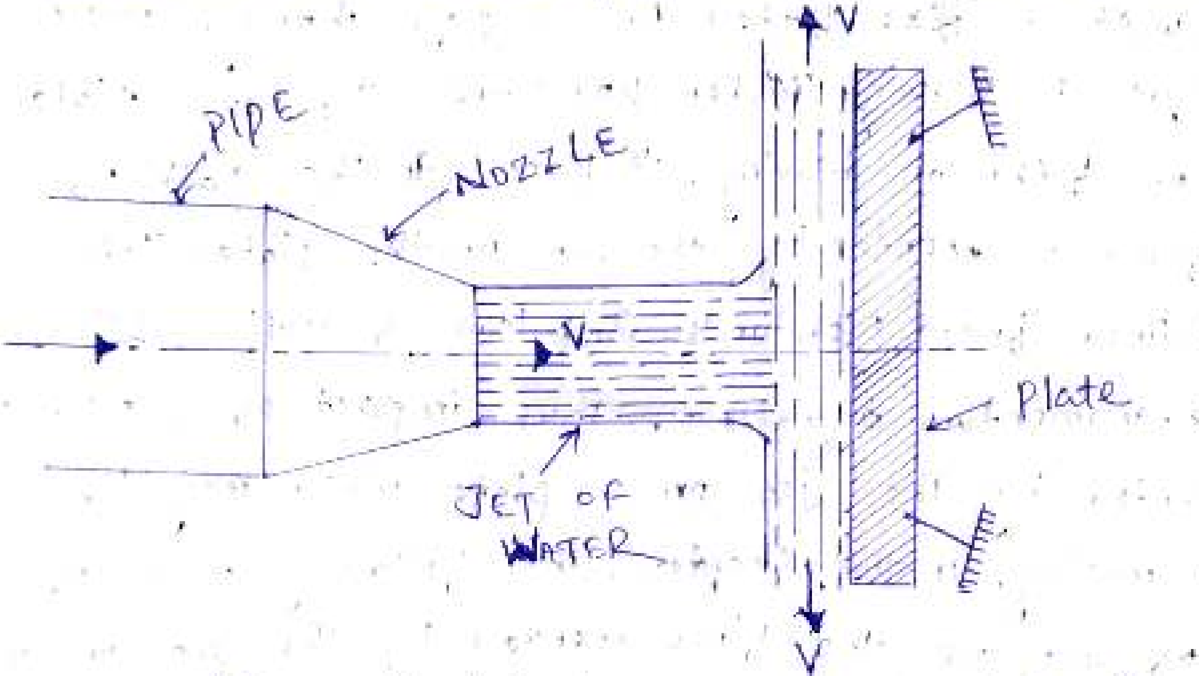
(a) plate is vertical to jet,

(b) plate is inclined to the jet and

(c) plate is curved.

Force Exerted By The Jet On a Stationary Vertical Plate :-

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in the below figure.



(Force exerted by jet on vertical plate)

Let V = velocity of the jet
d = diameter of the jet
a = area of cross section of the jet $= \frac{\pi}{4} d^2$

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet.
Hence the jet after striking, will get deflected through 90°.
Hence the component of the velocity of jet in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet.

F_x = Rate of change of momentum in the direction of force

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} (\text{Initial velocity} - \text{final velocity})$$

$= (\text{Mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$

$= \rho a V (V - 0)$ ($\because$ mass/sec $= \rho \times aV$)

$= \rho a V^2$ ——(1)

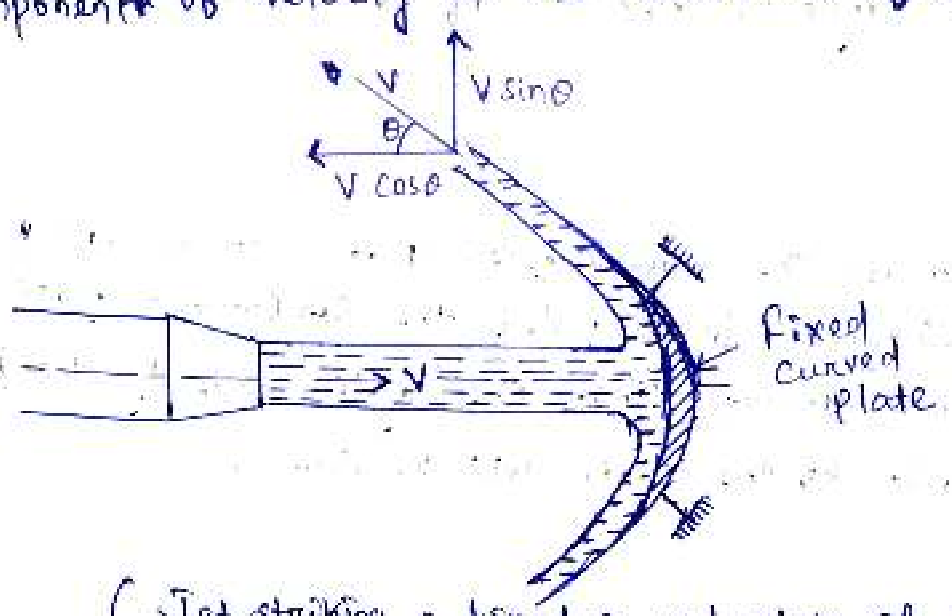
Force Exerted by a jet on Stationary Curved plate →

(A) Jet strikes the curved plate at the Centre :-

Let a jet of water strikes a fixed curved plate at the Centre as shown in the below figure. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at outlet of the plate can be resolved into two components.

One in the direction of jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of jet $= -V\cos\theta$



(Jet striking a fixed curved plate at centre)

(-ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from Nozzle).

Component of velocity perpendicular to the jet $= V \sin\theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{mass per sec} \times [V_{1x} - V_{2x}]$$

where, V_{1x} = Initial velocity in the direction of jet $= V$

V_{2x} = Final velocity in the direction of jet $= -V\cos\theta$

$$\therefore F_x = \rho a V[V - (-V\cos\theta)]$$

$$= \rho a V[V + V\cos\theta]$$

$$= \rho a V^2[1 + \cos\theta] \quad \text{--- (2)}$$

Similarly, $F_y = \text{mass per sec} \times [V_{1y} - V_{2y}]$

where, V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V\sin\theta$

$$\therefore F_y = \rho a V[0 - V\sin\theta]$$

$$= -\rho a V^2 \sin\theta \quad \text{--- (3)}$$

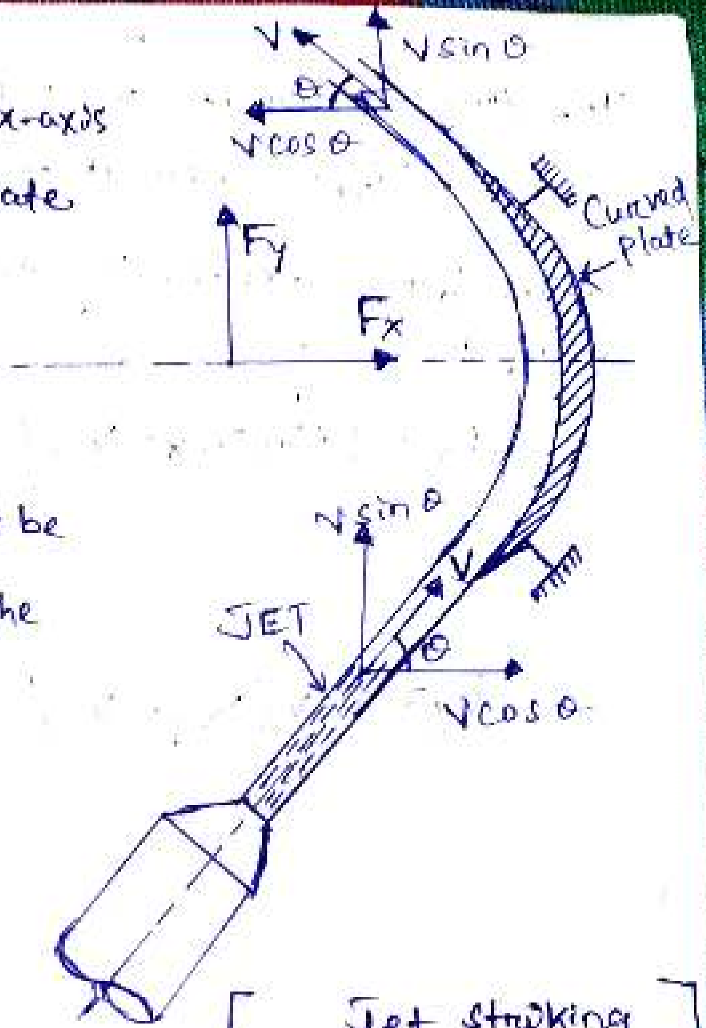
-ve sign means that force is acting in the downward direction. In this case the angle of deflection of jet $= (180° - \theta)$

(B) Jet strikes the curved plate at one end tangentially when the plate is symmetrical :-

Let the jet strikes the curved fixed plate at one end tangentially as shown in figure. Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let V = velocity of jet of water

θ = Angle made by jet with x-axis at inlet tip of the curved plate



If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the dimensions of x and y are

$$F_x = (\text{mass/sec}) \times [V_{1x} - V_{2x}]$$

$$= \rho a V[V\cos\theta - (-V\cos\theta)]$$

$$= \rho a V[V\cos\theta + V\cos\theta]$$

$$= 2\rho a V^2 \cos\theta \quad \text{——— (4)}$$

$$F_y = \rho a V[V_{1y} - V_{2y}]$$

$$= \rho a V[V\sin\theta - V\sin\theta] = 0$$

[Jet striking curved fixed plate at one end]

(C) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical :-

When the curved plate is unsymmetrical about x-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angle made by tangent at inlet tip with x-axis.

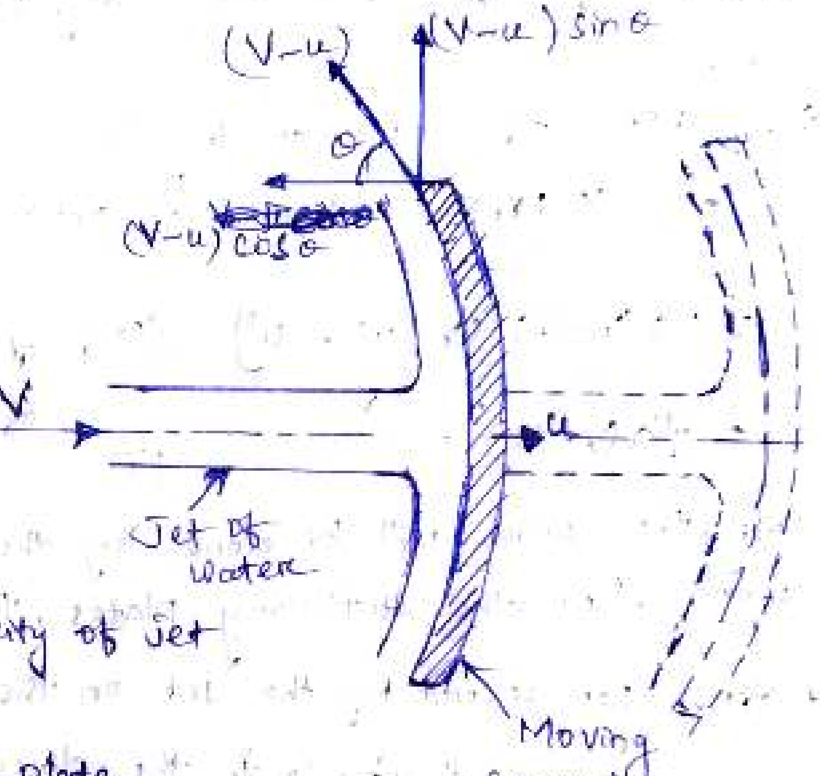
φ = angle made by tangent at outlet tip with x-axis

The two components of the velocity at inlet are

$$V_{1x} = V\cos\theta \text{ and } V_{1y} = V\sin\theta$$

Force on the Curved plate when the plate is moving in the direction of Jet :-

Let a jet of water strikes a Curved plate at the Centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in the figure.



(Jet striking a Curved moving plate)

Let V = Absolute velocity of jet

a = area of Jet

u = velocity of the plate in the direction of the jet

- Relative velocity of the jet of water or the velocity with which jet strikes the curved plate = $(V-u)$.
- If plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the Curved vane = $(V-u)$.
- This velocity can be restored into two components, one in the direction of the jet and other perpendicular to the direction of the Jet

Component of the velocity in the direction of Jet

$= -(V-u)\cos\theta$

(-ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of the velocity in the direction perpendicular to the direction of the jet $= (V-u)\sin\theta$

Mass of the water striking the plate

$= \rho \times a \times$ Velocity with which jet strikes the plate

$= \rho a(V-u)$.

∴ Force exerted by the jet of water on the curved plate in the direction of the jet,

F_x = mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet − Final velocity]

$= \rho a(V-u)[(V-u)-(-(V-u)\cos\theta)]$

$= \rho a(V-u)[(V-u)+(V-u)\cos\theta]$

$= \rho a(V-u)^2[1+\cos\theta]$ —— (9)

Workdone by the jet on the plate per second

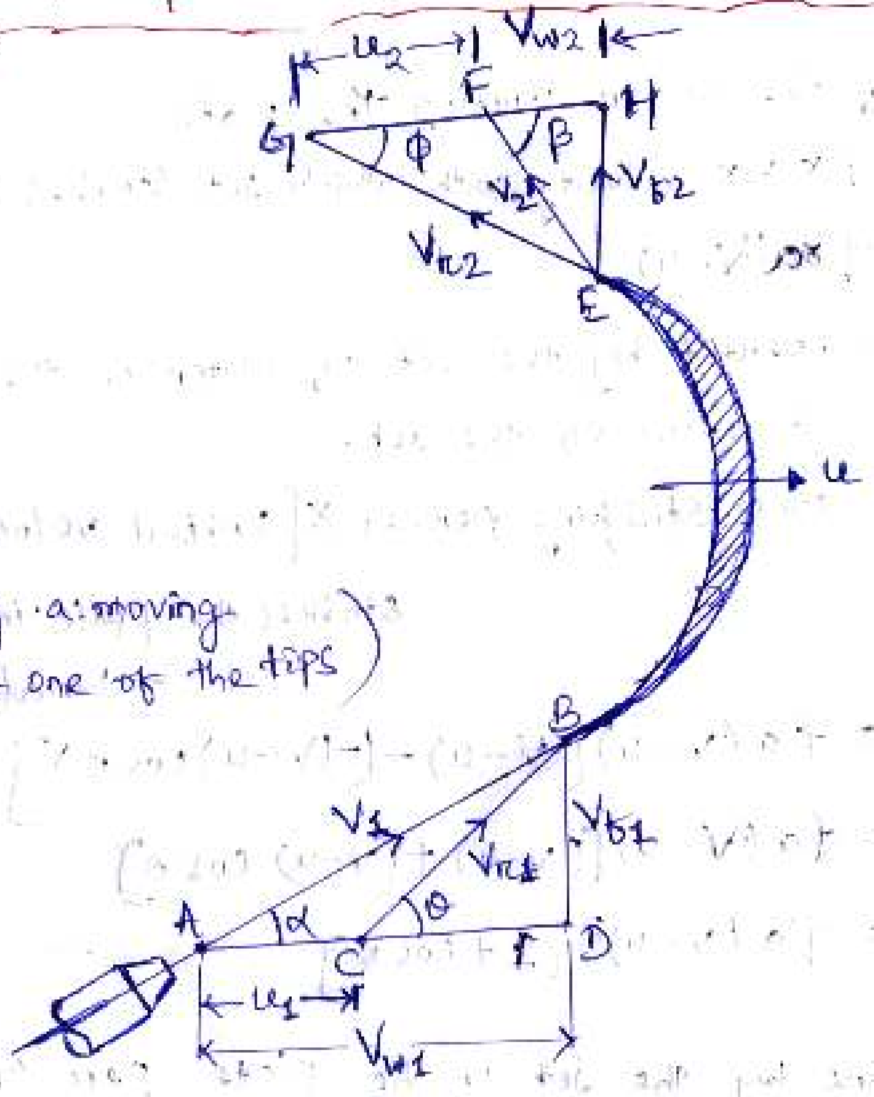
$= F_x \times$ Distance travelled per second in the direction of x

$= F_x \times u$

$= \rho a(V-u)^2[1+\cos\theta]u$

$= \rho a(V-u)^2 \times u[1+\cos\theta]$ —— (10)

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved plate when Jet strikes Tangentially at one of the Tips :-



(Jet striking a moving Curved Vane at one of the tips)

The above figure shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

Let V_1 = Velocity of the jet at inlet

u_1 = Velocity of the plate (Vane) at inlet

V_{r1} = Relative velocity of jet and plate at inlet

α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle

θ = Angle made by the relative velocity (V_{r1}) with the direction of motion at inlet also called Vane angle at inlet

V_{w1} and V_{f1} = The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the Vane respectively.

V_{w1} = It is also known as velocity of whirl at inlet

V_{f1} = It is also known as velocity of flow at inlet

V_2 = velocity of the jet, leaving the vane or velocity of jet at outlet of the vane.

u_2 = velocity of the vane at outlet

V_{r2} = Relative velocity of jet with respect to the vane at outlet

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet.

ϕ = Angle made by the relative velocity (V_{r2}) with the direction of motion of the vane at outlet and also called vane angle at outlet

V_{w2} and V_{f2} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet

V_{w2} = It is also called the velocity of whirl at outlet

V_{f2} = Velocity of flow at outlet

The triangles ABD and EGH are called the velocity triangles at inlet and outlet. These velocity triangles are drawn as given below :-

(1) Velocity Triangle at Inlet :-

Take any point A and draw a line AB = V_1 in magnitude and direction which means line AB makes an angle α with the horizontal line AD. Next draw a line AC = u_1 in magnitude. Join C to B. Then CB represents the relative velocity of the jet at inlet. If the loss of energy at inlet due to impact is zero, then CB must be in the tangential direction to the vane at inlet. From B draw a vertical line BD in the downward direction to meet the horizontal line AC produced at D.

Then BD = Represents the velocity of flow at inlet = V_{f1}

AD = Represents the velocity of whirl at inlet = V_{w1}

$\angle BCD$ = Vane angle at inlet = θ

$\angle BAC = \alpha$

(2) Velocity Triangle at Outlet :-

If the vane surface is assumed to be very smooth, the loss of energy due to friction will be zero. The water will be gliding over the surface of the vane with a relative velocity equal to V_{r1} and will come out of the vane with a relative velocity V_{r2}. This means that the relative velocity at outlet $V_{r2} = V_{r1}$. And also the relative velocity at outlet should be in tangential direction to the vane at outlet.

Draw EG in the tangential direction of the Vane at outlet and cut $EG = V_{r2}$.

From G, draw a line GF in the direction of vane at outlet and equal to u_2, the velocity of the vane at outlet.

Join EF, then EF represents the absolute velocity of the jet at outlet in magnitude and direction. From E draw a vertical line EH to meet the line GF produced at H.

Then EH = velocity of flow at outlet $= V_{f2}$

FH = velocity of whirl at outlet $= V_{w2}$

$\angle EGF = \phi$ = Angle of vane at outlet

$\angle EFH = \beta$ = Angle made by V_2 with the direction of motion of vane at outlet.

If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal then we have

$u_1 = u_2 = u$ = velocity of vane in the direction of motion

and $V_{r1} = V_{r2}$.

Now mass of water striking vane per sec $= \rho a V_{r1}$ —(1)

where a = Area of jet of water

V_{r1} = Relative velocity at inlet

∴ Force exerted by the jet in the direction of motion

F_x = mass of water striking per sec × [Initial velocity with which jet strikes in the direction of motion − Final velocity of jet in the direction of motion] —(2)

But initial velocity with which jet strikes the vane $= V_{r1}$

The component of this velocity in the direction of motion

$= V_{r1} \cos\theta = (V_{w1} - u_1)$

Similarly, the components of the relative velocity at outlet in the direction of motion $= -V_{r2}\cos\phi$

$$= -[u_2 + V_{w2}]$$

−ve sign is taken as the component of V_{r2} in the direction of motion is in the opposite direction.

Substituting the equation (1) and all above values of the velocities in equation (2), we get

$$F_x = \rho a V_{r1}[(V_{w1} - u_1) - \{-(u_2 + V_{w2})\}]$$

$$= \rho a V_{r1}[V_{w1} - u_1 + u_2 + V_{w2}]$$

$$= \rho a V_{r1}[V_{w1} + V_{w2}] \quad (\because u_1 = u_2) \quad \text{— (3)}$$

Equation (3) is true only when angle β shown in figure is an acute angle. If $\beta = 90°$, the $V_{w2} = 0$, then equation (3) becomes as, $F_x = \rho a V_{r1}[V_{w1}]$

If β is an obtuse angle, the expression for F_x will become

$$F_x = \rho a V_{r1}[V_{w1} - V_{w2}]$$

Thus in general, F_x is written as $F_x = \rho a V_{r1}[V_{w1} \pm V_{w2}]$

Work done per second on the vane by the jet

= Force x Distance per second in the direction of force

$= F_x \times u$

$$= \rho a V_{r1}[V_{w1} \pm V_{w2}] \times u \quad \text{— (4)}$$

∴ Work done per second per unit weight of fluid striking per second

$$= \frac{\rho a V_{r1}[V_{w1} \pm V_{w2}] \times u}{\text{weight of fluid striking/s}} \quad \frac{Nm/s}{N/s}$$

$$= \frac{\rho a V_{r1}[V_{w1} \pm V_{w2}] \times u}{g \times \rho a V_{r1}} \quad \frac{Nm}{N}$$

$$= \frac{1}{g}\left[V_{w_1} \pm V_{w_2}\right] \times u \;\; Nm/N \qquad \text{---(5)}$$

Work done/sec per unit mass of fluid striking per second

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\text{Mass of fluid striking/s}} \quad \frac{Nm/s}{kg/s}$$

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\rho a V_{r_1}} \quad \frac{Nm}{kg}$$

$$= \left(V_{w_1} \pm V_{w_2}\right) \times u \quad Nm/kg \qquad \text{--- (6)}$$

(3) Efficiency of JET →

The work done by the jet on the vane given by equation (4) is the output of the jet whereas the initial kinetic energy of the jet is the input. Hence, the efficiency (η) of the jet is expressed as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second on the vane}}{\text{Initial Kinetic Energy per second of the jet}}$$

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\frac{1}{2} m V_1^2}$$

where m = mass of the fluid per second in the jet $= \rho a V_1$

V_1 = initial velocity of jet

$$\therefore \eta = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2} \qquad \text{--- (7)}$$