

Chapter-1

Introduction

Mechanics of soil → properties
 → Tests
 → Application.

Soil:

Soil is the unaggregated or uncemented deposits of minerals or organic particles or fragments covering large portion of earth crust. It includes,

- Gravels & boulders
- Sands
- clays
- silts

Soil mechanics:

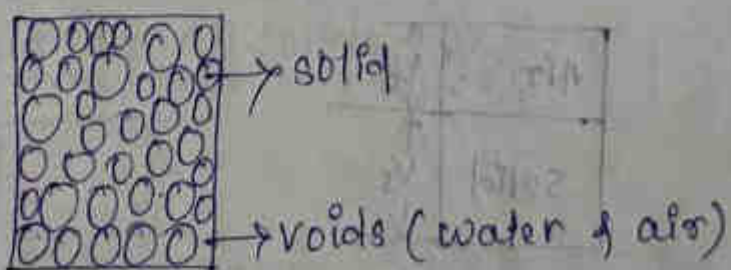
Soil mechanics is the application of laws of mechanics and hydraulics mechanics to engineering problems dealing with the sediments and other unconsolidated accumulations of solid particles produced by the mechanical & chemical ^(increase in natural growth) disintegration of rocks.

Chapter-2

Preliminary definitions and Relationship

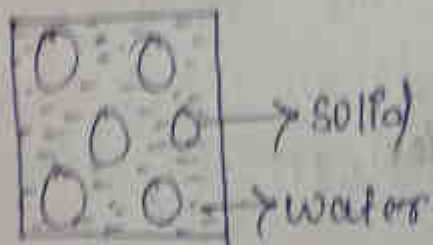
Phase system of soil : → 2 phase system
 → 3 phase system

2 phase system →

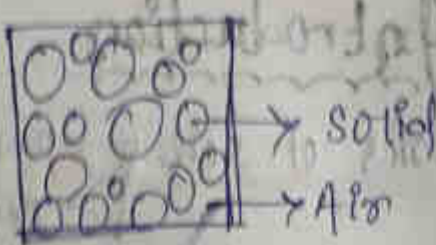


(2-phase system)

(e.g. fully saturated soil)



(Wet soil)



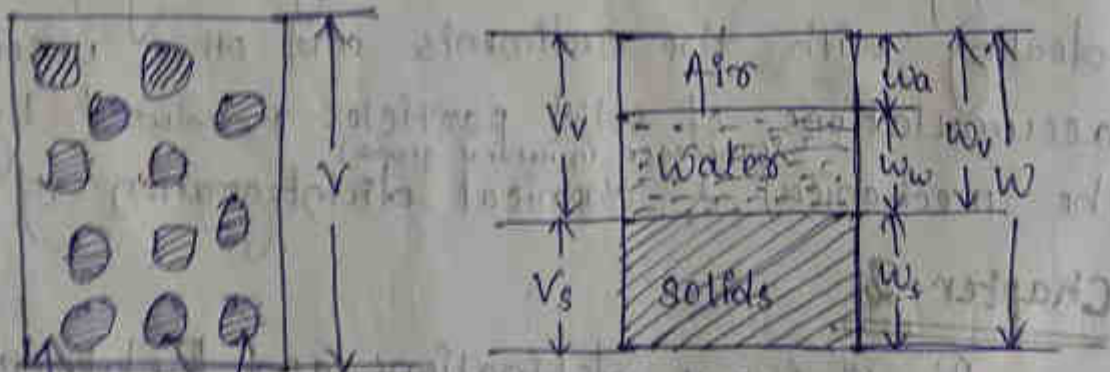
(Dry soil)

3-phase system:

A soil mass is a 3-phase system consisting of solid particles (soil grains), water & air. The void space between the soil grains is filled partially with water and partially with air.

→ If it is dry soil mass the voids are filled with air only.

→ In case of a fully saturated soil the voids are completely filled with water.

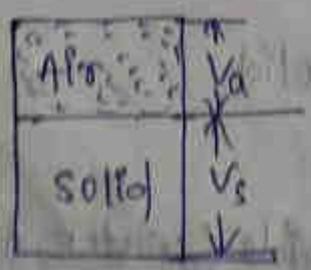


Voids Solids ① (Vol^m) ② (weight)

(Dry soil mass)

$$V_v = V_a$$

$$V_v = V_w$$



Weight - Volume relationship: —

(a) Weight relations →

① Water content / moisture content (w):

It is defined as the ratio of weight of water to the ~~ratio~~ weight of solids in a given mass of soil.

$$w = \frac{W_w}{W_s} \times 100 \quad (\text{Unit: Unitless})$$

② Density of soil (f):

$$f = \frac{\text{Mass}}{\text{Volume}} = \text{gm/cm}^3, \text{kg/m}^3$$

③ Bulk density (f) / moist density: —

→ It is the total mass of the soil per unit of its total volume.

$$f = \frac{M}{V} = \frac{\text{Total mass of soil}}{\text{Total vol}^m \text{ of soil.}}$$

Unit: kg/m^3

④ Dry density (f_d): —

It is the mass of soil solids per unit of its total volume of the soil mass.

$$f_d = \frac{M_d}{V}$$

⑤ Density of solids (f_s): —

$$f_s = \frac{M_s}{V_s}$$

⑥ Saturated density (f_{sat}): —

When the soil mass is saturated its bulk density

is called as saturated density.

$$\rho_{sat} = \frac{\text{Mass of saturated soil}}{\text{Total vol}^m \text{ of soil.}}$$

⑦ Submerged density (ρ_{sub}): — (ρ')

It is the submerged mass of soil solid per unit of total volume of soil mass.

$$\rho_{sub} = \frac{(M_s)_{sub}}{V}$$

⑧ Unit weight of soil (γ): — (Bulk)

→ It is the total weight of soil mass per its total volume of soil mass.

Bulk unit weight / moist unit weight

$$\gamma = \frac{W}{V}$$

⑨ Dry unit weight (γ_d): —

It is the weight of dry soil solid per unit of total volume of soil.

$$\gamma_d = \frac{W_s}{V}$$

⑩ Unit weight of solid (γ_s): —

$$\gamma_s = \frac{W_s}{V_s}$$

⑪ Saturated unit weight (γ_{sat}): —

$$\gamma_{sat} = \frac{\text{Weight of saturated soil}}{\text{Total vol}^m \text{ of soil}} = \frac{W_s + W_w}{V}$$

(12) Submerged unit weight (γ_{sub}) :-

$$\gamma' = \gamma_{sub} = \frac{(W_s)_{sub}}{V}$$

↳ Specific Gravity (G_s) :-

It is defined as the ratio of the weight of a given volume of soil solid at a given temp. to a weight of an equal volume of water at that temp.

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{W_s/V_s}{W_w/V_w}$$

↳ Volume relationship :-

(1) Voids ratio (e) :-

It is the ratio of volume of voids to the vol^m of soil solids in a given soil mass.

$$e = \frac{V_v}{V_s} = \frac{\text{Vol}^m \text{ of voids}}{\text{Vol}^m \text{ of solids}}$$

(2) Porosity (n) :-

It is the ratio of volume of voids to total vol^m of the given soil mass.

$$n = \frac{V_v}{V}$$

* Relation between e & n :-

$$e = \frac{V_v}{V_s}, \quad n = \frac{V_v}{V} = \frac{V_v}{V_s + V_v} \quad (\because V = V_s + V_v)$$

$$\Rightarrow n = \frac{V_v/V_s}{V_s/V_s + V_v/V_s} = \frac{e}{1+e} \Rightarrow \boxed{n = \frac{e}{1+e}}$$

$$e = \frac{V_v}{V_s}, \quad \eta = \frac{V_v}{V}$$

$$= \frac{V_v}{V - V_v} \quad (\because V = V_s + V_v \Rightarrow V_s = V - V_v)$$

$$\Rightarrow e = \frac{V_v}{V - V_v} \Rightarrow \frac{1}{e} = \frac{V - V_v}{V_v} = \frac{V}{V_v} - \frac{V_v}{V_v}$$

$$\Rightarrow \frac{1}{e} = \frac{1}{\eta} - 1 \Rightarrow \boxed{e = \frac{\eta}{1 - \eta}}$$
$$= \frac{1 - \eta}{\eta}$$

* Degree of Saturation (S) :-

→ In a given vol^m of voids of a sample some space is occupied by water and raised by air.

→ In a fully saturated sample the voids that completely filled with water.

→ Degree of saturation is defined as the ratio of vol^m of water present in a soil mass to the total volume of voids in it.

$$\boxed{S = \frac{V_w}{V_v}}$$

→ For a fully saturated sample, $\boxed{S = 1}$ ($V_w = V_v$)

→ For a perfectly dry sample, $\boxed{S = 0}$ ($V_w = 0$)

* Percentage of air voids (η_a) :-

It is the ratio of volume of air voids to the total volume of soil mass.

$$\boxed{\eta_a = \frac{V_a}{V}}$$

* Air Content (a_c):—

It is the ratio of volume of air voids to the total volume of voids.

$$a_c = \frac{V_a}{V_v}$$

$$\Rightarrow V_a = V_v - V_w$$

$$\because V = V_s + V_v, \quad \boxed{V_w = V_a + V_w}$$

* Relation between 's' & 'a_c':—

$$s = \frac{V_w}{V_v}, \quad a_c = \frac{V_a}{V_v} = \frac{V_v - V_w}{V_v} \quad (\because V_v = V_a + V_w)$$

$$= 1 - \frac{V_w}{V_v} = 1 - s$$

$$\Rightarrow \boxed{a_c = 1 - s}$$

↳ Functional Relationship:—

① Relation between e, G, w & s:

$$e = \frac{V_v}{V_s}, \quad G = \frac{\gamma_s}{\gamma_w}, \quad w = \frac{W_w}{W_s}, \quad s = \frac{V_w}{V_v}$$

$$\Rightarrow G = \frac{\gamma_s}{\gamma_w} = \frac{W_s/V_s}{W_w/V_w} = \frac{W_s \times V_w}{W_w \times V_s} = \frac{V_w}{V_s \times \frac{W_w}{W_s}}$$

$$= \frac{V_w}{V_s \times w} = \frac{V_w/V_v}{V_s \times w/V_v}$$

$$= \frac{s}{w \times 1/e} = \frac{se}{w}$$

$$\Rightarrow \boxed{G = \frac{se}{w}}$$

$$\Rightarrow \boxed{wG = se}$$

② Relation between e, s & η_a :

$$e = \frac{V_v}{V_s}, \quad s = \frac{V_w}{V_v}, \quad \eta_a = \frac{V_a}{V}$$

$$\Rightarrow \eta_a = \frac{V_a}{V} = \frac{V_a}{V_s + V_v} \quad (\because V = V_s + V_v)$$

$$= \frac{V_v - V_w}{V_s + V_v} \quad (\because V_v = V_a + V_w)$$

$$\Rightarrow \eta_a = \frac{V_v/V_v - V_w/V_w}{V_s/V_v + V_v/V_v} = \frac{1-s}{1+e}$$

$$\Rightarrow \eta_a = \frac{1-s}{\frac{1+e}{e}} = \frac{(1-s)e}{1+e}$$

$$\Rightarrow \boxed{\eta_a = \frac{e(1-s)}{1+e}}$$

③ Relation between η_a , a_c , η :

$$\eta_a = \frac{V_a}{V}, \quad a_c = \frac{V_a}{V_v}, \quad \eta = \frac{V_v}{V}$$

$$\Rightarrow a_c = \frac{V_a}{V_v} = \frac{V_a/V}{V_v/V} = \frac{\eta_a}{\eta}$$

$$\Rightarrow \boxed{a_c \times \eta = \eta_a}$$

④ Relation between γ_d , G , e (or η):

$$\gamma_d = \frac{W_s}{V}, \quad G = \frac{\gamma_s}{\gamma_w}, \quad e = \frac{V_v}{V_s}$$

$$\gamma_d = \frac{W_s}{V} = \frac{\gamma_s \times V_s}{V}$$

$$\Rightarrow \gamma_d = \frac{G \gamma_w \times V_s}{V}$$

$$= \frac{G \gamma_w \times V_s}{V_s} \times \frac{V_s}{V} = \frac{G \gamma_w}{V/V_s} = \frac{G \gamma_w}{\frac{V_s + V_v}{V_s}}$$

$$\Rightarrow \boxed{\gamma_d = \frac{G \gamma_w}{1+e}}$$

$$\boxed{\gamma_d = \frac{G \gamma_w}{1+e}}$$

$$\gamma_d = \frac{W_s}{V}, \quad G = \frac{\gamma_s}{\gamma_w}, \quad \eta = \frac{V_v}{V}$$

$$\Rightarrow \gamma_d = \frac{W_s}{V} = \frac{\gamma_s \times V_s}{V}$$

$$= \frac{G \gamma_w \times V_s}{V} = \frac{G \gamma_w [V - V_v]}{V} = G \gamma_w \left[1 - \frac{V_v}{V} \right]$$

$$= G\gamma_w(1-n)$$

$$\Rightarrow \boxed{\gamma_d = G\gamma_w(1-n)}$$

⑤ Relation between γ_{sat} , G , e (or n):

$$\gamma_{sat} = \frac{W_{sat}}{V} = \frac{W_s + W_w}{V}, \quad G = \frac{\gamma_s}{\gamma_w}, \quad e = \frac{V_v}{V_s}$$

$$\Rightarrow \gamma_{sat} = \frac{W_{sat}}{V} = \frac{W_s + W_w}{V} = \frac{(\gamma_s \times V_s) + (\gamma_w \times V_w)}{V} = \frac{(G\gamma_w \times V_s) + \gamma_w V_w}{V}$$

$$= \frac{G\gamma_w V_s + \gamma_w V_w}{V} = \frac{G\gamma_w V_s + \gamma_w V_w}{V_s + V_v}$$

$$= \frac{G\gamma_w V_s / V_s + \frac{\gamma_w V_w}{V_s}}{V_s / V_s + V_v / V_s} = \frac{G\gamma_w + \gamma_w e}{1 + e}$$

$$\Rightarrow \boxed{\gamma_{sat} = \frac{G\gamma_w + \gamma_w e}{1 + e}}, \quad G = \frac{\gamma_s}{\gamma_w}, \quad \eta = \frac{V_v}{V}$$

$$\therefore \gamma_{sat} = \frac{W_s + W_w}{V} = \frac{(\gamma_s \times V_s) + (\gamma_w \times V_w)}{V}$$

$$= \frac{G\gamma_w \times V_s + \gamma_w V_w}{V} = \frac{G\gamma_w V_s + \gamma_w V_w}{V}$$

$$= \frac{G\gamma_w (V - V_v) + \gamma_w V_v}{V} = \frac{G\gamma_w (1 - \eta) + \gamma_w \eta}{1}$$

$$\Rightarrow \boxed{\gamma_{sat} = G\gamma_w(1-\eta) + \gamma_w \eta}$$

⑥ Relation between γ , G , e & S :

$$\gamma = \frac{W}{V}, \quad G = \frac{\gamma_s}{\gamma_w}, \quad e = \frac{V_v}{V_s}, \quad S = \frac{V_w}{V_v}$$

$$\Rightarrow \gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s + (W_w^0 + W_w)}{V}$$

$$= \frac{(\gamma_s \times V_s) + (\gamma_w \times V_w)}{V}$$

$$= \frac{G\gamma_w V_s + \gamma_w V_w}{V_s + V_w} = \frac{G\gamma_w V_s/V_v + \gamma_w V_w/V_v}{V_s/V_v + V_w/V_v}$$

$$= \frac{G\gamma_w \left(\frac{1}{e}\right) + \gamma_w}{\left(\frac{1}{e}\right) + 1}$$

$$= \left(\frac{G\gamma_w + \gamma_w e}{e}\right) / \left(\frac{1+e}{e}\right) = \frac{G\gamma_w + \gamma_w e}{1+e}$$

$$\Rightarrow \boxed{\gamma = \frac{(G+e)\gamma_w}{1+e}}$$

(7) Relation between γ' , G & e :

$$\boxed{\gamma' = \gamma_{sat} - \gamma_w}, \quad G = \frac{\sigma_s}{\gamma_w}, \quad e = \frac{V_w}{V_s}$$

$$\Rightarrow \gamma' = \gamma_{sat} - \gamma_w$$

$$= \frac{\gamma_w(G+e)}{1+e} - \gamma_w$$

$$= \frac{G\gamma_w + e\gamma_w - \gamma_w - e\gamma_w}{1+e} = \frac{G\gamma_w - \gamma_w}{1+e}$$

$$\Rightarrow \boxed{\gamma' = \frac{G\gamma_w - \gamma_w}{1+e}}$$

$$\Rightarrow \gamma' = \frac{(G-1)\gamma_w}{1+e}$$

(8) Relation between γ_d , γ & w :

$$\gamma_d = \frac{w_s}{V}, \quad \gamma = \frac{w}{V}, \quad w = \frac{w_w}{w_s}$$

$$\Rightarrow \gamma = \frac{w}{V} = \frac{w_s + w_w}{V} = \frac{w_s}{V} + \frac{w_w}{V}$$

$$= \gamma_d + \frac{w_w}{V}$$

$$= \gamma_d + \frac{w_w}{w_s/\gamma_d} = \gamma_d + \frac{w_w}{w_s} \times \gamma_d$$

$$= \gamma_d [1 + w]$$

$$\Rightarrow \gamma = \gamma_d (1 + w)$$

$$\Rightarrow \boxed{\gamma_d = \frac{\gamma}{(1+w)}}$$

9) Relation between r_d, n, w & S :-

$$r_d = \frac{w_s}{V}, \quad n = \frac{r_s}{r_w}, \quad w = \frac{w_w}{w_s}, \quad S = \frac{V_w}{V_s}$$

$$\Rightarrow r_d = \frac{w_s}{V} = \frac{r_s \times V_s}{V_s + V_w}$$

$$= \frac{n r_w \times V_s}{V_s + V_w} = \frac{n r_w V_s / V_s}{V_s / V_s + V_w / V_s} = \frac{n r_w}{1 + e}$$

$$\Rightarrow \boxed{r_d = \frac{n r_w}{1 + \frac{w_w}{S}}}$$

10) Relation between r', r_d & n :-

$$r' = \frac{(n-1)r_w}{1+e} = \frac{(n-1)r_w}{1 + \frac{D}{1-n}} \quad \left(\because e = \frac{n}{1-n} \right)$$

$$= \frac{(n-1)r_w}{\frac{1-n+n}{1-n}} = (n-1)r_w(1-n)$$

$$= \left(\frac{r_s}{r_w} - 1 \right) r_w (1-n) \quad \left(\because n = \frac{r_s}{r_w} \right)$$

$$= \frac{r_s - r_w}{r_w} \times r_w (1-n) = (r_s - r_w)(1-n)$$

$$\Rightarrow r' = \frac{(n-1)r_w}{1+e} = \frac{n r_w}{1+e} - \frac{r_w}{1+e}$$

$$= r_d - \frac{r_w}{1+e} = r_d - \frac{r_w}{1 + \frac{D}{1-n}}$$

$$= r_d - \frac{r_w}{\frac{1-n+n}{1-n}} = r_d - r_w(1-n)$$

$$\Rightarrow \boxed{r' = r_d - (1-n)r_w}$$

11) Relation between r_{sat}, r, r_d & S :-

$$r = \frac{(n+e)r_w}{1+e} = \frac{n r_w}{1+e} + \frac{e r_w}{1+e}$$

$$= r_d + \frac{e r_w}{1+e}$$

$$\left[r_{sat} = \frac{w_s + w_w}{V}, \quad r = \frac{w}{V}, \quad r_d = \frac{w_s}{V}, \quad s = \frac{w_w}{V} \right]$$

$$\Rightarrow r_{sat} = \frac{(r + e) r_w}{1 + e} = \frac{r r_w}{1 + e} + \frac{e r_w}{1 + e}$$

$$= r_d + \frac{e r_w}{1 + e}$$

$$\therefore r = r_d + s \left[\frac{e r_w}{1 + e} \right]$$

$$\Rightarrow \boxed{r = r_d + s (r_{sat} - r_d)}$$

(2) Relation between r_d, G, w, η_a :-

$$r_d = \frac{w_s}{V}, \quad G = \frac{r_s}{r_w}, \quad w = \frac{w_w}{w_s}, \quad \eta_a = \frac{V_a}{V}$$

$$\therefore V = V_a + V_w + V_s$$

$$\Rightarrow V = V_a + \frac{w_w}{r_w} + \frac{w_s}{r_s}$$

$$\Rightarrow 1 = \frac{V_a}{V} + \frac{w_w}{r_w V} + \frac{w_s}{r_s V}$$

$$\Rightarrow 1 = \eta_a + \frac{w \times w_s}{r_w V} + \frac{r_d}{r_s} \quad \left(\because r_d = \frac{w_s}{V} \right)$$

$$\left(w = \frac{w_w}{w_s} \right)$$

$$\Rightarrow w_w = w \times w_s$$

$$\Rightarrow (1 - \eta_a) = \frac{w \times r_d}{r_w} + \frac{r_d}{r_s}$$

$$\Rightarrow (1 - \eta_a) \times r_w = \frac{w \times r_d}{r_w} \times r_w + \frac{r_d}{r_s} \times r_w$$

$$\Rightarrow (1 - \eta_a) r_w = w r_d + \frac{r_d}{r_s / r_w}$$

$$= w r_d + \frac{r_d}{G}$$

$$\Rightarrow (1 - \eta_a) G r_w = (w + 1) r_d \quad G w r_d + r_d$$

$$= r_d (G w + 1)$$

$$\Rightarrow \boxed{r_d = \frac{(1 - \eta_a) G r_w}{1 + w G}}$$

Density Relations :-

$$(1) f_d = \frac{G f_w}{1+e}$$

$$(2) f_d = (1-n) G f_w$$

$$(3) f_{sat} = G f_w (1-n) + f_w$$

$$(4) f_{sat} = \frac{(G+e) f_w}{1+e}$$

$$(5) f = \frac{(G+eS) f_w}{1+e}$$

$$(6) f_d = \frac{f}{1+w}$$

$$(7) f' = \frac{(G-1) f_w}{1+e}$$

$$(8) f' = f_d - (1-n) f_w$$

$$(9) f = f_d + S (f_{sat} - f_d)$$

$$(10) f_d = \frac{G f_w}{1 + \frac{wG}{S}}$$

$$(11) f_d = \frac{(1-n_a) G f_w}{1+wG}$$

Q.1

A soil sample has a porosity of 40%. The specific gravity of solids is 2.7. Calculate

(a) void ratio.

(b) Dry density, dry unit weight.

(c) Unit weight if the soil is 50% saturated.

(d) Unit wt. of the soil is completely saturated.

Solⁿ: $n = 40\% = 0.4$

$$G = 2.7$$

$$(a) e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.66$$

$$(b) f_d = \frac{G f_w}{1+e} = \frac{2.7 \times 1}{1+0.66} = 1.626 \text{ g/cm}^3$$

$$(c) \gamma_d = \frac{G \gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.66} = 15.95 \text{ kN/m}^3$$

$$(d) \gamma = \frac{(G+e) \gamma_w}{1+e}$$

$$S = 50\% = 0.5$$

$$\Rightarrow \gamma = \frac{(2.7 + 0.66 \times 0.5) 9.81}{1+0.66} = 17.9$$

$$(d) S = 100\% = 1.00$$

$$\gamma = \frac{(2.7 + 0.66 \times 1) 9.81}{1+0.66} = 19.85 \text{ (Ans)}$$

Q-2) An undisturbed sample of soil has a volume of 100cm^3 & a mass of 190g on oven drying for 24 hrs. The mass is reduced to 160g . If the specific gravity of soil is 2.68 . Determine the water content, void ratio & degree of saturation of soil.

Solⁿ Data given \rightarrow

$$V = 100\text{cm}^3, \quad G_s = 2.68$$

$$M = 190\text{g}, \quad M_s = 160\text{g}$$

$$M_w = 190 - 160 = 30\text{g}$$

$$\therefore w = \frac{M_w}{M_s} = \frac{M_w \times g}{M_s \times g} = \frac{30}{160} = 0.1875$$

$$= 18.75\%$$

$$\rho = \frac{M}{V} = \frac{190}{100} = 1.9\text{g/cm}^3$$

$$\rho_d = \frac{\rho}{1+w} = \frac{1.9}{1+0.1875} = 1.6$$

$$\rho_d = \frac{G_s \rho_w}{1+e} \Rightarrow 1.6 = \frac{2.68 \times 1}{1+e}$$

$$\Rightarrow e = \frac{2.68 \times 1}{1.6} - 1 = 0.675$$

S = ?

$$\Rightarrow eS = wG_s \Rightarrow S = \frac{wG_s}{e} = \frac{0.1875 \times 2.68}{0.675} = 0.75 \text{ (Ans)}$$

Q-3 The in-situ density of an embankment compacted at a water content of 12% was determined with the help of core cutter. The empty mass of the cutter was 1286g & the cutter full of soil had a mass of 3195g , the volume of the cutter being 1000cm^3 . Determine the bulk density, dry density & degree of saturation.

\rightarrow If the embankment becomes loose

rains, what would be its water content and saturated unit weight. Assume no volume change in soil on saturation. Take the sp. gravity of soil as 2.7.

Solⁿ:

Given \rightarrow $M_{\text{cutter}} = 1286 \text{ gm}$, $w = 12\% = 0.12$

$M_{\text{(cutter+soil)}} = 3195 \text{ gm}$, $G_s = 2.7$

$M = 3195 - 1286 = 1909 \text{ gm}$

$V = 1000 \text{ cm}^3$

Bulk density (ρ) = $\frac{M}{V} = \frac{1909}{1000} = 1.909 \text{ gm/cm}^3$

Dry density (ρ_d) = $\frac{\rho}{1+w}$

= $\frac{1.909}{1+0.12} = 1.704 \text{ gm/cm}^3$

$\therefore S_d = \frac{G_s w}{1 + \frac{w G_s}{S}}$

$\Rightarrow \frac{w G_s}{S} = \frac{G_s w}{\rho_d} - 1$

$\Rightarrow \frac{0.12 \times 2.7}{S} = \frac{2.7 \times 1}{1.704} - 1$

$\Rightarrow S = \frac{0.12 \times 2.7 \times 1.704}{2.7 \times 1 - 1.704} = 0.55$

$e = \frac{G_s w}{\rho_d} - 1 = 0.584$

If, $S = 100\%$, $w = ?$

$e_s = w G_s$

$\Rightarrow w_{\text{sat}} = \frac{e_s}{G_s} = \frac{0.584 \times 1}{2.7} = 0.216 = 21.6\%$

$\gamma_{\text{sat}} = \frac{(G_s + e_s) \gamma_w}{1 + e}$

= $\frac{(2.7 + 0.584) \times 9.81}{1 + 0.584} = 20.338 \text{ kN/m}^3$

$\Rightarrow \boxed{\gamma_{\text{sat}} = 20.338 \text{ kN/m}^3}$ (Ans)

Q-4: The specific gravity of soil solids is 2.7 & the dry unit weight is given as 16.09 kN/m^3 . Determine the void ratio under the assumptions that the soil is perfectly dry. What would be the voids ratio if the sample is assumed to have a water content of 8% & unit weight of 16.09 kN/m^3 .

Solⁿ: Given $\Rightarrow G_s = 2.7$

$$\gamma_d = 16.09 \text{ kN/m}^3$$

$$w = 8\% = 0.08, \quad \gamma = 16.09 \text{ kN/m}^3$$

$$\gamma_d = \frac{G_s \gamma_w}{1+e} \Rightarrow 16.09 = \frac{2.7 \times 9.81}{1+e}$$

$$\Rightarrow e = \frac{2.7 \times 9.81}{16.09} - 1 = 0.646$$

$$\therefore \gamma_d = \frac{\gamma}{1+w}$$

$$\Rightarrow \gamma_d = \frac{16.09}{1+0.08} = 14.898 \text{ kN/m}^3$$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{14.898} - 1 = 0.778 \text{ (Ans)}$$

Q-5: A soil sample is partially saturated its natural water content was found to be 22% & bulk density 2 g/cm^3 . If the specific gravity of solids is 2.65 find out the degree of saturation & void ratio.

Solⁿ: $w = 22\% = 0.22$

$$\gamma = 2 \text{ g/cm}^3, \quad G_s = 2.65$$

$$\Rightarrow \gamma_d = \frac{\gamma}{1+w} = \frac{2}{1+0.22} = 1.639 \text{ g/cm}^3$$

$$\gamma_d = \frac{G_s \gamma_w}{1+e} \Rightarrow e = \frac{G_s \gamma_w}{\gamma_d} - 1$$

$$= \frac{2.65 \times 1}{1.63} - 1 = 0.62$$

$$\text{Degree of saturation, } (s) = \frac{wG_s}{e} = \frac{0.22 \times 2.65}{0.62} = 0.93 = 93\%$$

Q-6: A sample with a volume of 45cc is filled with a soil sample. When the soil poured into a graduated cylinder it displaced 25cc of water. What is the porosity and void ratio of the soil.

Solⁿ: Given, vol^m of soil (V) = 45cm³

Volume of displaced water = Volume of soil solid (V_s)
= 25cm³

⇒ Volume of voids = V - V_s = 45 - 25 = 20cm³

$$\text{Void ratio } (e) = \frac{V_v}{V_s} = \frac{20}{25} = 0.8$$

$$\text{Porosity } (n) = \frac{e}{1+e} = \frac{0.8}{1+0.8} = 0.44\% \text{ (Ans)}$$

Q-7: The void ratio and specific gravity of a sample of clay are 0.73 & 2.7 respectively. If the voids are 92% saturated, find the bulk density, dry density and the water content. What would be the water content for complete saturation, the void ratio remaining the same.

Solⁿ: Given data →

$$\text{Void ratio } (e) = 0.73$$

$$\text{Specific gravity } (G) = 2.7$$

$$\text{Degree of saturation } (S) = 92\% = 0.92$$

$$\text{Bulk density } (\rho) = \frac{(G+e) \rho_w}{1+e}$$

$$= \frac{(2.7 + 0.73 \times 0.92) \times 1}{1 + 0.73} = 1.94 \text{ g/cm}^3$$

$$\text{Dry density, } \rho_d = \frac{G \rho_w}{1+e}$$

$$= \frac{2.7 \times 1}{1 + 0.73} = 1.56 \text{ g/cm}^3$$

w = ?

$$\rho_d = \frac{\rho}{1+w} \Rightarrow w = \frac{\rho}{\rho_d} - 1 = \frac{1.94}{1.56} - 1 = 0.24$$

$$S = 100\% = 1$$

$$\therefore w = \frac{eS}{G} = \frac{0.73 \times 1}{2.7} = 0.2703$$

$$\Rightarrow \boxed{w = 27.03\%} \text{ (Ans)}$$

8) The total unit weight of soil is 16 kN/m^3 , sp. gravity of soil particle is 2.67. The water content is 17%. Calculate.

(i) Dry unit weight

(ii) porosity

(iii) void ratio

(iv) Degree of saturation

Ans Given data :

$$\text{Total unit weight } (\gamma) = 16 \text{ kN/m}^3$$

$$\text{Specific gravity } (G) = 2.67$$

$$\text{Water content } (w) = 17\%$$

$$\text{(i) } \gamma_d = \frac{\gamma}{1+w} = \frac{16}{1+0.17} = 13.67 \text{ kN/m}^3$$

$$\text{(iii) } e = \frac{G \gamma_w}{\gamma_d} - 1 = \frac{2.67 \times 9.81}{13.67} - 1 = 0.91$$

$$\text{(ii) Porosity } (n) = \frac{e}{1+e} = \frac{0.91}{1+0.91} = 0.4764 = 47.64\%$$

$$\text{(iv) Degree of saturation } (S) = \frac{wG}{e} = \frac{0.17 \times 2.67}{0.91} = 0.4987 = 49.87\% \text{ (Ans)}$$

9) A fully saturated clay has mass of 101.5 gm & a volume of 50 cm^3 . After oven drying the clay has mass of 84.5 gm. Assuming that the volume does not change during drying. Determine specific gravity, void ratio, porosity & dry unit weight.

Ans: In a fully saturated clay,

$$\text{Mass, } M = 101.5 \text{ gm, } S = 1$$

$$\text{Volume, } V = 50 \text{ cm}^3$$

$$\text{After drying, } M_d = 84.5 \text{ gm}$$

$$\text{Bulk density } (\rho) = \frac{M}{V} = \frac{101.5}{50} \\ = 2.03 \text{ g/cm}^3$$

$$\text{Dry density } (\rho_d) = \frac{M_d}{V} = \frac{84.5}{50} \\ = 1.69 \text{ g/cm}^3$$

$$\rho_d = \frac{\rho}{1+w}$$

$$\Rightarrow w = \frac{\rho}{\rho_d} - 1 = \frac{2.03}{1.69} - 1 = 0.201$$

$$\therefore \rho_d = \frac{G \gamma_w}{1 + \frac{wG}{S}} \Rightarrow 1.69 = \frac{G \times 1}{1 + \frac{0.201 \times G}{1}}$$

$$\Rightarrow 1.69 = \frac{G}{1 + 0.201G}$$

$$\Rightarrow 1.69 (1 + 0.201G) = G \Rightarrow 1.69 + 0.339G = G$$

$$\Rightarrow G - 0.339G = 1.69$$

$$\Rightarrow G(1 - 0.339) = 1.69 \Rightarrow G = \frac{1.69}{1 - 0.339} = \boxed{2.556}$$

$$\text{(i) void ratio } (e) = \frac{wG}{S} \\ = \frac{0.201 \times 2.556}{1} = 0.51 = \boxed{51\%}$$

$$\text{(ii) porosity } (n) = \frac{e}{1+e} = \frac{0.51}{1+0.51} = 0.3377 = \boxed{33.77\%}$$

$$\text{(iii) Dry unit weight } (\gamma_d) = \frac{G \gamma_w}{1+e}$$

$$= \frac{2.556 \times 9.81}{1+0.51} = \boxed{16.60 \text{ kN/m}^3} \quad (\text{Ans})$$

Chapter 3 DETERMINATION OF INDEX PROPERTIES

Index Properties : [Index properties are the simple physical properties of the soils, which are used for classification of soils for various engineering applications.]

① Water content

② Specific Gravity

③ Particle size distribution

→ Sieve analysis

→ Hydrometer analysis

④ Consistency Limits

⑤ Density → Mass
→ Weight

[Engineering properties of soil are bearing capacity, shear strength, permeability, Compressibility etc.]

④ Water contents :

Methods →

① Oven drying method

② Sand bath method

③ Alcohol method

④ Pycnometer method

⑤ Calcium carbide method

⑥ Torsion balance method

⑦ Radiation method

① Oven drying method :

→ This is the most accurate method of determining water content used in the Laboratory.

→ A specimen of soil sample is kept in a clean container and put in a thermostatically control oven with interior of non-corroding material to maintain the temperature between 105°C to 150°C.

→ For complete drying, sandy soils takes about 4 hrs & clay take about 14 to 16 hrs.

Procedure:

- A clean, non-corrodible container is taken and its mass is found with its lid (M_1).
- A specimen of moist soil is placed in the container & the lid is replaced. The mass of the container & contents is determined. (M_2)
- With the lid removed, the container is then placed in the oven for drying.
- After drying the container is removed from the oven & allowed to cool in a desiccator.
- The lid is then replaced & the mass of container & dry soil is found (M_3).
- The water content is calculated from the following expression,

$$\omega = \frac{M_2 - M_3}{M_3 - M_1} \times 100$$

M_1 = mass of container with lid
 M_2 = Mass of (container with lid + wet soil)
 M_3 = Mass of (container + lid + dry soil)

② Pycnometer method:

- It is a quick method of determining water content of those soils whose specific gravity 'G' is accurately known.
- Pycnometer is a large size density bottle of about 500ml capacity.
- A conical brass cap, having a 6mm dia. hole at its top, is screwed to the open end of the pycnometer.
- A rubber washer is placed between conical cap & the rim of the bottle, so that there is no leakage.

of water.

Procedure:

- Take a clean, dry pycnometer & find its mass with its cap & washer (M_1).
- Put about 200-400gm of wet soil sample and find its mass with its cap & washer. (M_2)
- Fill pycnometer to half its height and vibrate it thoroughly with a glass rod. Add more water and stirring, replace the screw of and with the hole in the conical cap. Dry the pycnometer from outside and find its mass. (M_3)
- Empty the pycnometer clean it thoroughly & fill it with clean water to the hole of the conical cap and find its mass. (M_4)

↳ The water content is calculated from the following expression:

$$w = \left[\left(\frac{M_2 - M_1}{M_3 - M_4} \right) \left(\frac{G - 1}{G} \right) - 1 \right] \times 100$$

M_1 = Mass of pycnometer with cap & washer.

M_2 = Mass of pycnometer with cap & washer + wet soil.

M_3 = Mass of pycnometer with cap & washer + wet soil + water.

M_4 = Mass of pycnometer with cap & washer + full of water.

↳ Specific Gravity:

The specific gravity of soil solids is determined by,

(a) some density bottle.

(b) some flask.

(c) Pycnometer.

- The density bottle method is the most accurate and is suitable for all type of soil.
- The flask or pycnometer is used only for coarsest grained soil.
- The density bottle method is the saturated method used in the laboratory.

Procedure:

- The mass of the empty, clean, dry, bottle, flask or pycnometer is taken (M_1).
- A sample of oven dry soil is put in the bottle & the mass (M_2) is taken.
- The bottle is then filled with distilled water or kerosene gradually, removing the entrapped air. The mass M_3 of the bottle, soil & water (full upto the top) is taken.
- Then the bottle is empty completely and thoroughly washed and clean water or kerosene is filled to the top & the mass (M_4) is taken.

The specific gravity can be calculated from the following.

$$G_s = \frac{M_2 - M_1}{(M_4 - M_1) - (M_3 - M_2)}$$

$$\Rightarrow G_s = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)}$$

$$\Rightarrow G_s = \frac{M_3}{M_3 - (M_3 - M_4)}$$

Q.: In order to determine the water content 370g. of a wet sandy sample was placed in a pycnometer. The mass of the pycnometer, sand & water was found to be 2143g. The mass of pycnometer full of ^{Water} was 1932g. Taking $G_s = 2.65$. Determine the

water content of the sample.

Ans Given data,

weight of soil sample = 370 gm

$$M_3 = 2148 \text{ gm}, \quad G = 2.65$$

$$M_4 = 1932 \text{ gm}, \quad M_2 - M_1 = 370 \text{ gm}$$

$$\text{Water content (w)} = \left[\left(\frac{M_2 - M_1}{M_3 - M_4} \right) \left(\frac{G - 1}{G} \right) - 1 \right] \times 100$$

$$= \left[\left(\frac{370}{2148 - 1932} \right) \left(\frac{2.65 - 1}{2.65} \right) - 1 \right] \times 100$$

$$= 6.5\% \text{ (Ans)}$$

Q: An oven dried soil having a mass of 200 gm is placed in a pycnometer which is then completely filled with water. The total mass of the pycnometer with water & soil is 1605 gm. The pycnometer filled with water alone has a mass of 1480 g. Calculate the sp. gravity of the soil.

Ans: Given data,

Mass of the dry soil = 200 gm

$$\Rightarrow M_2 - M_1 = 200 \text{ gm} \text{ (Equation 1)}$$

$$M_3 = 1605 \text{ g}$$

$$M_4 = 1480 \text{ g}$$

$$\therefore G = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)}$$

$$= \frac{200}{200 - (1605 - 1480)} = 2.66 \text{ (Ans)}$$

PARTICLE SIZE DISTRIBUTION :

→ The percentage of various sizes of particle in a given dry soil sample is found by a particle size analysis or mechanical analysis.

→ It is meant for the separation of a soil sample into its different size fraction. It is performed in two stages.

i) Sieve analysis

ii) sedimentation analysis / wet mechanical analysis.

i) Sieve Analysis :

→ In Indian standards the sieves are designated by the size of opening in mm. The complete sieve analysis can be divided into 2 parts.

(a) Coarse analysis

(b) Fine analysis.

→ An oven dried sample of soil is separated into 2 fractions by sieving it through a 4.75 mm IS sieve.

→ The portion retained on 4.75 mm sieve is termed as gravel fraction & is kept for coarse analysis, while the portion passing through 4.75 mm sieve is subjected to fine sieve analysis.

↳ Set of sieves :

Coarse analysis → (150 mm, 60 mm, 42.5 mm, 20 mm, 12 mm, 10 mm, 4.75 mm etc.)

Fine analysis → (4.75 mm, 2.0 mm, 1.18 mm, 600 μ , 300 μ , 150 μ , 75 μ etc.)

→ By sieving is performed by arranging the various sieves one over the other in order of their mesh opening.

- The largest aperture sieve being kept at the top & the smallest at the bottom. A receiver is kept at the bottom & a cover is kept at the top of the whole assembly.
- The soil sample is put on the top sieve & the whole assembly is fitted on a sieve shaking machine.
- The amount of shaking depends upon the shape & the no. of particles. At least 10 minutes of shaking is desirable of soil with small particles.
- The portion of the soil sample retained on each sieve is weighted. The %age of soil retained on each sieve is calculated on the basis of the total mass of soil sample taken & from these result %age passing through each sieve is calculated.

ii) Sedimentation analysis :

- In sedimentation analysis the soil fraction finer than 75 μ size is kept in suspension in a liquid medium.
- The analysis is based on Stoke's law. According to which the velocity at which grains settle out of suspension, all other fractions being equal is dependent upon weight & size of the grain.
- The coarser particle settle more quickly than finer particle.

$$V = \frac{2}{9} r^2 \frac{\gamma_s - \gamma_w}{\eta}$$

$$\Rightarrow V = \frac{2}{9} \left(\frac{D}{2}\right)^2 \frac{\gamma_s - \gamma_w}{\eta}$$

$$\Rightarrow V = \frac{1}{18} D^2 \frac{\gamma_s - \gamma_w}{\eta}$$

r = Radius of the particle.

D = Diameter of the particle.

η = Viscosity of water

V = velocity

γ_s = Unit weight of soil particle.

γ_w = Unit weight of water.

$$\Rightarrow V = \frac{1}{18} D^2 \frac{(\gamma_s - 1) \gamma_w}{\eta}$$

$$\Rightarrow D = \sqrt{\frac{18 V \eta}{(\gamma_s - 1) \gamma_w}}$$

$$\Rightarrow D = \frac{18 \times 10^6 \times \eta V}{(\gamma_s - 1) \gamma_w}$$

↳ Particle size distribution curve :

A particle size distribution curve gives an idea about the type & gradation of the soil. Acc. to this curve a soil sample is categorised into 3 types.

i) well graded

ii) Poorly graded

iii) Gap graded/skip graded.

(i) A soil is said to be well graded when it has a good representation of particles of all sizes.

(ii) A soil is said to be poorly graded if it has an excess of certain particles & deficiency of other.

(iii) A soil is said to be gap graded or skip graded when it has some of intermediate size particles are missing.

⇒ For coarse grain soil certain particle sizes such as D_{10} , D_{30} , D_{60} are important.

⇒ " D_{10} " represents a sizes in "mm" such that 10% of the particles are finer than this size.

" D_{10} " → Effective size / Effective diameter.

Uniformity Coefficient (C_u):

It is a measure of particle size range.

$$C_u = \frac{D_{60}}{D_{10}}$$

- For uniformly graded soil, $C_u = 1$
- For gravels, $C_u > 4$
- For sand, $C_u > 6$

Coefficient of Curvature (C_c):

C_c → Shape of the particle size distribution curve.

$$C_c = \frac{(D_{10})^2}{D_{30} \times D_{60}}$$

Well graded soil, $[1 < C_c < 3]$

Consistency of soils:

→ Consistency is defined as the relative ease with which a soil can be deformed.

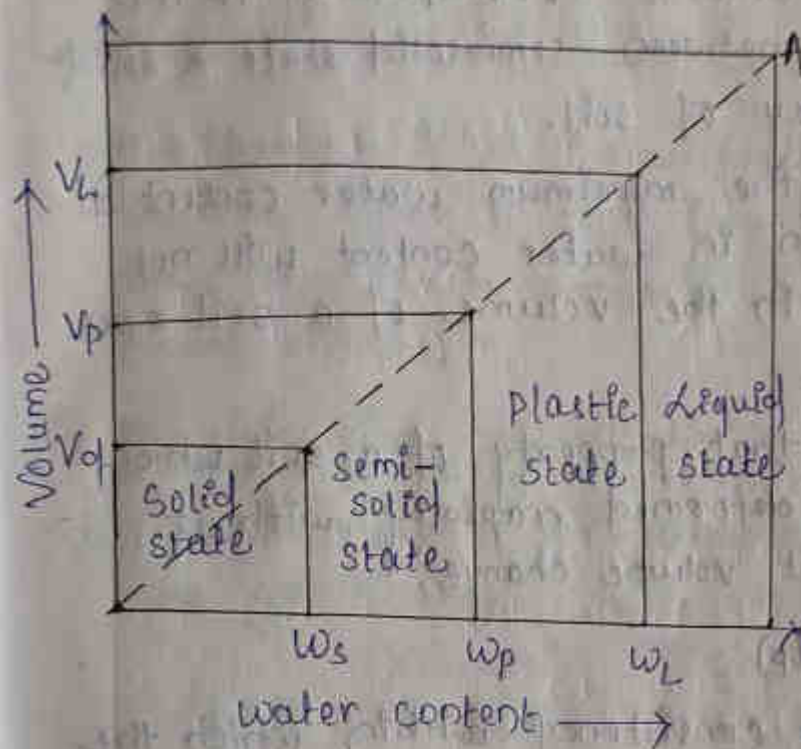
→ Consistency is used for fine grade soil for which it is related to a large extent to water content. Consistency measures the degree of firmness of the soil.

Consistency limits: (Atterberg's Limits)

→ An agriculturist Atterberg divided the entire range of soil from liquid to solid state into 4 stages.

- i) Liquid state
- ii) plastic state
- iii) semisolid state
- iv) Solid state.

→ He said arbitrary limits known as consistency limits or Atterberg's limit for this division in terms of water content. Thus, the consistency limits can be defined as the water content in which the soil mass passes from one state to the next.



1) Liquid limit (w_l):

→ It is the water content corresponding to the arbitrary limits between the liquid state and plastic state of consistency of soil.

→ It is defined as the minimum water content at which the soil is still in the liquid state but has a small shearing strength against flowing which can be measured by standard available means.

→ If a part of soil is cut by a groove then it will flow together for a distance of $\frac{1}{2}$ inch under a effect of 25 blows.

2) Plastic limit (w_p):

It is the water content corresponding to the arbitrary limits between plastic state & semi-solid state of consistency of soil.

→ It is defined as the minimum water content at which a soil will just begin to crumble when rolled into a thread approximately 3mm in diameter.

3) Shrinkage Limit (w_s):

→ It is the water content corresponding to the arbitrary limits between semi-solid state & solid state of consistency of soil.

→ It is defined as the maximum water content at which a reduction in water content will not cause decrease in the volume of a soil mass.

* Plasticity:

It is defined as that property of a soil which allows it to be deformed rapidly without rupture & without volume change.

① Plasticity Index (I_p):

→ It is the range of consistency within which the soil exhibit plastic properties.

$$I_p = w_L - w_p$$

= Liquid limit - plastic limit

② Liquidity Index (I_L): (Water plasticity ratio)

→ It is the ratio expressed as a %age of natural water content - plastic limits to its plasticity index.

$$I_L = \frac{w - w_p}{I_p}$$

③ Consistency Index (I_c):

$$I_c = \frac{w_L - w}{I_p}$$

* It is the ratio of liquid limit - natural water content to its plasticity index of soil.

CLASSIFICATION OF SOILS

- ① Particle size classification.
- ② Textural classification.
- ③ Highway Research Board (HRB) classification.
- ④ Unified soil classification system (USCS).
- ⑤ Indian standard classification system (ISCS).

* The purpose of soil classification is to arrange various types of soils into groups, according to their engineering properties & other various characteristics.

① Particle size classification:

* In this system soils are arranged according to the grain size.

i) Gravel iii) silt } These are used to indicate the grain size.
 ii) Sand iv) clay }

		0.0485	2.00		20mm	60mm	300mm	
Clay size	silt size	Fine	Medi-um	Coarse	Fine	Coarse	Cobble	B D L P R
		Sand			Gravel			
		0.002 mm	0.075 mm	4-75 mm				

(All sizes are in mm)

② Textural classification:

(Diagram)

→ In this classification system soils are classified based on particle size distribution (different %age of different size). Soils are composed of different %age of sand, silt & clay size particle.

③ Highway Research Board classification (HRB) :-

- This system is based on both the particle size composition as well as the plasticity characteristics.
- Soils are divided into 7 groups.

[A-1, A-2, ..., A-7]

↓ ↓
2 grp, 4 grp

→ A characteristic group index is used to describe the performance of the soils when used for pavement construction.

$$[\text{Group Index (G.I.)} = 0.2a + 0.005ac + 0.01bd]$$

- * a = That portion of %age passing 75 μ 15 sieve, greater than 35 and not exceeding 75 expressed as a whole no. (0-40).
- * b = That portion of %age passing 75 μ 15 sieve greater than 15 & not exceeding 55 expressed as a whole no. (0-40).
- * c = That portion of the numerical Liquid Limit greater than 40 & not exceeding 60, expressed as a +ve whole no. (0-20).
- * d = That portion of the numerical plasticity index greater than 10 & not exceeding 30, expressed as a ^{+ve} whole no. (0-20).

Q: Laboratory test on a soil sample reveals that 56% of soil passes 75 μ sieve. The liquid limit & plastic limit of the soil are 36% & 23% respectively. Determine the group index of the soil.

Solⁿ: Given data,

% age passing 75 μ sieve = 56%.

$$w_L = 36\%$$

$$w_p = 23\%$$

$$I_p = w_L - w_p = 36 - 23 = 13\%$$

$$G.I. = 0.2a + 0.005ac + 0.01bd$$

$$a = 56 - 35 = 21$$

$$b = 56 - 15 = 41 > 40$$

$$b \approx 40$$

$$c = 0, d = 13 - 10 = 3$$

$$\begin{aligned} \Rightarrow G.I. &= (0.2 \times 21) + (0.005 \times 21 \times 0) + (0.01 \times 40 \times 3) \\ &= 5.4 \text{ (Ans)} \end{aligned}$$

④ Unified soil classification system (USCS):-

According to USCS, soils are classified into 4 groups

i) Coarse grained soils.

ii) Fine grained soils.

iii) Organic soils.

iv) Peat

* The coarse grain soils are classified on the basis of their grain size distribution, while fine grained soils are classified on the basis of their plasticity.

* There are 15 groups of soils, 8 groups are of coarse grained soils & 6 fine grained soil (including organic soil) & one peat.

Prefix & suffix of Uses :-

Soil type	prefix	sub group	suffix
i) Gravel \rightarrow	G	① Well graded \rightarrow	w
ii) Sand \rightarrow	S	② poorly graded \rightarrow	P
iii) silt \rightarrow	M	③ silty \rightarrow	M
iv) clay \rightarrow	C	④ clayey \rightarrow	C
v) Organic \rightarrow	O	⑤ $w_L < 50\%$ \rightarrow	L
vi) peat \rightarrow	Pt	⑥ $w_L > 50\%$ \rightarrow	H

① Coarse grained soil :-

* It more than 50% of the soil is retained on ~~4.75~~ 4.75 mm IS sieve it is designated as coarse grained soil.

* Coarse grained soil is of 2 types.

(a) Gravel

(b) sand

* A coarse grained soil is designated as gravel (G), if 50% or more coarse fraction is retained on 4.75 mm IS sieve, otherwise it is termed as sand.

* Coarse grain soil containing less than 5% fine are designated by symbols Gw, ~~G~~sw, Gp, sp.

* $< 5\%$ Fines = Gw, sw, Gp, sp.

* 5-12% Fines = Gm, sm, Gc, sc.

* Coarse grain soil containing 5-12% of fines are designated by the symbol Gm, sm, Gc, sc.

④ Fine grained soil :-

- A soil is known as fine grained if more than 50% of the soil sample passes 75 μ sieve.
- Fine grained soil are divided into two types. These are silt & clay based on their liquid limit & plasticity Index. Organic soils are also included in fine grained soils.

→ The fine grained soil (silt, clay, organic soil) are further subdivided into two groups.

i) Low plasticity (L) $\Rightarrow W_L < 50\%$.

ii) High plasticity (H) $\Rightarrow W_L > 50\%$.

Ex: ML, MH, CL, CH, OL, OH.

⑤ Indian standard classification system (ISCS) :-

- It is the revised version of USCS, with the modification of that the fine grained soils are subdivided into 3 groups (Low, medium & High plasticity) as against only two groups in USCS.

→ The ISCS classifies the soil into 18 groups as against 15 groups of USCS.

i) coarse grained soil.

① Gravel \rightarrow well graded.

② sand \rightarrow poorly graded.

ii) Fine grained soil.

- (a) silt
- (b) clay
- (c) organic soils.

→ The fine grained soils are further divided into three groups based on their liquid limit or compressive limit.

- (a) Low compressible (L) $\Rightarrow w_L < 35\%$
- (b) Medium compressible (I) $\Rightarrow 35 < w_L < 50\%$
- (c) Highly compressible (H) $\Rightarrow w_L > 50\%$

- Permeability is defined as the property of a porous material which permits the passage or seepage of water through its inter connecting voids.
- A material having continuous voids is called permeable. Gravels are highly permeable while clay is the least permeable.
- The flow of water through soils may be a laminar flow or a turbulent flow. In laminar flow each fluid particle travels along a definite path which never crosses the path of any other particle.
- While in turbulent flow the paths are irregular, twisting, crossing & recrossing at random.
- * In soil mechanics the flow of water through soils is assumed to be laminar.



(Gravel) (coarse sand) (fine sand) (clay)

Darcy's Law :-

Darcy's Law states that for laminar flow conditions in a saturated soil, the rate of flow or the discharge per unit time is proportional to the hydraulic gradient.

$$\therefore V \propto i \Rightarrow \frac{q}{A} \propto i$$

$$\Rightarrow \frac{q}{A} = ki \Rightarrow \boxed{q = kiA}$$

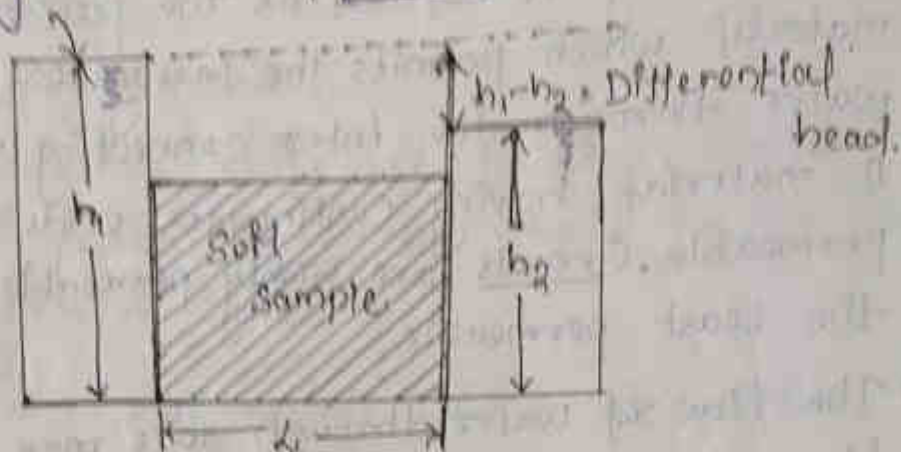
Where, k is the coefficient of permeability.

q = Discharge per unit time.

i = Hydraulic gradient

A = Total cross sectional area of soil mass.

V = velocity of flow. $\rightarrow V = Ki$



\rightarrow If a soil sample of length 'L' & a cross sectional area 'A' is subjected to differential head of water ($h_1 - h_2$), then the hydraulic gradient 'i' will be equal to,

$$i = \frac{h_1 - h_2}{L}$$

\rightarrow The co-efficient of permeability is defined as the average velocity of flow that will occur through the total cross sectional area of soil under unit hydraulic gradient.

Factors affecting permeability:

- i) Grain size or particle size
- ii) Properties of pore fluid.
- iii) voids ratio of the soil.
- iv) structural arrangement of the soil particle.
- v) Entrapped air.

\rightarrow The factors affecting permeability can be illustrated based on poiseuille's law.

$$K_v = D_{10}^2 \frac{\gamma_w}{\eta} \times \frac{e^3}{1+e} \times C$$

i) Grain size or particle size:

permeability varies approximately as the square of the grain size.

$$k \propto D_{10}^2 \Rightarrow k = CD_{10}^2$$

ii) properties of pore fluid:

The above equation indicates that the permeability is directly proportional to the unit weight of water & inversely proportional to its viscosity.

$$\therefore k \propto \gamma_w \Rightarrow \frac{k_1}{k_2} = \frac{\gamma_{w1}}{\gamma_{w2}}$$

$$k \propto \frac{1}{\eta} \Rightarrow \frac{k_1}{k_2} = \frac{\eta_2}{\eta_1}$$

iii) Voids ratio:

permeability of soil is directly proportional to voids ratio.

$$\therefore k \propto e^2 \Rightarrow \frac{k_1}{k_2} = \frac{e_1^2}{e_2^2}$$

iv) Structural arrangement of particles:

→ The structural arrangement of particle may vary depending upon the method of deposition or compacting the soil.

→ An undisturb sample is highly permeable as compare to a disturb soil sample.

v) Entrapped air / Degree of samp saturation:

The permeability is greatly reduced if air is entrapped in the voids thus reducing the degree of saturation.

↳ Seepage velocity:

The velocity of flow 'v' is the rate of discharge of water per unit of total cross sectional area 'A' of the soil. This total area of cross section is composed of the area of solids (A_s) & the area of voids (A_v).

→ Since the flow takes through the voids the actual or true velocity of flow will be more than discharge velocity. This actual velocity is called as seepage velocity (V_s).

→ Seepage velocity is defined as the rate of discharge of percolating water per unit cross-sectional area of voids.

$$v = \frac{q}{A} \quad \text{--- (1)}$$

$$\Rightarrow V_s = \frac{q}{A_v} \quad \text{--- (2)}$$

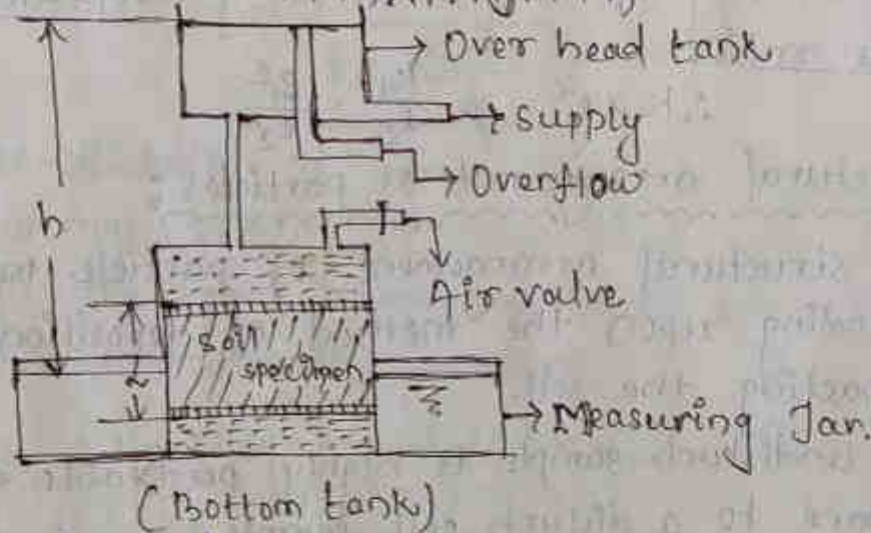
$$\Rightarrow \frac{V_s}{v} = \frac{A}{A_v} \Rightarrow \boxed{V_s = v \times \frac{A}{A_v}}$$

Area of int. of soil

$$\Rightarrow q = Av = A_v V_s \Rightarrow v = \frac{V_s A_v}{A} = v \times \frac{V_s}{v} = n \times \frac{1}{n}$$

$$\Rightarrow \boxed{V_s = \frac{v}{n}}$$

↳ Determination of permeability test: —



→ Water flows from the over head tank consisting of three tips, the inlet tube, over flow tube & the outlet tube.

→ The constant hydraulic gradient 'i' causing the flow is the head 'h' divided by the length of the sample.

$$\boxed{i = \frac{h}{L}}$$

→ If the length of the sample is large, the head const. lost over a length of specimen is measured by inserting piezometric tube.

→ If Q is the total quantity of flow in a time interval 't', from Darcy's law.

$$\boxed{q = \frac{Q}{t}}$$

$$q = k i t$$

$$\Rightarrow \frac{Q}{t} = k i a \Rightarrow Q = k i a t \quad \left(\text{Unit of } k = \frac{\text{mm}}{\text{sec}} \text{ or } \frac{\text{cm}}{\text{sec}} \right)$$

$$= k \frac{b}{L} a t \quad \left(i = \frac{b}{L} \right)$$

$$\Rightarrow \boxed{k = \frac{Q L}{A h t}}$$

Ques Calculate the coefficient of permeability of a soil sample 6cm in height & 50cm² in cross sectional area, if a quantity of water equal to 430ml pass down in 10 minutes under an effective constant head of 40cm. On oven drying the test specimen has mass of 498gm. Taking the specific gravity of soil solids as 2.65. Calculate the seepage velocity of water during the test.

Solⁿ: Given data,
 Length of the sample, $L = 6\text{cm}$.
 Area, $A = 50\text{cm}^2$, $h = 40\text{cm}$,

$$Q = 430\text{ml} = 430\text{cm}^3$$

$$t = 10\text{minutes} = 600\text{sec}$$

$$k = \frac{Q L}{A h t} = \frac{430 \times 6}{50 \times 40 \times 600} = 2.15 \times 10^{-3} \text{cm/sec}$$

On oven drying,

Mass of the sample (M) = 498gm
 Specific gravity (G) = 2.65

$$V_s = \frac{V}{n}$$

$$q = A \times v \Rightarrow \frac{Q}{t} = 50 \times v \Rightarrow v = \frac{430}{600 \times 50} = 0.01433 \dots$$

$$= 1.43 \times 10^{-2} \text{cm/sec}$$

$$\Rightarrow M_s = 498\text{gm}$$

$$\text{Volume (V)} = 50 \times 6 = 300\text{cm}^3$$

$$\text{Dry density (}\rho_d\text{)} = \frac{M_s}{V} = \frac{498}{300} = 1.66 \text{g/cm}^3$$

$$\rho_d = \frac{G \rho_w}{1+e} \Rightarrow e = \frac{G \rho_w}{\rho_d} - 1 = \frac{2.65 \times 1}{1.66} - 1$$

$$\Rightarrow \boxed{e = 0.59}, \quad n = \frac{e}{1+e} = \frac{0.59}{1+0.59} = 0.37$$

$$V_s = \frac{V}{n} = \frac{0.0143}{0.37} = 3.85 \times 10^{-2} \text{ cm/sec. (Ans)}$$

2) Calculate the co-efficient of permeability of a soil sample 6cm in height, 50cm² in cross-sectional area. If a quantity of water equal to 450ml passed down in 10min. under an effective constant head of 40cm. On oven drying the test specimen weighs 495gm. Taking the specific gravity of soil solid as 2.65. calculate the seepage velocity of water during the test.

Solⁿ: Given data,

$$L = 6 \text{ cm, } Q = 450 \text{ ml} = 450 \text{ cm}^3$$

$$A = 50 \text{ cm}^2, \quad t = 10 \text{ min} = 600 \text{ sec, } h = 40 \text{ cm.}$$

$$K = \frac{QL}{Aht} = \frac{450 \times 6}{50 \times 40 \times 600} = 2.25 \times 10^{-3} \text{ cm/sec.}$$

$$M = 495 \text{ gm}$$

$$G = 2.65$$

$$V = Ki \Rightarrow V = 2.25 \times 10^{-3} \times \frac{h}{L}$$

$$= 2.25 \times 10^{-3} \times \frac{40}{6} = 0.015 \text{ cm/sec.}$$

$$q = Av$$

$$\Rightarrow \frac{Q}{t} = Av \Rightarrow \frac{450}{600} = 50 \times v$$

$$\Rightarrow v = 0.015 \text{ cm/sec.}$$

$$M_s = 498 \text{ gm}$$

$$V = 50 \times 6 = 300 \text{ cm}^3$$

$$\rho_d = \frac{M_s}{V} = \frac{498}{300} = 1.66 \text{ gm/cm}^3$$

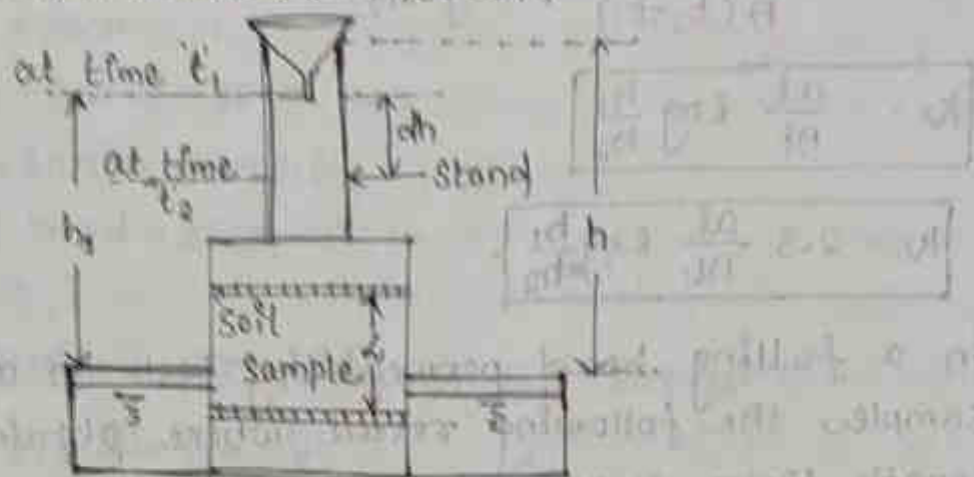
$$\therefore e = \frac{G \rho_w}{\rho_d} - 1$$

$$\Rightarrow e = \frac{2.65 \times 1}{1.66} - 1 = 0.596$$

$$\therefore n = \frac{e}{1+e} = \frac{0.596}{1+0.596} = 0.373$$

$$V_s = \frac{V}{n} = \frac{0.015}{0.373} = 0.040 \text{ cm/sec. (Ans)}$$

Falling head permeability test:



→ The falling head test is used for less permeable soil where the discharge is small. A stand pipe of known cross sectional area a is fitted over the permeant & the water is allow to run down the water level in the stand pipe constantly falls as water flows the head at any time instant t is equal to the difference in the water level in the stand pipe & the bottom tank.

→ Let h_1 & h_2 be the heads at time interval t_1 & t_2 respectively.

→ Let h be the head at any intermediate time interval t & dh be the change in head in a smaller time interval dt . Hence from darcy's law the rate of flow q is given by →

$$q = k i A = \frac{-dh}{dt} \times a$$

$$\Rightarrow k i A = \frac{-dh}{dt} a$$

$$\Rightarrow k \frac{h}{L} \times A = \frac{-dh}{dt} a$$

$$\Rightarrow \frac{kA}{La} = \frac{-dh}{dt} \times \frac{1}{h}$$

$$\Rightarrow \frac{kA}{La} \int_{t_1}^{t_2} dt = - \int_{h_1}^{h_2} \frac{dh}{h}$$

$$\Rightarrow \frac{kA}{La} \times [t]_{t_1}^{t_2} = - [\log h]_{h_1}^{h_2}$$

$$\Rightarrow \frac{kA}{La} \times (t_2 - t_1) = - (\log h_2 - \log h_1)$$

$$\Rightarrow \frac{kA}{La} \times (t_2 - t_1) = \log h_1 - \log h_2 = \log \frac{h_1}{h_2}$$

$$\Rightarrow K_v = \frac{al}{A(t_2 - t_1)} \log \frac{h_1}{h_2}$$

$$\Rightarrow K = \frac{al}{At} \log \frac{h_1}{h_2}$$

$$\Rightarrow K = 2.3 \frac{al}{At} \log_{10} \frac{h_1}{h_2}$$

Q) In a falling head permeability test on a silty clay sample, the following result were obtained sample length 12mm, sample diameter 80mm, Initial head 1200mm, Final head 400mm time for fall in head 6min, stand pipe diameter 4mm. Find the coefficient permeability of sample.

Solⁿ:

$$L = 1.2 \text{ m}, D = 8 \text{ cm},$$

$$A = \pi/4 \times (8)^2 = 50.265 \text{ m}^2$$

$$h_1 = 1200 \text{ mm} = 120 \text{ cm}$$

$$h_2 = 400 \text{ mm} = 40 \text{ cm},$$

$$t = 6 \text{ min} = 360 \text{ sec.}$$

$$\text{dia. of stand pipe} = d = 4 \text{ mm} = 0.4 \text{ cm},$$

$$a = \pi/4 \times (0.4)^2 = 0.1257 \text{ m}^2$$

$$K = 2.3 \frac{al}{At} \log_{10} \frac{h_1}{h_2}$$

$$= 2.3 \times \frac{0.1257 \times 1.2}{50.265 \times 360} \times \log_{10} \left(\frac{120}{40} \right)$$

$$= 9.147 \times 10^{-6} \text{ cm/sec. (Ans)}$$

Q) In a falling head permeability test the initial head is 40cm (at $t=0$) the head drops by 5cm in 10min. Calculate the time required to run the test for the final head to be at 20cm. If the sample is 6cm in height & 50 cm^2 in cross-sectional area. Calculate the coefficient of permeability taking area of stand pipe = 0.5 cm^2 .

Given data:

$$h_1 = 40 \text{ cm}$$

$$h_2 = (40 - 5) = 35 \text{ cm}$$

$$t = 10 \text{ min} = 600 \text{ sec}$$

$$\text{Final head} = 20 \text{ cm}$$

$$t = ?$$

$$A = 50 \text{ cm}^2, d = 6 \text{ cm}, A_{\text{stand pipe}} = 0.5 \text{ cm}^2$$

$$\Rightarrow K_v = 2.3 \frac{al}{AE} \log_{10} \frac{h_1}{h_2}$$

$$\Rightarrow t = 2.3 \frac{al}{AK} \log_{10} \frac{h_1}{h_2}$$

$$\Rightarrow 600 = 2.3 \frac{al}{AK} \log_{10} \frac{40}{35}$$

$$\Rightarrow 600 = 2.3 \frac{al}{AK} \times 0.057 \quad \Rightarrow \frac{600}{0.057} = 2.3 \frac{al}{AK}$$

$$\Rightarrow 2.3 \times \frac{0.5 \times 6}{50 \times K} = 10526.3157$$

$$\Rightarrow \frac{3}{50K} = \frac{10526.3157}{2.3}$$

$$\Rightarrow K_v = \frac{3}{228832.95} = 1.34 \times 10^{-5}$$

$$\therefore K_v = 2.3 \frac{al}{AE} \log_{10} \frac{h_1}{h_2}$$

$$\Rightarrow 1.34 \times 10^{-5} = 2.3 \times \frac{0.5 \times 6}{50 \times t} \log_{10} \left(\frac{40}{20} \right)$$

$$\Rightarrow t = \frac{2.3 \times 0.5 \times 6}{50 \times 1.34 \times 10^{-5}} \log_{10} \left(\frac{40}{20} \right) = 3168 \text{ sec.}$$
$$= 52.8 \text{ min. (Ans)}$$

COMPACTION

Compaction is a process by which soil particles are artificially be arranged & pack together into a closer state of contact by mechanical means in order to decrease the porosity or void ratio of the soil & to increase its dry density.

→ The compaction process can be done by rolling, tamping or vibration.

Compaction test:① Standard proctor test:

The standard proctor test was develop by R.R. proctor the state of California. The test equipment consist of →

- (a) Cylindrical metal mould having an internal diameter of 10.15cm or 4inch. And internal effective height of 11.7cm & a capacity of 0.945 liter.
- (b) Detachable based plate.
- (c) Collar, 5cm in effective height.
- (d) Rammer, 2.5kg in mass falling through a height of 30.5cm.

* The test consist of compacting soil at various water contents in the mould in 3 equal layers. Each layers being given 25 blows of the 2.5kg rammer dropped from a height of 30.5cm.

* The dry density obtain in each test is determine by knowing the mass of the compacted soil & its water content.

* The ~~can~~ comparative energy used for this test is $6065 \text{ kg/cm} / 1000 \text{ ml}$ of soil.

* The bulk density & the corresponding dry density for the compacted soil are calculated from the following relation.

M = Mass of wet compacted soil.

V = Volume of cylindrical mould = 945 ml.

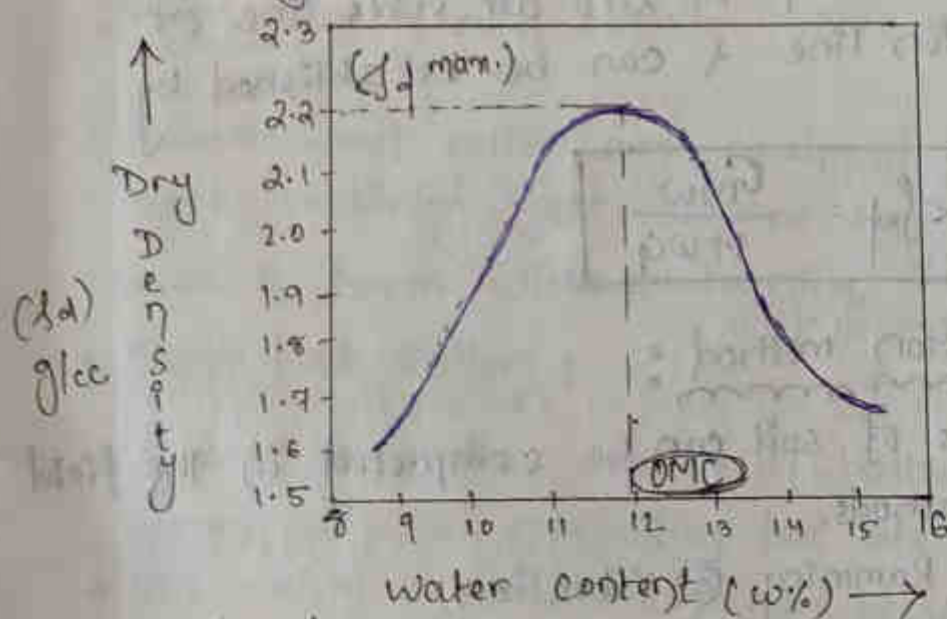
w = water content of the soil.

→ The compacted soil is taken out of the mould broken with hand & remixed with raised water content, 2-4%. The test is repeated on soil samples with increasing water content & corresponding dry density, & is determine.

→ A compaction curve is plotted in between water content as x-axis & dry density as y-axis.

→ The dry density goes on increasing as the water content is increase till maximum density is reached.

→ The water content corresponding to the maximum density is known as optimum moisture content.



② Modified Proctor Test: —

→ The modified proctor test is develop to give a higher standard of compaction. In this test the soil is compacted in the standard proctor mould (capacity 945ml) but in 5 layers.

→ Each layer being given 25 blows, of a 4.5kg rammer dropped through a height of 18 inch (25.72cm).

→ The comparative energy given to the soil in this test is $27260 \text{ kg/cm} \cdot / 1000 \text{ cm}^3$ of soil which is about 4.5 times that of standard proctor test.

↳ Air voids line:

A line which shows the water content, dry density relation for the compacted soil containing a constant percentage air voids is known as an air voids line & can be determined from the relation,

$$\gamma_d = \frac{(1 - D_a) G_s W_w}{1 + W G_s}$$

↳ Zero air voids line:

The line which shows the dry density, water content relation for compacted soil containing no air voids is called as zero air voids line or the saturation line & can be established by the relation,

$$\gamma_d = \frac{G_s W_w}{1 + W G_s}$$

Field compaction method:

Various types of soil can be compacted in the field by three methods.

- ① Rolling
- ② Ramming
- ③ Vibration.

Corresponding to these methods the compacting equipment can be grouped under 3 categories: rollers, rammers & vibrators.

Rolling equipment:

① Smooth wheel rollers.

② pneumatic type rollers.

③ sheep foot rollers.

Hamming equipment:

- ① Dropping weight type equipment
- ② Internal combustion type equipment
- ③ Pneumatic type.

Vibrating equipment:

- ① Dropping weight type
- ② Hydraulic type.

* Smooth wheel rollers:

→ Smooth wheel rollers are of three types.

① The conventional three wheel type with 2 large smooth faced steel wheels in the rear & one smaller smooth faced drum in the front (20-150kN).

② Rollers weighing from (10-140kN).

③ The three axle tandem rollers weighing from 120-180kN.

→ Smooth wheel roller are equipped with a clutch type receiving gear, so that they can be operated back & forth without turning.

* Sheep foot rollers:

→ Sheep foot rollers consists of hollow cylindrical steel drum on which projecting feet are mounted.

→ The weight of the drum can be varied by filling it partly or fully with water or sand.

→ The loaded weight per drum ranges from (15-130kN).

Pneumatic tyred rollers:

→ It consist of a box or platform mounted between two axles. The rear of which has one more wheel than the front.

→ The weight of pneumatic tyred rollers ranges from 7.5kN for smaller rollers to 100-500kN for bigger

rollers per tyred.

Rammers:

Rammers comprise of pneumatic & internal combustion type weighing from 300-1500N.

Vibrators:

It consist of a vibrating unit of either dropping weight type or hydraulic type mounted on a plate or roller.

Factors affecting compaction:

(a) Water content: As the water content is increase the compacted density goes on increasing, till a maximum dry density is achieved. After which a further addition of water decreases the density.

(b) Amount of compaction: The effect of increasing the compactive energy results in an increase in the maximum dry density & decrease in the optimum water content.

(c) Method of compaction:

The density obtained during compaction greatly depends upon the type of compaction or the manner in which the compactive efforts is applied.

(d) Type of soil:

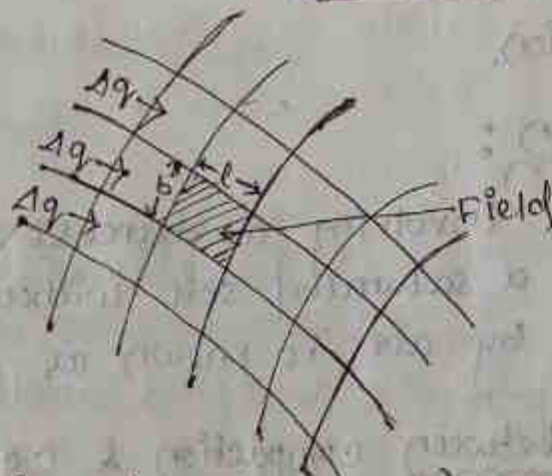
Well graded coarse grained soils attain a much higher density at lower optimum water content than fine grained soils which require more water.

(e) Addition of admixtures:

The compaction properties can be modified by a no. of admixtures other than soil material.

Flow Net : —

- The quantity of water flowing through a saturated soil mass as well as the distribution of water can be estimated by the theory of two dimensional flow of fluids through porous medium.
- The solution of two dimensional flow given by Laplace equation gives two sets of curves known as equipotential lines & stream lines mutually orthogonal to each other.
- The equipotential lines represent the lines of equal head & the stream lines or flow lines represent the path along which the individual particles of water seeps (seepage) through the soil.
- The graphical representation of equipotential lines & flow lines are known as flow net.



(Portion of a flow net)

Properties of a flow net : —

- The flow lines & equipotential lines meet at right angles to each other.
- The fields are approximately squares so that a circle can be drawn touching all the four sides of the square.
- The quantity of water flowing through each flow channel is same.
- Smaller the dimensions of the field greater will be the hydraulic gradient & velocity of flow through it.

→ In a homogenous soil every transition in the shape of curves is smooth being either elliptical or parabolic in shape.

[CONSOLIDATION]

↳ Compressibility:

→ When a compressive load is applied to soil mass, it decreases in its volume takes place. The decrease in the volume of soil mass under stress is known as compression & the property of soil mass pertaining to its susceptibility (resistance) to decrease in the volume under pressure is known as "compressibility".

→ There are two methods of compressibility.

① Compaction

② consolidation.

Consolidation:

→ The process involving a decrease in the water content of a saturated soil without replacement of water by air is known as consolidation.

Imp Difference between compaction & consolidation.

Compaction

i) Expulsion of air from voids with replacement of water.

ii) Compaction is a rapid process.

iii) Reduction in volume under loading for short duration.

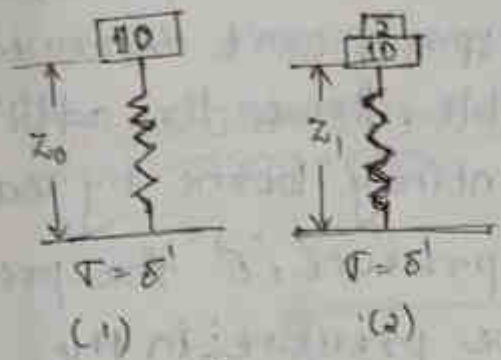
consolidation

i) Expulsion of air from voids without replacement of water.

ii) It is a gradual process.

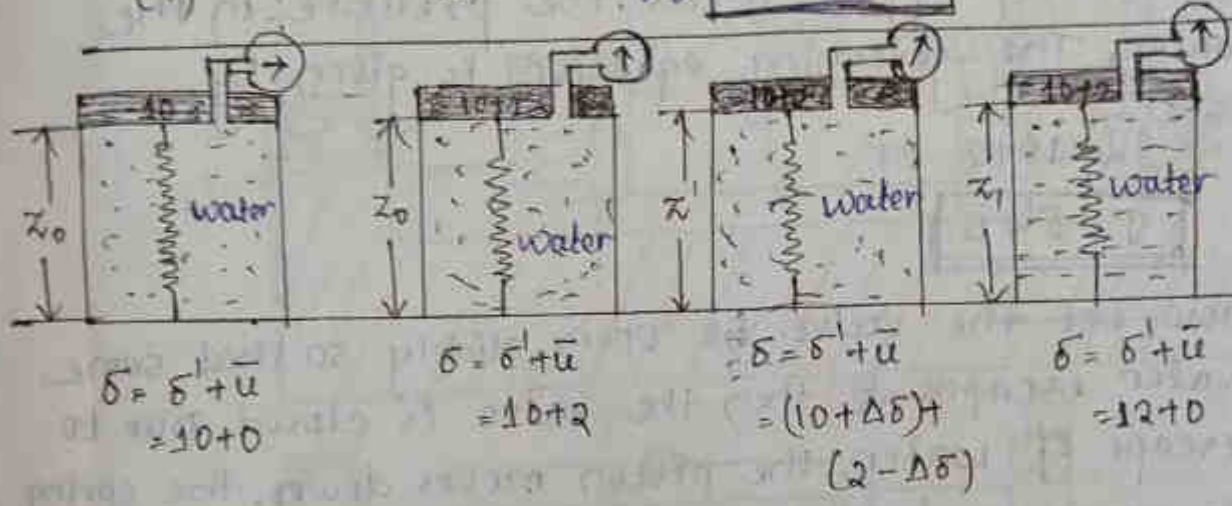
iii) Reduction in volume under sustained loading.

Consolidation process (Spring Analogy) :-



σ = Total pressure
 σ' = Effective pressure
 (Pr. absorbed by spring)

$\therefore \sigma = \sigma' + \bar{u}$



- ① → Valve close
- ② → Valve partially open.
- ③ → Valve open.

Spring Analogy (Terzaghi's) :-

- The mechanics of consolidation was given by "Terzaghi" by means of piston & spring analogy.
- Let a spring is attached with a piston on its top. the length of the spring be " z_0 " under a pressure of 10 units. If 12 units of pressure are added to its top, the spring will be compressed immediately to a length " z_1 ".
- A further application of load will result in further decrease in the length of the spring.
- If the spring & piston is placed in a cylinder containing water upto the bottom of the piston & a valve that its bottom, water will be free of stress since the whole load is carried by the spring

alone.

→ If the pressure on the piston is increased to 12 units & the valve is close, the spring can't deformed, since water is incompressible. Hence the additional pressure of two units is entirely borne by water.

→ If σ denotes the total pressure, δ the pressure in the spring & u as the pressure in the water. The governing equation is given by,

$$\therefore 12 = 10 + 2 \text{ or}$$

$$\boxed{\sigma = \delta + u}$$

→ Now, let the valve be open slightly so that some water escapes & then the valve is closed. Due to escape of water, the piston moves down, the spring is compressed & hence some pressure out of pressure of 2 units is now transfer to the spring. Thus at any intermediate stage, the pressure equation form,

$$\Rightarrow \boxed{12 = (10 + \Delta\delta) + (2 - \Delta\delta)}$$

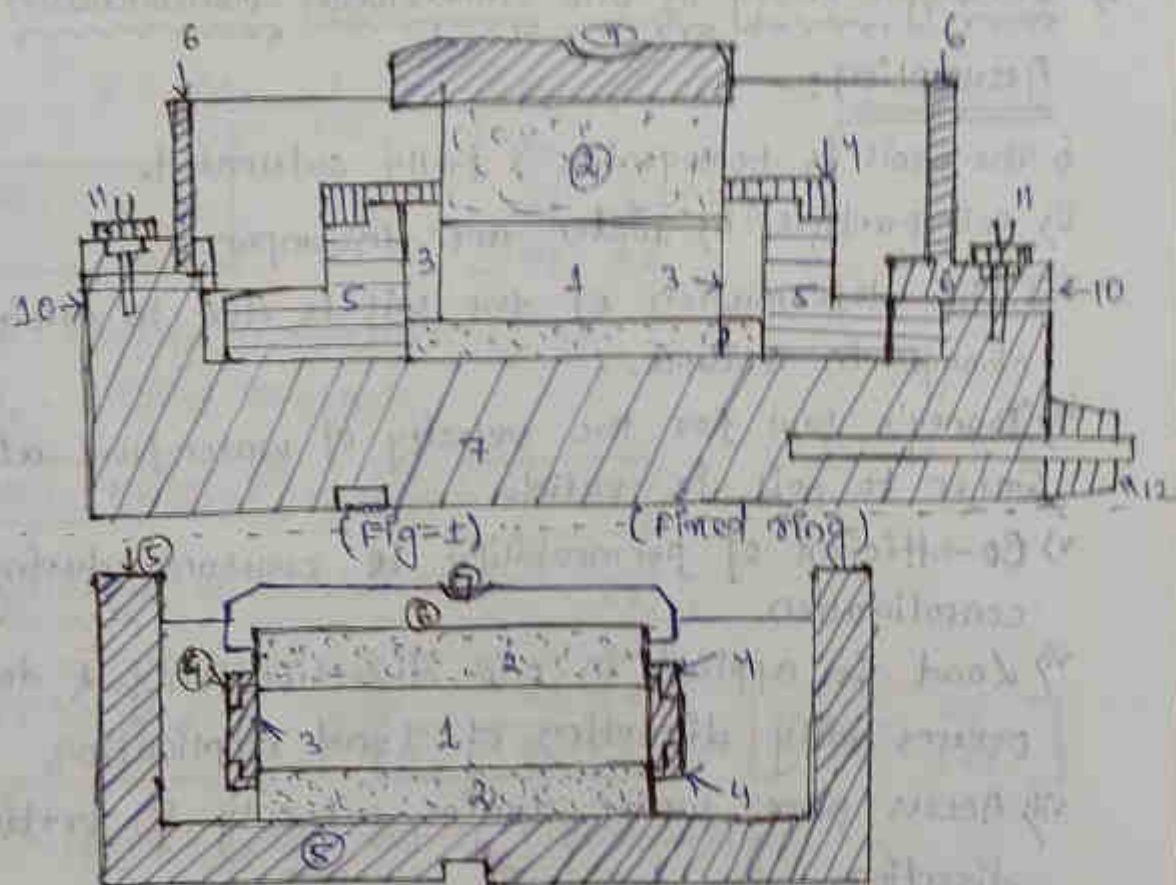
→ If the valve is fully opened sufficient water will escape till the length of the spring is reduced to a height of z_1 . Thus the whole of 2 units of pressure is transfer from water to the spring. The water become free of pressure. Thus the pressure equation at its stage,

$$\boxed{12 = 12 + 0}$$

→ This analogy can be applied to the consolidation process of a soil mass consisting of soil water system. The grain structure represents the spring while the voids filled with water represent the cylinder.

The valve opening is represented by the permeability of the soil mass & the rate of load transfer from water to soil depends upon the permeability.

Consolidation test : (lab test)



- Figure-1 : (Fixed ring)
- Figure-2 : (Floating ring consolidation cell)
- | | | |
|-------------------|-------------------|--------------------|
| 1 = Soil specimen | 5 = Outer ring | 9 = Pressure ball |
| 2 = Porous stone | 6 = Water channel | 10 = Rubber gasket |
| 3 = Specimen ring | 7 = Base | 11 = Bone |
| 4 = Guide ring | 8 = Pressure bed | 12 = Drain tube |

→ The lab consolidation test is conducted with an apparatus known as consolidometer consisting of a loading frame & consolidation cell in which the specimen is kept. Porous stone are put on the top & bottom ends of the specimen.

* In fixed ring cell only the top porous stone is permitted to move downwards as the specimen compresses.

* In floating ring cell both the top & bottom porous stones are free to compress the specimen towards

-the middle.

→ Direct measurement of permeability of the specimen at any stage of loading can be made in the fixed ring type.

↳ Terzaghi theory of one dimensional consolidation:

Assumption:-

i) The soil is homogenous & fully saturated.

ii) Soil particle in water are incompressible.

iii) The deformation of the soil is due to entirely change in volume.

iv) Darcy's law for the velocity of water flow at water to soil is valid.

v) Co-efficient of permeability is constant during consolidation.

vi) Load is applied in one direction only & deformation occurs only direction of load application.

vii) Access pore water drains out only in vertical direction.

viii) The boundary is a fine free surface offering no resistance to flow of water from the soil.

ix) The change in thickness of the layer during consolidation is insignificant.

x) The time lag in consolidation is due to the permeability of the soil.

↳ Coefficient of compressibility (a_v):-

$$a_v = \frac{-\Delta e}{\Delta \sigma'} = \frac{e_0 - e}{\sigma'_1 - \sigma'_0}$$

→ It is defined as the decrease in voids ratio per unit increase of pressure.

→ '-ve' sign indicates the decrease in void ratio.

Co-efficient of volume change (m_v):

* It is defined as the change in volume of a soil per unit of initial volume due to a given unit increase in the pressure.

$$m_v = -\frac{\Delta e}{1+e_0} \cdot \frac{1}{\Delta \sigma}$$

$$= \frac{-\Delta e}{\Delta \sigma} \cdot \frac{1}{1+e_0} = \frac{a_v}{1+e_0}$$

$$\Rightarrow \boxed{m_v = \frac{a_v}{1+e_0}} \quad m_v = \frac{-\Delta H}{H_0} \cdot \frac{1}{\Delta \sigma}$$

ΔH = change in thickness of soil

H_0 = Initial thickness

$$\Rightarrow \boxed{\Delta H = -m_v H_0 \Delta \sigma} \quad \Rightarrow \boxed{\Delta F = m_v H \Delta \sigma}$$

Co-efficient of consolidation (c_v):

$$\boxed{c_v = \frac{k}{m_v \gamma_w}} \quad \Rightarrow \quad \boxed{c_v = \frac{k(1+e_0)}{a_v \gamma_w}} \quad \left[\because m_v = \frac{a_v}{1+e_0} \right]$$

where, k = Co-efficient of permeability

m_v = co-efficient of volume change.

γ_w = Unit weight of water.

a_v = co-efficient of compressibility.

e_0 = Initial voids ratio.

Time Factor (T_v):

$$\boxed{T_v = \frac{c_v t}{d^2}}$$

where, d = Drainage path (It represents the maximum distance which the water particles have to travel for reaching the free drainage layer)

$\therefore d = H$ (For single drainage)

$d = H/2$ (For double drainage)

Degree of consolidation (U):

$$U = \frac{s}{s_f} \times 100$$

s = Initial settlement
 s_f = Final settlement

Expression for T_v :

① When, $U < 60\%$

$$\Rightarrow T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2$$

② When, $U > 60\%$

$$\Rightarrow T_v = -0.9332 \log_{10} \left(1 - \frac{U\%}{100} \right) - 0.0851$$

$$\text{or } T_v = 0.17813 - 0.9332 \log_{10} (100 - U\%)$$

Q1 An undisturbed sample of clay 24mm thick consolidated 50% in 20 minutes when tested in the lab with drainage allow at top & bottom. The clay layer from which the sample was obtained is 4m thick in the field. How much time will it take to consolidate 50% with double drainage, if the clay layer has only single drainage. Calculate the time to consolidate 50%. Assume uniform distribution of consolidation pressure.

Solⁿ: Given data,

Degree of consolidation (U) = 50%

Soil sample

$$U_2 = 50\%$$

$$t_2 = 20 \text{ min}$$

$$H_2 = 24 \text{ mm}$$

$$d_2 = \frac{24}{2} = 12 \text{ mm}$$

$$T_{v1} = C_{v1} \times \frac{t_1}{d_1^2} \quad \& \quad T_{v2} = C_{v2} \times \frac{t_2}{d_2^2}$$

$$\therefore \frac{T_{v1}}{T_{v2}} = \frac{C_{v1} \times \frac{t_1}{d_1^2}}{C_{v2} \times \frac{t_2}{d_2^2}} \quad \& \quad \frac{T_{v1}}{T_{v2}} =$$

Soil in field

$$U_1 = 50\%$$

$$t_1 = ?$$

$$H_1 = 4 \text{ m}$$

$$d_1 = \frac{4}{2} = 2 \text{ m}$$

$$\rightarrow \frac{t_1}{d_1^2} = \frac{t_2}{d_2^2}$$

$$\rightarrow t_1 = \frac{t_2}{d_2^2} \times d_1^2 = \frac{20}{(12 \times 10^{-3})^2} \times 2^2 = 55555.55 \text{ min} \\ = 385.8 \text{ days}$$

For single drainage,

$$\left(\frac{t_1}{d_1^2} = \frac{t_2}{d_2^2} \right) \rightarrow t_1 = \frac{t_2}{d_2^2} \times d_1^2 \\ = \frac{20}{(12 \times 10^{-3})^2} \times 4^2 = 222222.22 \text{ min} \\ = 1543.2 \text{ days (Ans)}$$

Q2 An undisturb sample of a clay stratum 2m thick was tested in laboratory & the avg. value of co-efficient of consolidation was found to be $2 \times 10^{-4} \text{ cm}^2/\text{sec}$.

If a structure is build on the clay stratum, how long will it take to attend half the ultimate settlement under the load of the structure. Assume double drainage

Solⁿ: Given data,

$$H = 2\text{m}$$

$$C_v = 2 \times 10^{-4} \text{ cm}^2/\text{sec}$$

$$U = 50\%$$

$$t = ? \quad \text{and } d = H/2 = 1\text{m} = 100\text{cm}$$

$$\therefore T_v = C_v \times \frac{t}{d^2} \quad \Rightarrow t = \frac{T_v \times d^2}{C_v}$$

$$\Rightarrow T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2 = \frac{\pi}{4} \left(\frac{50\%}{100} \right)^2$$

$$\Rightarrow t = \frac{0.196 \times 100\text{cm}^2}{2 \times 10^{-4} \text{ cm}^2/\text{sec}} = 9800000 = 9.8 \times 10^6 \text{ sec (Ans)}$$

Q3 A layer of clay 2m thick subjected to a loading of 0.5 kg/cm^2 . 1 year after loading the apparent consolidation is 50%. The layer has double drainage.

(i) what is the co-efficient of consolidation.

(ii) If the co-efficient of permeability is 3mm per year what is the settlement after 1 year & how much time will the layer take to reach 90% consolidation.

Solⁿ Given data,

$$H = 2 \text{ m} \quad t = 1 \text{ year}$$

$$\delta = 0.5 \text{ kg/cm}^2, \quad d = 1 \text{ m} = 100 \text{ cm}$$

$$U = 50\%$$

$$\textcircled{1} C_v = T_v \times \frac{d^2}{t}$$

$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2 = \frac{\pi}{4} \left(\frac{50\%}{100} \right)^2 = 0.196 \quad (\because U < 60\%)$$

$$\therefore C_v = 0.196 \times \frac{(100 \text{ cm})^2}{1 \text{ year}} = 1960 \text{ cm}^2/\text{year}$$

$$\textcircled{2} K = 8 \text{ mm/year} = 0.8 \text{ cm/year}$$

$$J_s = m_v \Delta S H_0$$

$$\Rightarrow m_v = \frac{K_v \times \Delta S}{C_v \times \gamma_w} = \frac{0.8 \times 10^{-2}}{1960 \times 10^{-4} \times 1000} = 15.3 \times 10^{-6}$$

$$\Delta S = 0.5 \text{ kg/cm}^2 = \frac{0.5 \text{ kg}}{10^{-4} \text{ m}^2} = 0.5 \times 10^4 \text{ kg/m}^2$$

$$J_s = m_v \Delta S H = 15.3 \times 10^{-6} \times 0.5 \times 10^4 \times 2 = 0.153 \text{ m}$$

$$\textcircled{3} U = 90\% > 60\%$$

$$T_v = -0.9332 \log_{10} \left(1 - \frac{U\%}{100} \right) = -0.0851$$

$$= -0.9332 \log_{10} \left(1 - \frac{90}{100} \right) = -0.0651$$

$$= 0.8481$$

$$\left(C_v = T_v \frac{d^2}{t} \right) \frac{t}{d^2} = \frac{T_v}{C_v}$$

$$\Rightarrow 1960 \times 10^{-4} = 0.8481 \times \frac{(100)^2}{t}$$

$$\Rightarrow t = \frac{0.8481}{1960 \times 10^{-4}} \times \frac{1}{(100)^2}$$

Q4 A soil stratum 10m thick with pervious stratum on top & bottom. Determine the time required for 50% consolidation. Given the following data, coefficient of permeability = 10^{-7} cm/sec, coefficient of compressibility (a_v) = $0.0003 \text{ cm}^2/\text{gm}$

Void ratio = 2

Time Factor = $(T_v) = 0.197$

Solⁿ Given data,

$$H = 10\text{m}$$

$$d = 10/2 = 5\text{m}$$

$$U = 50\%$$

Co-efficient of permeability $(k) = 10^{-9}\text{cm/sec.}$

co-efficient of compressibility $(a_v) = 0.0003\text{cm}^2/\text{gm.}$

Void ratio = 2 = (e_0)

Time factor $(T_v) = 0.197$

$$C_v = \frac{k(1+e_0)}{a_v \gamma_w} = \frac{10^{-9} \times (1+2)}{0.0003 \times 1} = 1 \times 10^{-3}\text{cm}^2/\text{sec.}$$

$$T_v = C_v \times \frac{t}{d^2} \Rightarrow t = \frac{T_v}{C_v} \times d^2$$
$$= \frac{0.197 \times 500^2}{1 \times 10^{-3}} = 49250 \times 10^3\text{sec.}$$

$\Rightarrow t \approx 570\text{days (Ans)}$

Q-5 The time to reach 40% consolidation on a two way drain laboratory 1cm thick saturated clayey soil sample is 35sec. Determine the time required for 60% consolidation of the same soil 10m thick on the top of a rocky surface subjected to the same loading condition as the laboratory sample.

Solⁿ Given data,

Lab

$$U_1 = 40\%$$

$$H_1 = 1\text{m}$$

$$d_1 = H_1/2 = 0.5\text{m}$$

$$t_1 = 35\text{sec.}$$

Field

$$H_2 = 10\text{m}$$

$$d_2 = 5\text{m}$$

$$U_2 = 60\%$$

$$t_2 = ?$$

$$T_{v1} = \pi/4 \left(\frac{U_1}{100} \right)^2$$
$$= \pi/4 \times \left(\frac{40}{100} \right)^2$$
$$= 0.1256$$

$$T_{v2} = \pi/4 \left(\frac{U_2}{100} \right)^2$$
$$= \pi/4 \times \left(\frac{60}{100} \right)^2 = 0.2827$$

$$Cv_1 = Tv_1 \frac{d_1^2}{t_1} = 0.1256 \times \frac{(0.5)^2}{35} = 8.97 \times 10^{-4} \text{ cm}^2/\text{sec.}$$

But, $Cv_1 = Cv_2 = 8.97 \times 10^{-4} \text{ cm}^2/\text{sec.}$

$$\Rightarrow Cv_2 = Tv_2 \times \frac{d_2^2}{t_2}$$

$$\Rightarrow t_2 = \frac{Tv_2 \times d_2^2}{Cv_2} = \frac{0.2827}{8.97 \times 10^{-4}} \times 500^2$$

$$= 78790412.49 \approx 911 \text{ days (Ans)}$$

[SHEAR STRENGTH]

Chapter-7

When a soil is loaded shearing stresses are induced in it. When the shearing stresses reach a limiting value shear deformation takes place leading to the failure of the soil mass. The failure may be in the form of sinking of a footing or movement of a wedge of a soil behind a retaining wall forcing it to move out or the slide in an earth embankment.

→ The shear strength of soil is defined as the resistance to deformation by continuous shear displacement of soil particles or on masses upon the action of a shear stress.

→ The shearing resistance of soil is composed of:-

- (i) The structural resistance to displacement of the soil because of interlocking of the particle.
- (ii) Frictional resistance to translocation between the individual soil particles & their contact point.
- (iii) Cohesion or adhesion between the surface of the soil particle.

Mohr's stress circle:

→ In a loaded soil mass innumerable planes pass through a point & stress components on each plane depends upon the direction of the plane. They are exist three typical plane mutually orthogonal to each other on which the stress is wholly normal & no shear stress acts. These planes are called principal planes & the normal stresses acting on these planes are called principal stresses.

→ In the order of decreasing magnitude these planes are called major, intermediate & minor principal planes & corresponding stresses are called major principal stress (σ_1), intermediate principal stress (σ_2),

✓ A minor principal stress (σ_3)
Mohr's-Coulomb's Failure theory: →

Assumptions:

- (i) Materials fails essentially by shear. The shear stress causing failure depends upon the properties of material as well as on normal stress on the failure plane.
 - (ii) The ultimate strength of material determined by stresses on the failure plane.
 - (iii) When the material is subjected to three dimensional principal material stresses $\sigma_1, \sigma_2, \sigma_3$, the intermediate principal stress does not have any influence on the strength of the material.
- * The Mohr-Coulomb's failure theory can be expressed by the equation,

$$\tau_f = S = F(\sigma)$$

$$\Rightarrow \tau_f \text{ (or } S) = c + \sigma \tan \alpha$$

Where, τ_f = shear stress on failure plane / shear resistance of the material.

c = cohesion (Intercept in the shear axis)

α = Angle of internal friction / slope of the Mohr's envelope.

Measurement of shear strength:

1. Direct shear test
2. Triaxial compression test
3. Unconfined compression test
4. Vane shear test.

* Again depending upon the drainage condition three type of shear test are there.

1. Undrained test
2. Consolidated undrained test
3. Drained test.

→ In the undrained or quick test no drainage of water is permitted. Therefore there is no dissipation of pore water pressure during the entire test.

→ In drained test drainage is permitted throughout the test.

→ In the consolidated undrained test, drainage is permitted during primary consolidation & no drainage is allowed afterwards.

1. Direct shear test: —

→ It is a simple & commonly used test performed in a shear box apparatus which consists of a 2 piece shear box of square or circular cross section. The lower half of the box is rigidly held in position in a container which rest over slides or rollers & which can be pushed forward at a constant rate by gear driven either by electric motor or by hand.

→ The upper half of the box against a proving ring. The soil sample is compacted in the shear box & it is held between metal grids & porous stone.

→ The upper half of the specimen is held in the upper box & the lower half in the lower box & the joint between the two parts of the box is the level of the centre of the specimen.

→ Normal load is applied on the specimen from a loading frame bearing upon steel ball of pressure plate. When a shearing force is applied to the lower box through the gear the movement of the lower part is transmitted through the specimen to the upper part of the box & hence on the proving ring. The deformation of the proving ring indicates the shear force.

2. Triaxial Compression test :-

- In triaxial compression test the solids specimen cylindrical in shape is subjected to direct stresses acting in three mutually perpendicular directions.
- In the common solid cylindrical specimen test, the major principal stress (σ_1) is applied in the vertical direction & the other two principal stresses σ_2 & σ_3 ($\sigma_2 = \sigma_3$) are applied on the horizontal direction by the fluid pressure round the specimen.
- The test equipment consist of a high pressure cylindrical cell made of perspex preated between the base & the top cap. Three outlet connection are provided through the base, cell fluid inlet, pore water outlet from the bottom of the specimen & the drainage outlets at the top of the specimen.
- A separate compression is used to apply fluid pressure in the cell. pore pressure developed in the specimen during the test can be measure with the help of a separate pore pressure measuring equipment.
- The cylindrical specimen is enclosed in a rubber membrane. A stainless steel piston running through the centre of the top cap applies the vertical compressive load on the specimen & the load is applied through a proving ring with the help of a mechanically operated load frame.

3. Unconfined compression test :-

- It is a special case of triaxial compression test in which $\sigma_2 = \sigma_3 = 0$ due to absence of confining cell pressure the uniaxial test is called the unconfined compression test.
- The cylindrical specimen of the soil is subjected to major principal stress " σ " by the

to shearing. The apparatus consist of a small and load frame fitted with a proving ring & the deformation of the sample is measure with the help of a dial gauge.

4. Drain shear test:

- It is a quick test to determine the undrained shear strength of cohesive soil. The drain shear test consist of four thin steel plates called vanes welded orthogonally with a steel rod. A torque measuring arrangement is attached to the rod which is rotated by a gear & wheel arrangement.
- After pushing the vane gently into the soil the top plate is rotated in a uniform speed ($1^\circ/\text{m}$). The rotation of the vane shears the soil along a cylindrical surface.
- The rotation in degree is indicated by pointer moving on a graduated tie attached to the wheel shaft. The torque can be calculated by multiplying the dial value in the spring constant.

$$T = \pi d^2 \tau_f \left(\frac{H}{2} + \frac{d}{12} \right)$$

Where,

H = Height of the vane

d = Diameter of the vane

τ_f = Shear strength of the soil.

Retaining Wall:

- A retaining wall or a retaining structure is used for maintaining the ground surfaces at different elevations on either side of it.
- The material retained or supported by the structure is called back fill which may have a top surface horizontal or inclined.
- The position of the backfill lying above a horizontal plane at the elevation of the top of the wall is called surcharge & its inclination to the horizontal is called surcharge angle.

Plastic equilibrium of soils:

- A body of the soil is said to be in plastic equilibrium if every point of it is on the verge of failure.
- The ratio of horizontal stress (σ_h) to vertical stress (σ_v) is called the co-efficient of earth pressure when the soil is in the active state of plastic equilibrium.

$$\sigma_h = \sigma_3 \quad \& \quad \sigma_v = \sigma_1$$

- Then, co-efficient of earth pressure.

$$K_a = \frac{\sigma_h}{\sigma_v} = \frac{\sigma_3}{\sigma_1}$$

$$K_a = \frac{\sigma_3}{\sigma_1} = \frac{1 - \sin \alpha}{1 + \sin \alpha}$$

- When the soil is in plastic state of equilibrium.

$$\sigma_h = \sigma_1 \quad \& \quad \sigma_v = \sigma_3$$

- Then, co-efficient of earth pressure.

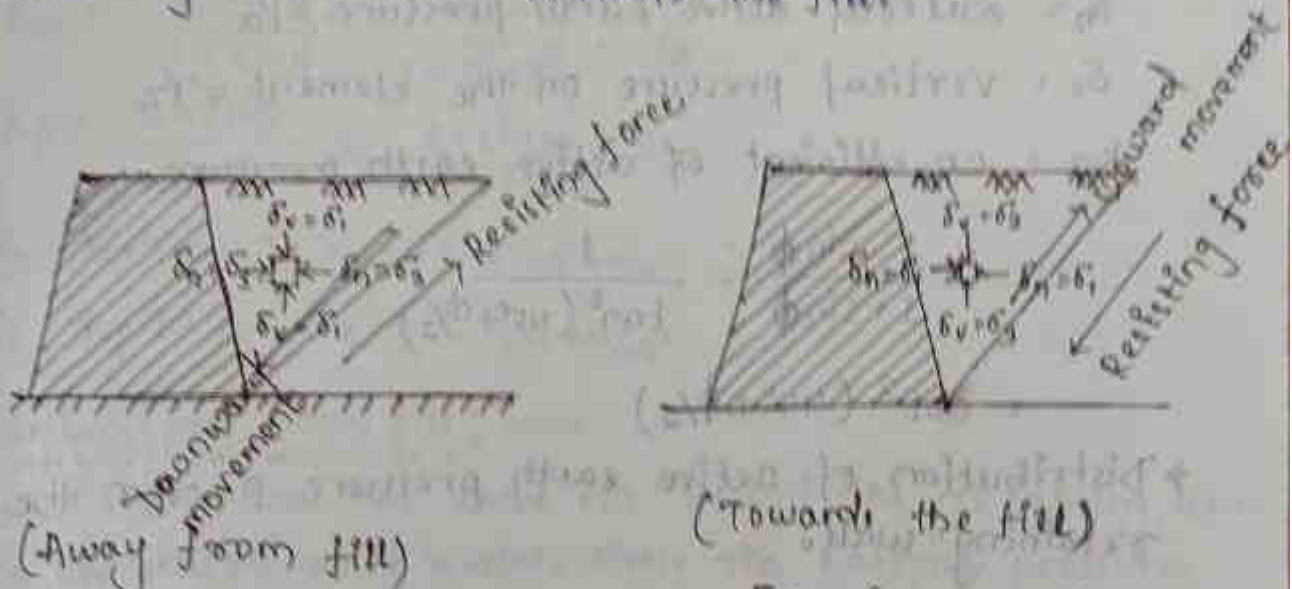
$$K_p = \frac{\sigma_h}{\sigma_v} = \frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \alpha}{1 - \sin \alpha}$$

- When the soil is at rest the ratio of horizontal to vertical stress is called the co-efficient of

earth pressure at rest.

$$K_0 = \frac{\delta_h}{\delta_v}$$

→ During the active state the retaining wall moves away from fill & a certain portion of the back fill located immediately behind the wall breaks away from the rest of the soil mass. During the passive state the retaining wall moves towards the fill.



(Away from fill)

(Towards the fill)

[Active state]

[Passive state]

Active earth pressure (Rankine's theory)

(Cohesionless soil only)

Assumptions: —

1. The soil mass is homogeneous dry & cohesionless.
2. The ground surface is a plane which may be horizontal or inclined.
3. The back of the wall is vertical & smooth.

Three cases of cohesionless backfill:

- 1) Backfill with no surcharge → soil
- 2) Submerged back fill → soil + water.
- 3) Backfill with uniform surcharge.

4) Backfill with no surcharge: —

consider an element at a depth 'x', below the ground surface, when the wall is at the point of moving

Outwards the active state of plastic eq. is established. The horizontal pressure " δ_h " is then the minimum principal stress δ_3 & the vertical pressure " δ_v " is the maximum principal stress δ_1 .

→ From the stress relationship,

$$k_a = \frac{\delta_h}{\delta_v} = \frac{\delta_3}{\delta_1}$$

δ_h = lateral active earth pressure = P_a

δ_v = vertical pressure on the element = γz

k_a = co-efficient of active earth pressure.

$$= \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1}{\tan^2(45^\circ + \phi/2)}$$

$$= \cot^2(45^\circ + \phi/2)$$

→ Distribution of active earth pressure " p " over the retaining wall.

At $z = H$ (the earth pressure)

$$P_a = k_a \gamma H$$

→ The total active pressure " P_a " or the resultant pressure per unit length of the wall is given by →

$$P_a = \int_0^H k_a \gamma H = k_a \gamma \left[\frac{H^2}{2} \right]_0^H = \frac{1}{2} k_a \gamma H^2$$

$$\Rightarrow P_a = \frac{1}{2} k_a \gamma H^2$$

(Again) at $H/3$ above the base of the wall)

$$P_p = k_p \gamma H$$

$$P_p = \frac{1}{2} k_p \gamma H^2$$

Q1 Calculate the intensity of active & passive earth pressure at depth 8m in a dry cohesionless sand with an angle of internal friction of 30° & unit weight of 18 kN/m^3 .

Given data,

$$z = 8 \text{ m}$$

$$P_a = K_a \gamma H$$

$$\phi = 30^\circ$$

$$P_p = K_p \gamma H$$

$$\gamma = 18 \text{ kN/m}^3$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} = 3$$

$$P_a = K_a \gamma H = \frac{1}{3} \times 18 \times 8 = 48 \text{ kN/m}^2$$

$$P_p = K_p \gamma H = 3 \times 18 \times 8 = 432 \text{ kN/m}^2$$

2) Submerged back fill: —

→ In this case the sand fill behind the retaining wall is saturated with water, then the lateral pressure is made up of two components.

(i) Lateral pressure due to submerged weight (γ') of the soil.

(ii) Lateral pressure due to water thus at any depth z' below the surface.

$$P_a = K_a \gamma' z + \gamma_w z$$

3) Backfill with uniform surcharge: —

→ If the back fill is horizontal & carries a surcharge of uniform intensity q per unit area. The vertical pressure at any depth z' , will increase by q . Hence the lateral pressure at any depth z' is given by →

$$P_a = K_a \gamma z + K_a q$$

Q2 What will be the intensities of active & passive earth pressure if the water level rises to the ground level. Take saturated unit weight of sand as 22 kN/m^3 .

Solⁿ $P_a = k_a \gamma' z + \gamma_w z$

$\gamma_{sat} = 22 \text{ kN/m}^3$

$\gamma_w = 9.81 \text{ kN/m}^3$

$\gamma' = 22 - 9.81 = 12.19 \text{ kN/m}^3$

$P_a = k_a \gamma' z + \gamma_w z = \frac{1}{3} \times 12.19 \times 8 + 9.81 \times 8$
 $= 110.98 \text{ kN/m}^2$

$P_p = k_p \gamma' z + \gamma_w z = 3 \times 12.19 \times 8 + 9.81 \times 8$
 $= 371.04 \text{ kN/m}^2 \text{ (Ans)}$

Q3 A retaining wall 4m height has a smooth vertical back. The back fill has a horizontal surface in level with the top of the wall. There is uniformly distributed surcharged load of 36 kN/m^2 intensity over the back fill. The unit weight of the back fill is 18 kN/m^3 , its angle of shearing resistance 30° & cohesion is zero. Determine the magnitude & point of application of active pressure per meter length of the wall.

Given data :

$\gamma = 18 \text{ kN/m}^3$

$q = 36 \text{ kN/m}^2$

$H = 4 \text{ m}$

$\phi = 30^\circ$

$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1}{3}$

$P_a = P_1 + P_2 = k_a q + k_a \gamma z$

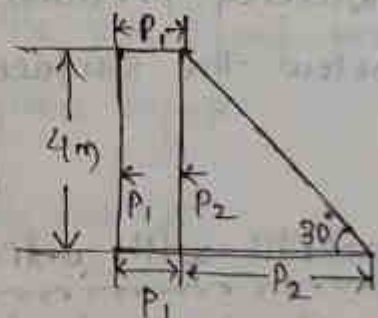
$P_1 = k_a q = \frac{1}{3} \times 36 = 12 \text{ kN/m}^2$

$P_2 = k_a \gamma z = \frac{1}{3} \times 18 \times 4 = 24 \text{ kN/m}^2$

$P_a = P_1 + P_2 = 12 + 24 = 36 \text{ kN/m}^2$

$\therefore P_1 = 12 \times 4 = 48 \text{ kN/m}$

$P_2 = \frac{1}{2} k_a \gamma H^2 = k_a \gamma H \times \frac{H}{2} = 24 \times \frac{4}{2} = 48 \text{ kN/m}$



P_1 acting at $H/2 = 4/2 = 2m$ from the base wall

$$\bar{x} = \frac{P_1 \times x_1 + P_2 \times x_2}{P_1 + P_2}$$

$$= \frac{48 \times 2 + 48 \times 4/3}{96} = 1.66m \text{ (Ans)}$$

Q11 - A soil mass is retained by a smooth backed vertical wall of 6m height. The soil has a bulk unit weight of $20kN/m^3$ & ϕ is 16° . The top of the soil is level with the top of the wall & is horizontal. If the soil surface carries a uniformly distributed load of $4.5kN/m^2$. Determine the total active pressure on the wall per meter length of the wall & its point of application.

Solⁿ Given data :-

$$\gamma = 20kN/m^3$$

$$q = 4.5kN/m^2$$

$$\phi = 16^\circ, H = 6m$$

Intensity of active earth pressure due to surcharge, q_1

$$P_1 = K_a q$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 16^\circ}{1 + \sin 16^\circ} = 0.56$$

$$\therefore P_1 = K_a q = 0.56 \times 4.5 = 2.52kN/m^2$$

Intensity due to back fill, $P_2 = K_a \gamma H$

$$P_2 = 0.56 \times 20 \times 6 = 67.2kN/m^2$$

Total resultant pressure due to surcharge per meter length of the wall, $P_1 = P_1 \times H$

$$= P_1 = 2.52 \times 6 = 15.12kN/m^2$$

Total resultant pr. due to back fill,

$$P_2 = \frac{1}{2} K_a \gamma H^2 = P_2 \times H/2$$

$$\Rightarrow P_2 = 67.2 \times 6/2 = 201.6kN/m^2$$

\therefore Total resultant pressure, $P = P_1 + P_2$

$$\Rightarrow P = 15.12 + 201.6 = 216.72$$

P_1 acts as a distance of $H/2$ above the base of the wall.

P_2 acts as a distance of $H/3$ above the base of the wall.

∴ The resultant pr. P acts as a distance of,

$$\bar{x}_0 = \frac{P_1 \times H/2 + P_2 \times H/3}{P_1 + P_2}$$

$$= \frac{15.12 \times 3 + 201.6 \times 2}{15.12 + 201.6}$$

$$= 2.069 \text{ m (Ans)}$$

BEARING CAPACITY :-

- (i) Footing → It is a portion of the foundation of a structure that transmits loads directly to the soil.
- (ii) Foundation → It is that part of the structure which is in direct contact with the ground & transmits loads to the ground.
- (iii) Foundation soil : It is the upper part of the soil mass carrying the load of the structure.
- (iv) Bearing capacity : It is the supporting power of soil.
- (v) Gross pressure intensity (q) :
It is the total pressure at the base of the footing due to the weight of the super-structure, self weight of the footing & weight of the earthfill.
- (vi) Net pressure intensity (q_n) :
It is the excess pressure or difference in intensities of gross pressure after the construction of the structure & the original over bottom pressure.

$$q_n = q - \bar{\sigma}$$

$$= q - \gamma D$$

where, D = Depth of the footing
 γ = Unit weight of the soil above the base of the foundation.

- (vii) Ultimate bearing capacity (q_f) :
It is the minimum gross pressure intensity at the base of the foundation at which the soil fails in shear.

- (viii) Net ultimate bearing capacity (q_{nf}) :
→ It is the minimum net pressure intensity causing shear failure of the soil.

$$q_{nf} = q_f - \bar{\sigma}$$

$$\Rightarrow q_{nf} = q_f - \gamma D$$

$$\Rightarrow q_f = q_{nf} + \bar{\sigma}$$

(ix) Effective surcharge ($\bar{\sigma}$):

It is the intensity of vertical pressure at which the base level of foundation.

(x) Safe bearing capacity (q_s):

It is the maximum pressure which the soil can carry safely without any risk of shear failure.

(xi) Net safe bearing capacity (q_{ns}):

It is the net ultimate bearing capacity, divided by a factor of safety.

$$q_{ns} = \frac{q_{nf}}{F}$$

F = Factor of safety

$$q_s = q_{ns} + \bar{\sigma}$$

$$= \frac{q_{nf}}{F} + \gamma D$$

Imp Types of bearing capacity : —

(i) General shear failure \rightarrow In this case continuous failure surfaces developed between the edges of footing of ground surfaces.

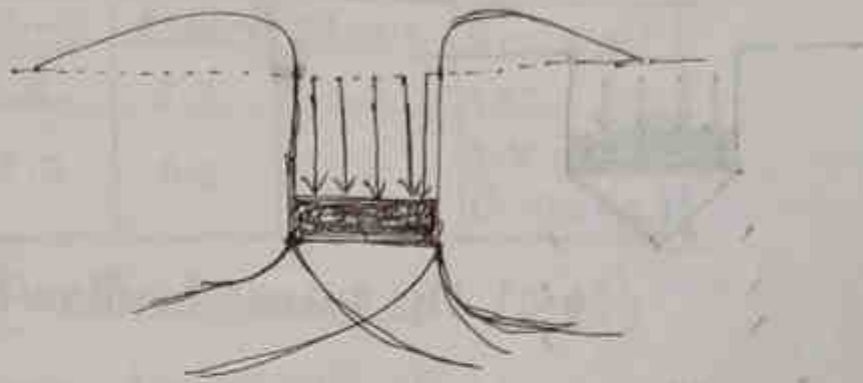
\rightarrow It has well define failure surface reaching upto the ground.

\rightarrow There is considerable bulging of soil adjacent of to the footing.

\rightarrow Failure is accompanied by tilting of the footing.

\rightarrow Failure is sudden.

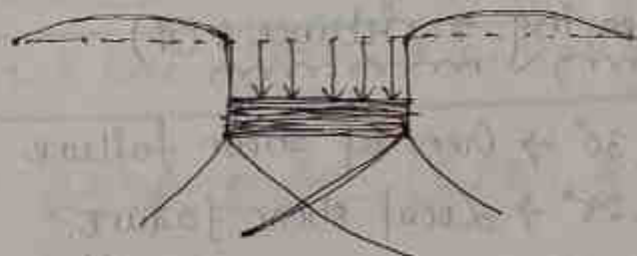
\rightarrow The ultimate bearing capacity is well defined.



(General shear failure)

ii) Local shear failure: →

- Failure pattern is clearly define only immediately below the footing.
- The failure surfaces do not reach the ground surface.
- There is only slight bulging of soil around the footing.
- Failure is not sudden & there is no tilting of footing.
- Ultimate bearing capacity is not well defined.
- Failure pattern is defined by large settlement.



(Local shear failure)

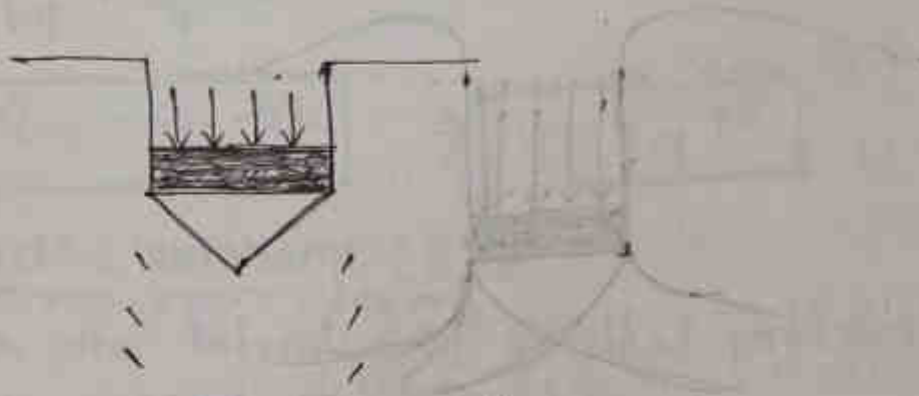
iii) Punching shear failure: →

It occurs where there is high compression of soil under the footing.

Characteristics →

- No failure pattern is observed.
- The failure surface is vertical.
- There is no bulging of soil around the footing.
- There is no tilting of footing.
- Failure is characterized in terms of very large settlements.

→ The ultimate bearing capacity is not well defined.



(punching shear failure)

↳ Terzaghi's analysis: —

Bearing capacity factors:

$$N_q = \frac{a^2}{2 \cos^2(45^\circ + \phi/2)}, \text{ where, } a = c$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_r = \frac{\tan \phi}{2} \left[\frac{k_{pr}}{\cos^2 \phi} - 1 \right]$$

k_{pr} = Passive earth pressure co-efficient

Angle of shearing resistance ϕ

For, $\phi > 30^\circ \rightarrow$ General shear failure

$\phi < 25^\circ \rightarrow$ Local shear failure.

Specialization of Terzaghi's equations:

$$q_f = c N_c S_c + \bar{\sigma} N_q + 0.5 \gamma B N_r S_r$$

N_c, N_q, N_r = Bearing capacity factors

S_c, S_r = shape factors

c = cohesion

$\bar{\sigma}$ = vertical pressure

Values of S_c & S_r :-

Shape	Strip	Round	Square	Rectangle
S_c	1.0	1.3	1.3	$1 + 0.3 B/L$
S_r	1.0	0.6	0.8	0.8 or $(1 - 0.3 B/L)$

Case-1 : (Functional cohesive soil ($c-\phi$))

(a) For square footing :-

$$q_f = 1.3 c N_c + \bar{\sigma} N_q + 0.5 \times 0.8 \gamma B N_r$$

$$q_f = 1.3 c N_c + \bar{\sigma} N_q + 0.4 \gamma B N_r$$

(b) Rectangular footing :-

$$q_f = c N_c [1 + 0.3 B/L] + \bar{\sigma} N_q + 0.4 \gamma B N_r$$

$$q_f = c N_c [1 + 0.3 B/L] + \bar{\sigma} N_q + \gamma B N_r [1 - 0.3 B/L]$$

(c) Circular footing :-

$$q_f = 1.3 c N_c + \bar{\sigma} N_q + 0.3 \gamma B N_r$$

Case-2 :- cohesive soil ($c > 0, \phi = 0$)

(a) Circular footing :-

$$q_f = 1.3 c N_c + \bar{\sigma}$$

(b) Rectangular & square footing :-

$$q_f = c N_c (1 + 0.3 B/L) + \bar{\sigma}$$

Case-3 :- Non-cohesive soil :-

(a) Circular footing :-

$$q_f = \bar{\sigma} N_q + 0.4 \gamma B N_r$$

(b) Rectangular & square footing :-

$$q_f = \bar{\sigma} N_q + 0.4 \gamma B N_r$$

Q.1) A square footing 2.5m / 2.5m is built in a homogeneous bed of sand of unit weight 20 kN/m³ & having an angle of shearing resist 36°. The depth of the base of the footing is 1.5m below the ground surface. Calculate the safe load that can be carried by a footing with a factor of safety 3. Against shear failure. Use Terzaghi's eqn.

Given, $N_c = 65.4$

$$N_q = 49.4$$

$$N_r = 54$$

Cohesion less soil: $c = 0$

$$\phi = 36^\circ$$

$$(c = 0, \phi > 0) \quad D = 1.5$$

$$\text{unit weight } \gamma = 20 \text{ kN/m}^3$$

$$(i) \quad q_f = c N_c S_c + \bar{\sigma} N_q + 0.5 \gamma B N_r$$

$$= 0 \times N_c S_c + \gamma D N_q + 0.5 \gamma B N_r \times 0.8$$

$$= 0 + 20 \times 1.5 \times 49.4 + 0.5 \times 20 \times 2.5 \times 54 \times 0.8$$

$$= 2562 \text{ kN/m}^2$$

$$(ii) \quad q_{nf}, \quad q_f = q_{nf} + \bar{\sigma}$$

$$\Rightarrow q_{nf} = q_f - \bar{\sigma}$$

$$= 2562 - \gamma D$$

$$= 2562 - 20 \times 1.5 = 2532 \text{ kN/m}^2$$

$$(iii) \quad q_s = q_{nf} + \bar{\sigma}$$

$$= \frac{q_{nf}}{F} + \gamma D$$

$$= \frac{2532}{3} + 20 \times 1.5 = 874 \text{ kN/m}^2$$

$$(iv) q_s = \frac{F_s}{A}$$

$$\Rightarrow F_s = q_s \times A = 874 \times 2.5^2 = 5462.5 \text{ kN (Ans)}$$

Q2. A rectangular footing $2\text{m} \times 3\text{m}$ rest on a c- ϕ soil with its base at 1.5m below the ground surface. Calculate the shape bearing capacity using a factor of safety of 3.

(i) Net ultimate bearing capacity

(ii) Ultimate bearing capacity.

The soil has the following parameter.

$$\gamma = 18 \text{ kN/m}^3, N_q = 22.5$$

$$c = 10 \text{ kN/m}^2, N_r = 19.7$$

$$N_c = 37.2 \quad (\text{Use Terzaghi's equation})$$

Given data :-

c- ϕ soil

Rectangular footing, $L = 3\text{m}, B = 2\text{m}$

$$c = 10 \text{ kN/m}^2, D = 1.5\text{m}$$

$$\phi = 30^\circ, F = 3$$

$$(\bar{\sigma} = \gamma D)$$

$$\therefore q_u = c N_c s_c + \bar{\sigma} N_q + 0.5 \gamma B N_r s_r$$

$$= 10 \times 37.2 \times [1 + 0.3 \times \frac{2}{3}] + 18 \times 1.5 \times 22.5 +$$

$$0.5 \times 18 \times 2 \times 19.7 \times 0.8$$

$$= 1337.58 \text{ kN/m}^2$$

$$q_{nf} = q_u - \bar{\sigma} = 1337.58 - 18 \times 1.5$$

$$= 1310.58 \text{ kN/m}^2$$

$$q_s = q_{nf} + \bar{\sigma}$$

$$= \frac{q_{nf}}{F} + \gamma D = \frac{1310.58}{3} + 18 \times 1.5$$

$$= 463.86 \text{ kN/m}^2$$

$$\Rightarrow q_s = \frac{q_u}{F} = \frac{1337.58}{3} = 445.86 \text{ kN/m}^2 \text{ (Ans)}$$

Q3) Determine the depth at which a circular footing of 2m diameter be founded to provide a factor of safety of 3. If it has to carry a safe load of 1600kN. The foundation soil has $c = 10 \text{ kN/m}^2$, $\phi = 30^\circ$ & unit weight 18 kN/m^3 . Use Terzaghi's analysis.

Given data,

$c - \phi$ soil, circular footing.

Diameter of footing, $B = 2 \text{ m}$

$$c = 10 \text{ kN/m}^2, \quad F = 3$$

$$\gamma = 18 \text{ kN/m}^3, \quad N_c = 37.2$$

$$\phi = 30^\circ, \quad N_q = 22.5$$

$$F_s = 1600 \text{ kN}, \quad N_\gamma = 19.7$$

$$(i) \quad q_s = \frac{F_s}{A} = \frac{1600}{\pi/4 \times 2^2} = 509.3 \text{ kN/m}^2$$

$$q_f = c N_c s_c + \bar{\sigma} N_q + 0.5 \gamma B N_\gamma s_\gamma$$

$$= (10 \times 37.2 \times 1.3) + (18 \times D \times 22.5) +$$

$$= 696.36 + 405D \quad (0.5 \times 18 \times 2 \times 19.7 \times 0.6)$$

$$q_{nf} = q_f - \gamma D$$

$$= 696.36 + 405D - 18D$$

$$= 696.36 + 387D$$

$$q_s = \frac{q_{nf}}{F} + \gamma D$$

$$\Rightarrow 509.3 = \frac{696.36 + 387D}{3} + 18D$$

$$\Rightarrow 509.3 \times 3 = 696.36 + 387D + (3 \times 18D)$$

$$\Rightarrow 1527.9 - 696.36 = 441D$$

$$\Rightarrow D = \frac{1527.9 - 696.36}{441} = 1.88$$

$$\approx 1.9 \text{ m (Ans)}$$

Q4 A square footing located at a depth of 1.3m below the ground has to carry a safe load of 800kN. Find the size of the footing if the desired factor of safety is 3. The soil has the following properties.

Void ratio = 0.55, $\phi = 30^\circ$
 Degree of saturation = 50%, $N_c = 37.2$
 Sp. gravity = 2.67, $N_q = 22.5$
 $c = 8 \text{ kN/m}^2$, $N_\gamma = 19.7$

Given data:

$D = 1.3 \text{ m}$, $N_c = 37.2$

$F_s = 800 \text{ kN}$, $N_q = 22.5$

$F = 3$, $N_\gamma = 19.7$

$c = 8 \text{ kN/m}^2$, void ratio (e) = 0.55

$\phi = 30^\circ$

Degree of saturation = 50% = 0.5

Sp. gravity (G) = 2.67

$$\gamma = \frac{(G + eS)\gamma_w}{1 + e}$$

$$= \frac{(2.67 + 0.55 \times 0.5) \times 9.81}{1 + 0.55} = 18.639 \text{ kN/m}^3$$

$$q_f = cN_c S_c + \bar{\sigma} N_q + 0.5 \gamma B N_\gamma$$

$$= (8 \times 37.2 \times 1.3) + (18.639 \times 1.3) + (0.5 \times 18.639 \times B \times$$

$$= 386.88 + 24.2307 + 146.8713$$

$$\Rightarrow q_f = 146.8713 + 411.11$$

$$q_f = \frac{F_s}{B}$$

$$\Rightarrow q_f = \frac{800}{B^2}$$

$$\Rightarrow 146.8713 + 411.11 = \frac{800}{B^2}$$

$$\Rightarrow 146.8713 B^3 + 411.11 B^2 = 800$$

$$\Rightarrow B = 1.17 \text{ m (Ans)}$$

IS code method :-

Strip Footing for $c-\phi$ soil :

① General shear failure \rightarrow

$$q_f = cN_c + \bar{\sigma}N_q + 0.5B\gamma N_\gamma$$

② Local shear failure \rightarrow

$$q_f = \frac{2}{3}cN_c + \bar{\sigma}N_q + 0.5\gamma B N_\gamma$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_q = \tan^2(45^\circ + \phi/2) e^{\pi \tan \phi}$$

$$N_\gamma = 2(N_q + 1) \tan \phi$$

Q1 What is the ultimate bearing capacity of a circular footing of 1.5m diameter resting on the surface of a surface saturated clay of cohesion 50 kN/m². what is the safe value if the factor of safety is 3.

Given data,

Circular Footing

$$N_c = 5.7$$

Diameter, $B = 1.5\text{m}$

$$N_q = 1$$

$$c = 50 \text{ kN/m}^2$$

$$N_\gamma = 0$$

$$F = 3$$

$$\therefore q_f = cN_c + \bar{\sigma}N_q + 0.5\gamma B N_\gamma$$

$$= (50 \times 5.7 + 0 \times 1.3) + (0.5 \times \gamma \times B \times 0 \times 1.5)$$

$$= 50 \times 5.7 \times 1.3$$

$$= 370.5 \text{ kN/m}^2$$

$$\therefore q_s = \frac{q_f}{F} = \frac{370.5}{3} = 123.5 \text{ kN/m}^2 \text{ (Ans)}$$

Q2 A square footing located at a depth of 1.5m from the ground surface carries a load of 150kN. The soil is submerged having an effective unit weight of 11 kN/m³ & an angle of shearing resistance

Of 30° . Find the size of the footing using Terzaghi's analysis.

$$F = 3, \quad Nq = 10$$

$$\phi = 30^\circ, \quad Nr = 6$$

Given data,

$$c = 0, \quad \gamma_b = 0, \quad Nq = 10$$

$$\gamma = 11 \text{ kN/m}^3, \quad Nr = 6$$

$$F = 3$$

$$D = 1.5 \text{ m}$$

$$P = 150 \text{ kN}$$

$$q_f = c N_c S_c + \bar{\sigma} N_q + 0.5 \gamma B N_r S_r$$

$$= 0 + 11 \times 1.5 \times 10 + 0.5 \times 11 \times B \times 6 \times 0.8$$

$$\Rightarrow q_f = 165 + 26.4B \quad \text{--- (1)}$$

$$q_{nf} = q_f - \gamma D$$

$$= 165 + 26.4B - 11 \times 1.5$$

$$= 148.5 + 26.4B$$

$$q_s = \frac{q_{nf}}{F} + \gamma D$$

$$= \frac{148.5 + 26.4B}{3} + 11 \times 1.5$$

$$= 49.5 + 8.8B + 16.5$$

$$= 66 + 8.8B \quad \text{--- (2)}$$

$$q_s = \frac{P}{A} = \frac{150}{B^2} \quad \text{--- (3)}$$

Equating (2) & (3),

$$\therefore 66 + 8.8B = \frac{150}{B^2}$$

$$\Rightarrow 66B^2 + 8.8B^3 = 150$$

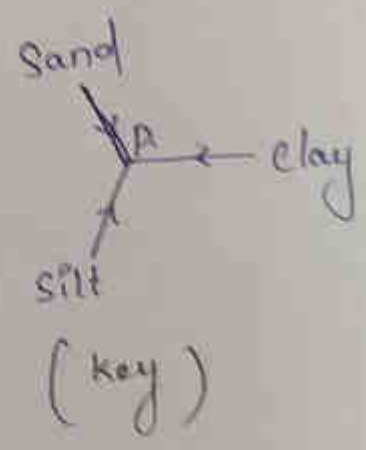
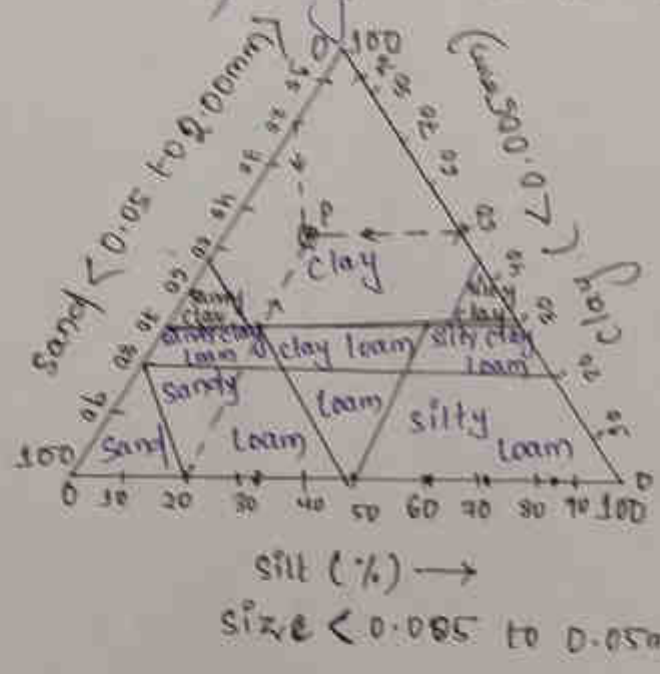
$$\Rightarrow B = 1.38 \text{ m}$$

$$\approx 1.4 \text{ m (Ans)}$$

② Textural classification of soil :-

→ The US Bureau of public Roads recommends triangular classification system for soil which is commonly called as the textural classification system. The figure-1 below shows the textural classification system, where the three sides of the equilateral triangle represent the percentage of sand, silt and clay. The sizes are →

- a) Sand = 0.05 - 2 mm
- b) Silt = 0.005 mm - 0.05 mm
- c) clay = size < 0.005 mm



(Textural classification system)

→ As shown above, the equilateral triangle has 10 zones. Each zone of the triangle will represent each type of soil. Hence by determining the zone, the type of soil can be classified.

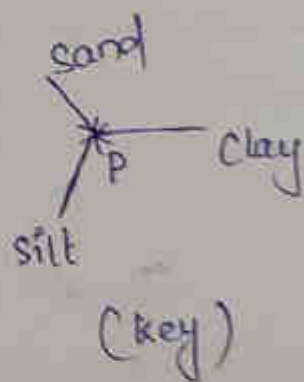
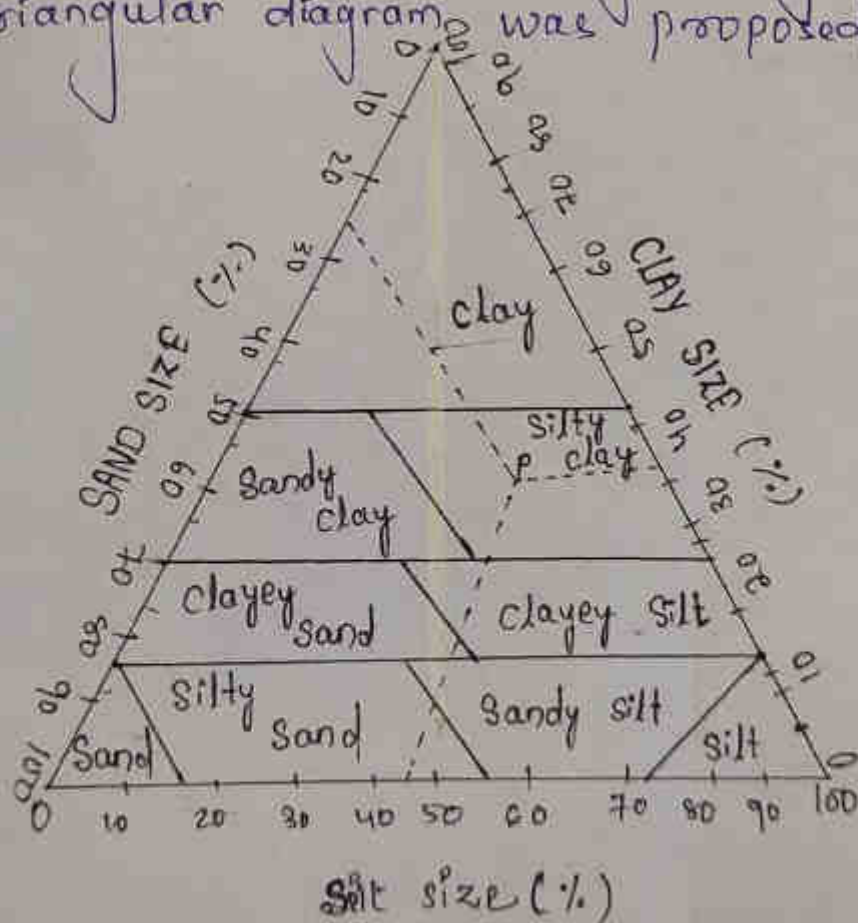
→ In order to locate the point a key is given. This key will give an indication about the direction at which the lines are to be drawn so that points can be located.

→ Let's take an example that, point 'p' is located

corresponding to 30% sand, 50% clay and 20% silt. Now the point 'p' will fall on the zone clay. Hence the soil is classified as clay.

→ This classification system ensures no particles greater than 2mm size is present. In cases where a certain amount of particles greater than 2mm is present a correction is required. Here the percentage of sand, silt and clay is increased to 100%.

→ Here loam is used to describe a mixture of sand, silt and clay in various proportions. The term loam is not used in soil engineering and a modified triangular diagram was proposed by USA.



(Modified triangle diagram)